On Speaking Languages

Luca Cardelli
Microsoft Research

MacQueen Fest, Chicago, 2012-05-12
http://lucacardelli.name
Outline

• The Language of Science

• The Language of Functions

• The Language of Objects

• The Language of Molecules
The Language of Science
The Language of Science

• Never much good at natural languages.

• Middle–school remedial lessons
  o English is the Language of Shakespeare Science!
  o Learned by Science Fiction, from Asimov to Zelazny.

• In Edinburgh, only two English speakers
  o Robin (textbook, slow English)
    • Lectures on Operational Semantics
  o Dave (American English has 5 vowels instead of 1)
    • Lectures on Hope
  o Corollary
    • Famous episode involving a Scottish milkman and 3 months worth of empty milk bottles stacked all over Gordon Plotkin’s flat.
The Language of Functions
The Language of Functions

• Ironically attracted to artificial languages
  o Pisa: \( \lambda \)-calculus (+ Fortran, Lisp, Algol68, Simula)
  o Edinburgh: ML & Hope (+ Pascal, VAX assembly)
  o Murray Hill: C and Unix
    (during a snow storm at Dave’s)

• And their semantics
  o Pisa: Scott–Strachey semantics.
  o Edinburgh: CPOs.
  o Murray Hill: the MacQueen–Sethi–Plotkin ideal model
    • More on this in a moment…
The Language of Objects
The Language of Objects

• The Practice of Objects
  o Pisa: Simula was my favorite language.
  o Edinburgh: Added records and variants to ML. Tried to add record subtyping by type inference, but gave up.
  o Pisa again: Galileo, an ML–inspired database programming language with records and variants and Simula–inspired subtyping.

• Where was the Theory of Objects?
  o Logic languages: --> Predicate logic
  o Database languages: --> Relational calculus
  o Functional languages: --> λ–calculus
  o Imperative languages: --> Hoare Logic / Weakest Preconditions
  o Modular languages: --> Algebraic semantics
  o Object–oriented languages: --> ???
Inspiration!

• The Ideal Model
  o In early Scott denotational models, types were “retracts” of the universal value set, which did not support subtyping.
  o The Ideal Model was designed as a semantics for polymorphism, which was modeled as a “big intersection” of domains.
  o So, it accidentally provided a subset-based denotational semantics of subtyping (via non-empty intersections between domains).
  o Therefore enabling:
    • records as functions (well-known lisp hack)
    • record types as domains (label-dependent function types)
    • record subtyping as set inclusion of function spaces
Result

• Paper:


A Semantics of Multiple Inheritance

Luca Cardelli

AT&T Bell Laboratories
Murray Hill, New Jersey 07974

1. Introduction

There are two major ways of structuring data in programming languages. The first and common one, used for example in Pascal, can be said to derive from structured branches of mathematics. Data is organized as coercion products (i.e. record types), disjoint sums (i.e. union or variant types) and function spaces (i.e. functions and procedures).

The second method can be said to derive from biology and taxonomy. Data is organized in a taxonomy of classes and subcategories, and data at any level of the hierarchy obtains all the attributes of data higher up in the hierarchy. The top level of this hierarchy is usually called the class of all "objects"; every datum is an object and every datum inherits the basic properties of objects, like the ability to tell whether two objects are the same or not. Functions and procedures are also considered as local actions of objects, as opposed to global operations.

• 1984 Semantics of Data Types Talk:
  ○ ... delivered by Dave
Followup

- Quantifiers and Modules
  - Still inspired by the Ideal model and the emerging ML module system, leading to Bounded Quantification
Persistence and Type Abstraction

Luca Cardelli
David MacQueen

AT&T Bell Laboratories
Murray Hill, NJ 07974

Introduction

Abstract types are a well-known and effective way of structuring programs. The basic idea of information hiding can however conflict with the need to store data for long periods of time, and make it accessible to different activities. In particular, a type-checker must be able to recognize the occurrence of the same abstract type during different activations, and must enforce the privacy of data representations.

To achieve this, the persistent storage of data must preserve type information, and must respect type abstraction. The use of type abstractions in the presence of persistent storage requires that abstract types be made persistent as well. Under these conditions, we can preserve type security across distinct activations of the type-checker.

The following is a brief account of how various models of abstraction and persistence interact. We start by sketching a simple polymorphic language and its types and showing various ways of modeling type abstraction in such a language. We then discuss some basic notions underlying persistent storage of typed objects, such as the intern and extern primitives and the special type dynamic, and describe three persistence strategies. Finally we discuss the particular problem of persistent abstract types.

