Reversibility in massive concurrent systems

Luca Cardelli Cosimo Laneve

MecBic 2011

programme

- study reversibility in DNA circuits
- define the algebra of DNA circuits – *reversible structures*
- analyze the interplay between reversibility and causal dependency
  + grasp causal dependency between coinitial reductions
  + investigate causal equivalent computations (permutation equivalence)
- measure the expressive power of reversible structures
  + implementation of asynchronous RCCS

“natural” reversibility?

- in computational systems, computations are sequence of *irreversible steps*
- implementations of these systems in physics or chemistry are usually reversible
- reversibility means undoing the computation *not in a deterministic way*:

  > *states reached during a backward computation are states that could have been reached during the forward computation by just performing independent actions in a different order*

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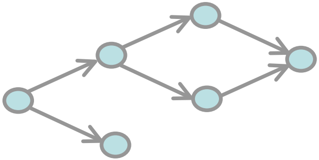
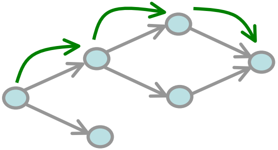
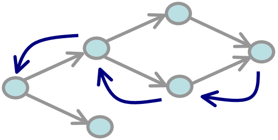


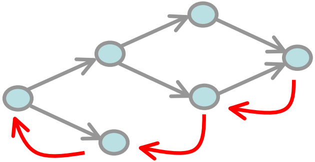
transition system



a forward computation



a correct backward computation



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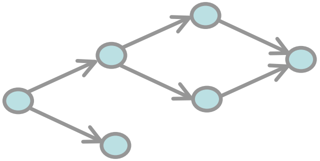

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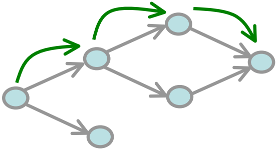

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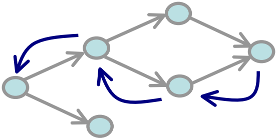

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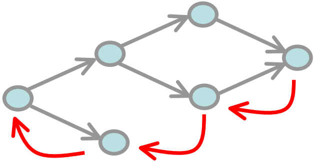

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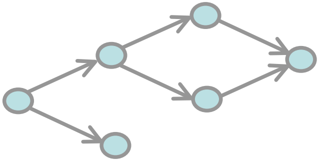

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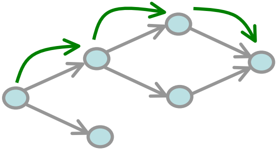

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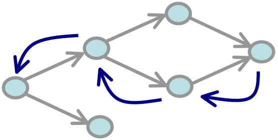

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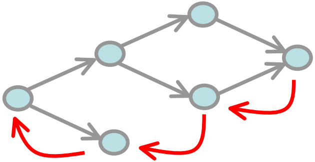

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since part of physics and chemistry is reversible,

what is the formal theory of reversibility in these fields?

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taking a natural reversible system, what properties may we prove?

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domains are independent of each other

- they cannot hybridize from any other domain except their complement

- there are very few short domains with reversible hybridizations

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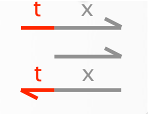

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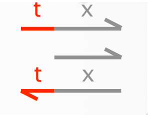

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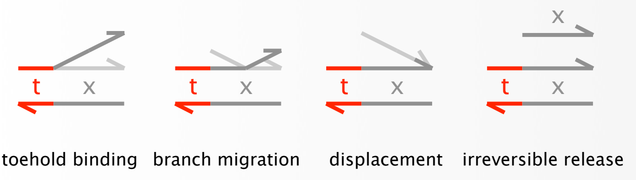

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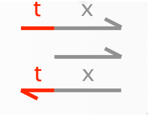

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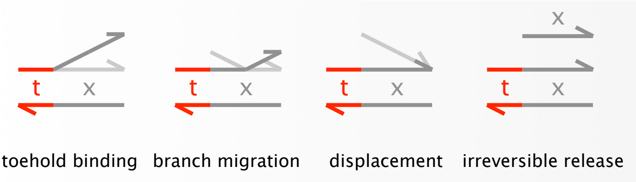

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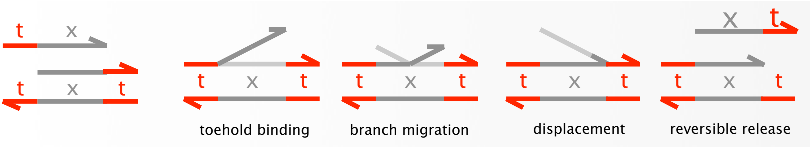
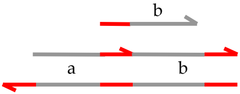


two-domains structures and causality

– a transducer



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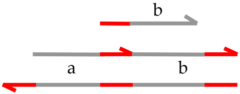

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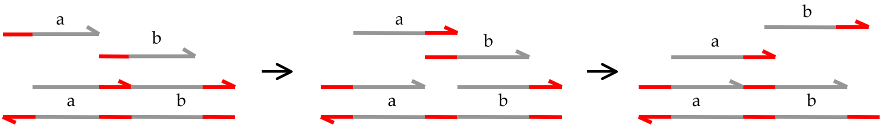

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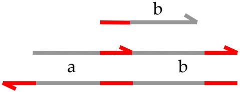

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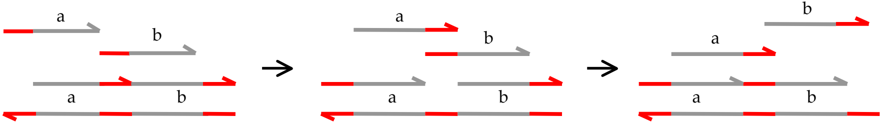

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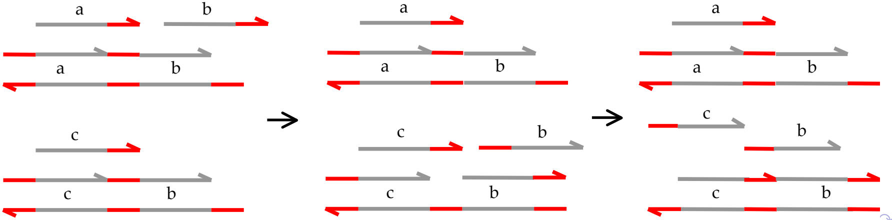

a better model for causality: three-domains structures

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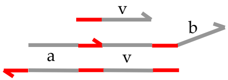

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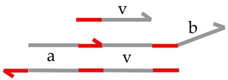

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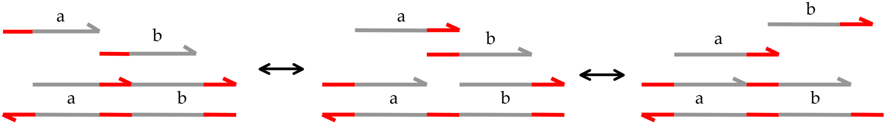

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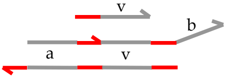

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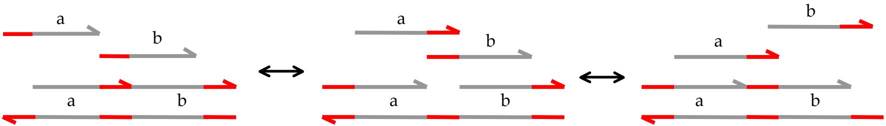

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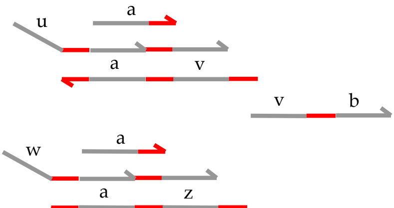

causality and massive concurrency

DNA circuits are *massively concurrent*:

- solutions consist of populations of species of strands and
- populations are not singletons

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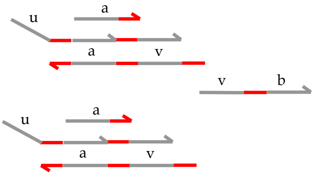

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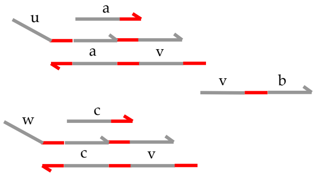
the situation may be even worse due to bad designs



a theory of massive concurrency?

reversibility/causality in massive concurrent systems has not been studied

- theories have been defined for reversible calculi where processes retain unique ids (Danos-Krivine, Phillips-Ulidowski, Lanese-Mezzina-Stefani)

question: *what is the distance between current theories of reversible algebras and reversibility in massive concurrent systems?*

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the algebra of reversible structures

notation: u, v, w: ids

$a, \overline{a}, b, \overline{b}$: names and conames

x, x': ids, names and conames

$\mathtt{A}, \mathtt{B}, \mathtt{C}$: sequences of names;