Values and Types

We will base our discussion on a simple polymorphic language in the tradition of ML [Milne 84] and Amber [Cardelli 84]. The simplified language we have in mind is closely related to the language SOL [Mitchell and Plotkin 85], variants of which are described in [Reynolds 85] and [Cardelli and Wegner 85].

The basis of this language is a slightly sugared typed lambda calculus that is adequate for expressing certain kinds of values. Type abstractions are added to the basic expressions in such a way that one can statically determine a type for each expression. This type is a structur

Cited by 34 - Related articles - All 8 versions
1982–03 “Basic Definitions and Facts” (?) 48 pages

**Basic Definitions and Facts**

**Lemma 1.** \( D, S \) is a (directed) complete partial order (cpo) if \( D \) is a partial order, and every directed subset \( D' \) of \( D \) is a pcpo.

**Lemma 2.** A cpo. \( D \) is directed closed \( \iff \) for any directed \( X \subseteq D \), \( \bigwedge X \in D \).

**Theorem:** Let \( X \subseteq D \) be a \( D \)-directed set. Then \( X \) is also \( E_D \)-directed (notion of directed set relativized). The \( \mathcal{U} \mathcal{U} \) and \( \mathcal{U} \mathcal{U} \) are closed.

**Lemma:** \( \mathcal{U} \mathcal{U} \) is the \( \mathcal{U} \mathcal{U} \)-least upper bound of \( X \).

**Corollary:** \( \mathcal{U} \mathcal{U} \) for any \( \gamma \in \mathcal{U} \mathcal{U} \).

**Example:** \( \mathcal{U} \mathcal{U} \) for any \( \gamma \in \mathcal{U} \mathcal{U} \).

**Condition 2:** Let \( D \) be a cpo. \( \mathcal{U} \mathcal{U} \)-directed closed \( \iff \) for any directed \( X \subseteq D \), \( \bigwedge X \leq \bigwedge X \).

**Note:** If \( D \) is a directed complete partial order, then \( \bigwedge X \subseteq \bigwedge X \) for every directed \( X \subseteq D \).

1982–04 “Semantics of Data Types”

**Definition of Data Types:**

\[ \delta(D) \rightarrow \delta(D) \]

\[ \gamma = \gamma \circ \text{id}_D \]

**Example:** \( C \subseteq D \) is a closure if \( C \subseteq C \) and \( \text{id}_D \subseteq C \).

**Theorem:** \( \text{close} \left( (D, D) \right) \) is given by

\[ \text{close}(f) = \bigcup_{n \geq 0} f^n(D) \]

and \( \Gamma \circ D \) by

\[ \Gamma = \Delta_X, \gamma \circ \text{close}(\gamma \gamma) \]

**Lemma 0:** \( \Gamma \) is \( (\forall \gamma \gamma) \) a continuous function.

**Lemma 1:** \( \forall \gamma \in D, \Gamma(D) = D \) and \( \Gamma(f) = D \).

**Lemma 0:** \( \forall C \in \mathcal{C}, \Gamma(C) = 0 \).

**Lemma 1:** \( \forall \gamma \in D, \Gamma(\gamma) = 0 \).

**Theorem:** \( C \subseteq \mathcal{C} \subseteq \mathcal{C} \)

(when \( \mathcal{C} \) is the set of all closed points of \( \Gamma \)).

**Condition 0:** \( C \) is a domain (i.e., a complete lattice). [König].

**Condition 1:** \( \Gamma \) is a closure. \( (\forall \gamma \gamma) \).

**Condition 2:** \( C \subseteq \mathcal{C} \).
1983–?? “Pattern Matching”

Pattern Matching
Choose low branching costs first to minimize number of branches and duplication of code.

Examples:

- type foo = A | a | b
- type bar = B | c | d
- type baz = C | e | f
- type quux = D | g | h

- The don't-care operators:
  - $\perp$
  - $\top$

- $\star$ or $\ast$

- $\ast\ast$

- $\ast\ast\ast$

1983–03 “Modified Damas Algorithm for Typechecking with References”

Modified Damas Algorithm for Typechecking with
References

- Type variable attributes:
  - $\mathit{weak}$: bool
  - $\mathit{binding\_level}$: nat

- If a type variable is weak, then it is involved in the type of the contents of a reference value.
  Weak variables can be generic when they occur

  (a) outside the “scope” of the ref value they are associated with, $\langle \mathit{scope\_extent} \rangle$

  (b) associated with a potential rather than actual ref value.

  $\langle \mathit{a} \rangle$ and $\langle \mathit{b} \rangle$ are probably two ways of saying the same thing.