$\overline{\mathtt{A}}, \overline{\mathtt{B}}, \overline{\mathtt{C}}$: sequences of elements $u : \overline{a}$

$\mathtt{A}^{\perp}, \mathtt{B}^{\perp}, \mathtt{C}^{\perp}$: sequences of elements $u : a$

signals : $u{:}\overline{a}$

gates : g *input part.output part +* ˆ

examples of gates:

$\hat{}a \,.\, a' \,.\, v{:}\overline{b}$ $\quad u{:}a \,.\, \hat{}a' \,.\, v{:}\overline{b}$

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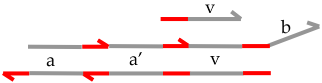

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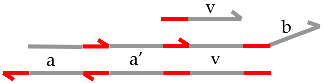

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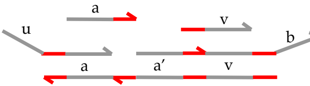

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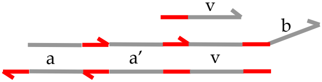

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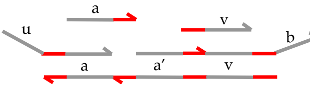

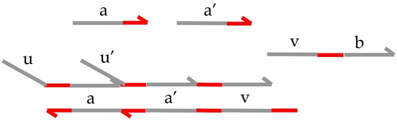

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the algebra of reversible structures – syntax

$$
\begin{array}{llll}
\textit{structures}: & \mathtt{S} & ::= & \\
 & & & \mathbf{0} & \text{(null)} \\
 & & \mid & u{:}\bar{a} & \text{(signal)} \\
 & & \mid & g & \text{(gate)} \\
 & & \mid & \mathtt{S} \mid \mathtt{S} & \text{(parallel)} \\
 & & \mid & (\mathtt{new}\ x)\mathtt{S} & \text{(new)}
\end{array}
$$

the algebra of reversible structures – reductions

input capture: $u{:}\bar{a} \mid \mathtt{A}^{\perp} \,.\, \hat{}\,a \,.\, \mathtt{B} \,.\, \overline{\mathtt{C}} \longrightarrow \mathtt{A}^{\perp} \,.\, u{:}a \,.\, \hat{}\,\mathtt{B} \,.\, \overline{\mathtt{C}}$

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(plus the standard contextual rules about `new`, $\mid$, and $\equiv$)

example:

$$w{:}\bar{a} \mid v{:}a \,.\, \hat{}\,u{:}\bar{b} \rightarrow w{:}\bar{a} \mid v{:}a \,.\, u{:}\bar{b}\,\hat{} \mid u{:}\bar{b}$$

$$\nearrow$$

$$v{:}\bar{a} \mid w{:}\bar{a} \mid \hat{}\,a \,.\, u{:}\bar{b}$$

$$\searrow$$

$$v{:}\bar{a} \mid w{:}a \,.\, \hat{}\,u{:}\bar{b}$$

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output capture: $u{:}\bar{a} \mid \mathtt{A}^{\perp} \,.\, \overline{\mathtt{B}} \,.\, u{:}\bar{a} \,.\, \hat{}\overline{\mathtt{C}} \longrightarrow \mathtt{A}^{\perp} \,.\, \overline{\mathtt{B}} \,.\, \hat{}u{:}\bar{a} \,.\, \overline{\mathtt{C}}$

(plus the standard contextual rules about `new`, $\mid$, and $\equiv$)

example:

$w{:}\bar{a} \mid v{:}a \,.\, \hat{}u{:}\bar{b} \rightarrow w{:}\bar{a} \mid v{:}a \,.\, u{:}\bar{b}\hat{} \mid u{:}\bar{b}$

$\nearrow$

$v{:}\bar{a} \mid w{:}\bar{a} \mid \hat{}a \,.\, u{:}\bar{b}$

$\searrow$

$v{:}\bar{a} \mid w{:}a \,.\, \hat{}u{:}\bar{b}$

the algebra of reversible structures – reductions

input capture: $u{:}\bar{a} \mid \mathtt{A}^{\perp} \,.\, \hat{}\,a \,.\, \mathtt{B} \,.\, \overline{\mathtt{C}} \longrightarrow \mathtt{A}^{\perp} \,.\, u{:}a \,.\, \hat{}\,\mathtt{B} \,.\, \overline{\mathtt{C}}$

input release: $\mathtt{A}^{\perp} \,.\, u{:}a \,.\, \hat{}\,\mathtt{B} \,.\, \overline{\mathtt{C}} \longrightarrow u{:}\bar{a} \mid \mathtt{A}^{\perp} \,.\, \hat{}\,a \,.\, \mathtt{B} \,.\, \overline{\mathtt{C}}$

output release: $\mathtt{A}^{\perp} \,.\, \overline{\mathtt{B}} \,.\, \hat{}\,u{:}\bar{a} \,.\, \overline{\mathtt{C}} \longrightarrow u{:}\bar{a} \mid \mathtt{A}^{\perp} \,.\, \overline{\mathtt{B}} \,.\, u{:}\bar{a} \,.\, \hat{}\,\overline{\mathtt{C}}$

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$\searrow$

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the algebra of reversible structures – reductions

input capture: $u{:}\overline{a} \mid \mathtt{A}^{\perp} \,.\, \hat{}\,a \,.\, \mathtt{B} \,.\, \overline{\mathtt{C}} \;\longrightarrow\; \mathtt{A}^{\perp} \,.\, u{:}a \,.\, \hat{}\,\mathtt{B} \,.\, \overline{\mathtt{C}}$

input release: $\mathtt{A}^{\perp} \,.\, u{:}a \,.\, \hat{}\,\mathtt{B} \,.\, \overline{\mathtt{C}} \;\longrightarrow\; u{:}\overline{a} \mid \mathtt{A}^{\perp} \,.\, \hat{}\,a \,.\, \mathtt{B} \,.\, \overline{\mathtt{C}}$

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$$\nearrow$$

$$v{:}\overline{a} \mid w{:}\overline{a} \mid \hat{}\,a \,.\, u{:}\overline{b}$$

$$\searrow$$

$$v{:}\overline{a} \mid w{:}a \,.\, \hat{}\,u{:}\overline{b}$$

the algebra of reversible structures – reductions

input capture: $u{:}\bar{a} \mid \mathtt{A}^{\perp} \,.\, \hat{}\,a \,.\, \mathtt{B} \,.\, \overline{\mathtt{C}} \;\longrightarrow\; \mathtt{A}^{\perp} \,.\, u{:}a \,.\, \hat{}\,\mathtt{B} \,.\, \overline{\mathtt{C}}$

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output capture: $u{:}\bar{a} \mid \mathtt{A}^{\perp} \,.\, \overline{\mathtt{B}} \,.\, u{:}\bar{a} \,.\, \hat{}\,\overline{\mathtt{C}} \;\longrightarrow\; \mathtt{A}^{\perp} \,.\, \overline{\mathtt{B}} \,.\, \hat{}\,u{:}\bar{a} \,.\, \overline{\mathtt{C}}$

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$$v{:}\bar{a} \mid w{:}\bar{a} \mid \hat{}\,a \,.\, u{:}\bar{b} \quad \nearrow \quad \searrow$$

$$v{:}\bar{a} \mid w{:}a \,.\, \hat{}\,u{:}\bar{b}$$

the algebra of reversible structures – reductions

$$\textit{input capture}: \quad u{:}\overline{a} \mid \mathtt{A}^{\perp}\,.\,\hat{}\,a\,.\,\mathtt{B}\,.\,\overline{\mathtt{C}} \;\longrightarrow\; \mathtt{A}^{\perp}\,.\,u{:}a\,.\,\hat{}\,\mathtt{B}\,.\,\overline{\mathtt{C}}$$

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(plus the standard contextual rules about `new`, $\mid$, and $\equiv$)

example:

$$v{:}\overline{a} \mid w{:}\overline{a} \mid \hat{}\,a\,.\,u{:}\overline{b} \;\nearrow\; w{:}\overline{a} \mid v{:}a\,.\,\hat{}\,u{:}\overline{b} \;\rightarrow\; w{:}\overline{a} \mid v{:}a\,.\,u{:}\overline{b}\,\hat{} \mid u{:}\overline{b}$$

$$v{:}\overline{a} \mid w{:}\overline{a} \mid \hat{}\,a\,.\,u{:}\overline{b} \;\searrow\; v{:}\overline{a} \mid w{:}a\,.\,\hat{}\,u{:}\overline{b}$$

the algebra of reversible structures – reductions

input capture: $u{:}\bar{a} \mid \mathtt{A}^{\perp} \centerdot \hat{}\, a \centerdot \mathtt{B} \centerdot \overline{\mathtt{C}} \longrightarrow \mathtt{A}^{\perp} \centerdot u{:}a \centerdot \hat{}\, \mathtt{B} \centerdot \overline{\mathtt{C}}$