- Ordinary
  - The $\mathit{binding\_level}$ of an $\mathit{type\_variable}$ represents the minimum nesting depth of the $\mathit{A\_bound\_variables}$ with which the $\mathit{type\_variable}$ is associated in the current $\mathit{type\_assignment}$. A generic (i.e., $\mathit{free}$ or universally $\mathit{bound\_variable}$) type variable has $\mathit{binding\_level} = 0$.

- An ordinary $\mathit{type\_variable}$ is non-generic when its $\mathit{binding\_level}$ (and the current $\lambda$-nesting depth) indicate that it is involved in the type of a
1984–05 “Algorithm for Compiling Pattern Matching”

(The one I failed to implement in my ML compiler.)
End of Story

- → California
  - The Language of Distributed Objects
    - Leading to ...

- → England
  - The Language of Mobile Processes
    - Leading to ...
  - The Language of Biological Processes
    - Leading to ...
The Language of Molecules
Molecular Programming Languages

- **Reaction-Based** \((A + B \rightarrow C + D)\) (Chemistry)
  - Limited to finite set of species (no polymerization)
  - Practically limited to small number of species (no run-away complexation)

- **Interaction-Based** \((A = !r; C)\) (Process Algebra)
  - Reduces combinatorial complexity of models by combining independent submodels connected by interactions.

- **Rule-Based** \((A{-}:B{p} \rightarrow A{p}:B{-})\) (Logic, Graph Rewriting)
  - Further reduces model complexity by describing molecular state, and by allowing one to ‘ignore the context’: a rule is a reaction in an unspecified (complexation/phosphorylation) context.
  - Similar to informal descriptions of biochemical events (“narratives”).

- **Different levels of representation efficiency**
  - The latter two can be translated (to each other and) to the first, but doing so may introduce an infinite, or anyway *extremely large*, number of species.
But what about Execution?

• Chemistry is not easily executable
  o Please Mr Chemist, execute me these reactions that I just made up.

• Description
  o Molecular languages used in systems biology are descriptive (modeling) languages

• Compilation
  o How can we compile arbitrary molecular programs?

• Execution
  o How can we actually execute molecular languages? With real molecules?
DNA as an Engineering Material

• This is why DNA/RNA is important: it is **programmable matter**.

• Not the only one, in principle, but the only one for which we have a well-developed manufacturing technology.

Sequence of Base Pairs (GACT alphabet)
Molecular Control Systems

- **Sensing**
  - Reacting to forces
  - Binding to molecules

- **Actuating**
  - Releasing molecules
  - Producing forces

- **Constructing**
  - Chassis
  - Growth

- **Computing**
  - Signal Processing
  - Decision Making

Nucleic Acids can do all this. And interface to biology.
“Embedded” DNA Computing

- Using bacterial machinery (e.g.) as the hardware. Using embedded gene networks as the software.

- MIT Registry of Standard Biological Parts

- GenoCAD
  - Meaningful sequences [Cai et al.]

- GEC
  - [Pedersen & Phillips]

\[
\text{r0040:prom; b0034:rbs; c0040:pcr; b0015:ter}
\]

\[
\text{prom<neg(C)>; rbs; pcr<codes(A)>; ter; }
\]

\[
\text{prom<neg(A)>; rbs; pcr<codes(B)>; ter; }
\]

\[
\text{prom<neg(B)>; rbs; pcr<codes(C)>; ter}
\]
“Autonomous” DNA Computing
(Nano-engineering with biological materials)

• Mix & go
  o All (or most) parts are synthesized
  o No manual cycling (cf. early DNA computing)
  o In some cases, all parts are made of DNA (no enzyme/proteins)

• Self-assembled and self-powered
  o Can run on its own (e.g. environmental sensing)
  o Or be embedded into organisms (in the future)
Curing

A doctor in each cell

*Fig. 1* Medicine in 2050: “Doctor in a Cell”
Modern DNA Computing

• Non–goals
  o Not to solve NP–complete problems.
  o Not to replace electronic computers.
  o Not necessarily using genes or to producing proteins.

• For general ‘molecular programming’
  o To precisely control the organization and dynamics of matter and information at the molecular level.
  o To interact algorithmically with biological entities.
Subsequences on a DNA strand are called **domains**. *Provided* they are “independent” of each other.

- I.e., differently named domains must not hybridize:
  - With each other
  - With each other’s complement
  - With subsequences of each other
  - With concatenations of other domains (or their complements)
  - Etc.