input release: $\mathtt{A}^{\perp} \centerdot u{:}a \centerdot \hat{}\, \mathtt{B} \centerdot \overline{\mathtt{C}} \longrightarrow u{:}\bar{a} \mid \mathtt{A}^{\perp} \centerdot \hat{}\, a \centerdot \mathtt{B} \centerdot \overline{\mathtt{C}}$

output release: $\mathtt{A}^{\perp} \centerdot \overline{\mathtt{B}} \centerdot \hat{}\, u{:}\bar{a} \centerdot \overline{\mathtt{C}} \longrightarrow u{:}\bar{a} \mid \mathtt{A}^{\perp} \centerdot \overline{\mathtt{B}} \centerdot u{:}\bar{a} \centerdot \hat{}\, \overline{\mathtt{C}}$

output capture: $u{:}\bar{a} \mid \mathtt{A}^{\perp} \centerdot \overline{\mathtt{B}} \centerdot u{:}\bar{a} \centerdot \hat{}\, \overline{\mathtt{C}} \longrightarrow \mathtt{A}^{\perp} \centerdot \overline{\mathtt{B}} \centerdot \hat{}\, u{:}\bar{a} \centerdot \overline{\mathtt{C}}$

(plus the standard contextual rules about `new`, $\mid$, and $\equiv$)

example:

$$v{:}\bar{a} \mid w{:}\bar{a} \mid \hat{}\, a \centerdot u{:}\bar{b} \nearrow w{:}\bar{a} \mid v{:}a \centerdot \hat{}\, u{:}\bar{b} \rightarrow w{:}\bar{a} \mid v{:}a \centerdot u{:}\bar{b}\,\hat{} \mid u{:}\bar{b}$$

$$v{:}\bar{a} \mid w{:}\bar{a} \mid \hat{}\, a \centerdot u{:}\bar{b} \searrow v{:}\bar{a} \mid w{:}a \centerdot \hat{}\, u{:}\bar{b}$$

pearls of expressive power

join input ($a \mid b \triangleright \overline{c}$)

$$\hat{\ } a.\ b.\ u{:}\overline{c}$$

input-guarded choice ($a.\overline{b} + a'.\overline{c}$)

$$(\text{new } v, e)(\ e.\ a.\ u{:}\overline{b} \ \mid\ e.\ a'.\ u'{:}\overline{c} \ \mid\ v{:}\overline{e}\ )$$

pearls of expressive power

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pearls of expressive power

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$$(\texttt{new}\ v, e)(\ e.\ a.\ u{:}\overline{b} \quad | \quad e.\ a'.\ u'{:}\overline{c} \quad | \quad v{:}\overline{e}\ )$$

weak coherence

remark: it is possible to restrict the arguments about the dynamics of reversible structures to structures without news (*this simplifies the theory*)

a structure S *is weak coherent whenever ids are uniquely associated to names and co-names*

(if $u : \alpha$ *and* $u : \alpha'$ *occur in* S' *then either* $\alpha = \alpha'$ *or* $\alpha = \overline{\alpha'}$*)*

proposition: *weak coherent reversible algebra may be implemented into three domains* DNA *circuits*

– the correspondence is consistent and complete wrt reductions

causality: the tour

1. define *causal dependence* on coinitial reductions
   - two reductions are dependent if they have either the signal or the gate in common
   - independent reductions can be swapped (diamond lemma)

2. define *permutation equivalence*, an equivalence on derivations that is insensible
   i. *to swapping of independent reductions*
   ii. *to the removal of reverse reductions*

3. study the theory of permutation equivalence

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causal independence

a reduction can be addressed by the signal/gate that move

remark: *in weak-coherent structures, names and co-names when prefixed by ids are useless*

– a signal $u : \overline{a}$ can be addressed by u

– a gate

$\hat{}a.a'.v : \overline{b}$ can be addressed by $\hat{}aa' \circ v$

$u : a.\hat{}a'.v : \overline{b}$ can be addressed by $u\hat{}a' \circ v$

$u : a.u' : a'.v : \overline{b}\hat{}$ can be addressed by $uu' \circ v\hat{}$

cf. Lévy labels in lambda calculus

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cf. Lévy labels in lambda calculus

causal independence/cont.