- Choosing domains (subsequences) that are suitably independent is a tricky issue that is still somewhat of an open problem (with a vast literature). But it can work in practice.
Short Domains

Reversible Hybridization
Long Domains

Irreversible Hybridization
Strand Displacement

“Toehold Mediated”
Strand Displacement

Toehold Binding
Strand Displacement

Branch Migration
Strand Displacement

Displacement
Strand Displacement

Irreversible release
Bad Match

t x z

t x y
Bad Match
Bad Match
Bad Match

Cannot proceed
Hence will undo
Two-Domain Architecture

- **Signals**: 1 toehold + 1 recognition region

  ![Diagram of signals]

- **Gates**: “top-nicked double strands” (or equivalently double strands with open toeholds)

  ![Diagram of gates]

Garbage collection “built into” the gates

---

Two-Domain DNA Strand Displacement

*Luca Cardelli*

Transducer $x \rightarrow y$
Transducer $x \rightarrow y$

$ta$ is a *private* signal (a different ‘a’ for each $xy$ pair)
Transducer $x \rightarrow y$
Transducer $x \rightarrow y$

Active waste

$t \rightarrow \text{waste} \rightarrow t$

$t \rightarrow a \rightarrow t$

$x \rightarrow t \rightarrow a \rightarrow t$

$y \rightarrow t$

$x \rightarrow t \rightarrow y \rightarrow t \rightarrow a \rightarrow t$
Transducer $x \rightarrow y$
Transducer $x \rightarrow y$

So far, a $tx$ signal has produced an $at$ cosignal. But we want signals as output, not cosignals.
Transducer $x \rightarrow y$
Transducer $x \rightarrow y$
Transducer $x \rightarrow y$
Transducer $x \rightarrow y$

Here is our output $ty$ signal.

But we are not done yet:
1) We need to make the output irreversible.
2) We need to remove the garbage.
We can use (2) to achieve (1).
Transducer $x \rightarrow y$
Transducer \( x \to y \)
Transducer $x \rightarrow y$
Transducer $x \rightarrow y$
Transducer $x \rightarrow y$
Transducer $x \rightarrow y$

Done.

N.B. the gate is consumed: it is the energy source.
Join \( x + y \) → \( z \)
General $n \times m$ Join–Fork

- Easily generalized to 2+ inputs (with 1+ collectors).
- Easily generalized to 2+ outputs.

![Diagram](image)

**Figure 9:** 3-Join $J_{wxy, \zeta} | tw | tx | ty \rightarrow tz$: initial state plus inputs $tw$, $tx$, $ty$. 
DNA Programming
Debugging

• Big Networks
  o Two-domain DNA gates for 1 Approximate Majority switch.
  o Initial species: 17
  o Total number of species: 85 (including run-time produced ones)
  o Total number of reactions: 104

• Analysis
  o Gate correctness
  o Circuit correctness
  o Compiler correctness
  o Currently, by simulation
  o Increasingly, by modelchecking:

  Design and Analysis of DNA Strand Displacement Devices using Probabilistic Model Checking

  Matthew R. Lakin †, David Parker ‡,
  Luca Cardelli*, Marta Kwiatkowska †,
  Andrew Phillips*
Experiments

Two-domain gate for $X+Y \rightarrow Y+B$

$X+Y \rightarrow Y+B$  
35C  
$1x = 50\text{nM}$

Yuan-Jyue Chen and Georg Seelig  
U.Washington.

<table>
<thead>
<tr>
<th>$X+Y \rightarrow Y+B$</th>
<th>Concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>LG1</td>
<td>1.5x</td>
</tr>
<tr>
<td>LG2</td>
<td>1.5x</td>
</tr>
<tr>
<td>input</td>
<td>1x</td>
</tr>
<tr>
<td>Catalyst</td>
<td>0x, 0.05x, 0.1x, 0.2x, 0.3x, 1x</td>
</tr>
<tr>
<td>$^*B$</td>
<td>2x</td>
</tr>
<tr>
<td>R1</td>
<td>2x</td>
</tr>
<tr>
<td>B readout</td>
<td>3x</td>
</tr>
</tbody>
</table>
Summary

• Executable chemistry
  o Given an arbitrary finite chemical network, compile it systematically and execute it. [D. Soloveichik, G. Seelig, E. Winfree. DNA as a Universal Substrate for Chemical Kinetics. PNAS 107 no. 12, 5393–5398, 2010.]
  o Finite chemical networks have the computing power of (stochastic) Petri Nets. Population protocols (such as AM) are also well-characterized. [D. Angluin, J. Aspnes, D. Eisenstat, E. Ruppert: The Computational Power of Population Protocols].

• Executable bio–chemistry
  o In addition, DNA supports polymerization, which gives the computing power of Turing Machines.
  o Then the programming language cannot be just chemical reactions, but has to be something more like process algebra or term-rewriting systems.
Conclusions
Conclusions

• The Language of Functions
• ...
• ...
• The Language of Life