an example

$$u{:}\overline{a} \mid \hat{}\,a \bullet v{:}\overline{b} \mid u{:}a \bullet \hat{}\,v{:}\overline{b}$$

$$u \mid \hat{}\,a{\circ}v \swarrow \qquad\qquad \searrow u{\circ}\hat{}\,v$$

$$u{:}a \bullet \hat{}\,v{:}\overline{b} \mid u{:}a \bullet \hat{}\,v{:}\overline{b} \qquad\qquad u{:}\overline{a} \mid \hat{}\,a \bullet v{:}\overline{b} \mid u{:}a \bullet v{:}\overline{b}\,\hat{} \mid v{:}\overline{b}$$

$$u{\circ}\hat{}\,v \searrow \qquad\qquad \swarrow u \mid \hat{}\,a{\circ}v$$

$$u{:}a \bullet \hat{}\,v{:}\overline{b} \mid u{:}a \bullet v{:}\overline{b}\,\hat{} \mid v{:}\overline{b}$$

two coinitial reductions are causally independent if the corresponding labels have no sublabel in common
(*causal dependency* is the opposite notion)

causal independence/cont.

an example

$$u{:}\overline{a} \mid {}^\wedge a \bullet v{:}\overline{b} \mid u{:}a \bullet {}^\wedge v{:}\overline{b}$$

$$u \mid {}^\wedge a \circ v \swarrow \qquad\qquad \searrow u \circ {}^\wedge v$$

$$u{:}a \bullet {}^\wedge v{:}\overline{b} \mid u{:}a \bullet {}^\wedge v{:}\overline{b} \qquad\qquad u{:}\overline{a} \mid {}^\wedge a \bullet v{:}\overline{b} \mid u{:}a \bullet v{:}\overline{b}{}^\wedge \mid v{:}\overline{b}$$

$$u \circ {}^\wedge v \searrow \qquad\qquad \swarrow u \mid {}^\wedge a \circ v$$

$$u{:}a \bullet {}^\wedge v{:}\overline{b} \mid u{:}a \bullet v{:}\overline{b}{}^\wedge \mid v{:}\overline{b}$$

two coinitial reductions are causally independent if the corresponding labels have no sublabel in common
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causal independence/cont.

an example

$$u{:}\overline{a} \mid \hat{}a \bullet v{:}\overline{b} \mid u{:}a \bullet \hat{}v{:}\overline{b}$$

$$u \mid \hat{}a \circ v \swarrow \qquad\qquad \searrow u \circ \hat{}v$$

$$u{:}a \bullet \hat{}v{:}\overline{b} \mid u{:}a \bullet \hat{}v{:}\overline{b} \qquad\qquad u{:}\overline{a} \mid \hat{}a \bullet v{:}\overline{b} \mid u{:}a \bullet v{:}\overline{b}\hat{} \mid v{:}\overline{b}$$

$$u \circ \hat{}v \searrow \qquad\qquad \swarrow u \mid \hat{}a \circ v$$

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an example

$$u{:}\overline{a} \mid \hat{\ }a \bullet v{:}\overline{b} \mid u{:}a \bullet \hat{\ }v{:}\overline{b}$$

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$$u{:}a \bullet \hat{\ }v{:}\overline{b} \mid u{:}a \bullet \hat{\ }v{:}\overline{b} \qquad\qquad u{:}\overline{a} \mid \hat{\ }a \bullet v{:}\overline{b} \mid u{:}a \bullet v{:}\overline{b}\hat{\ } \mid v{:}\overline{b}$$

$$u{\circ}\hat{\ }v \searrow \qquad\qquad \swarrow u \mid \hat{\ }a{\circ}v$$

$$u{:}a \bullet \hat{\ }v{:}\overline{b} \mid u{:}a \bullet v{:}\overline{b}\hat{\ } \mid v{:}\overline{b}$$

two coinitial reductions are causally independent if the corresponding labels have no sublabel in common
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causal independence/cont.

an example

$$u{:}\overline{a} \mid \hat{}a \bullet v{:}\overline{b} \mid u{:}a \bullet \hat{}v{:}\overline{b}$$

$$u \mid \hat{}a \circ v \swarrow \qquad\qquad \searrow u \circ \hat{}v$$

$$u{:}a \bullet \hat{}v{:}\overline{b} \mid u{:}a \bullet \hat{}v{:}\overline{b} \qquad\qquad u{:}\overline{a} \mid \hat{}a \bullet v{:}\overline{b} \mid u{:}a \bullet v{:}\overline{b}\hat{} \mid v{:}\overline{b}$$

$$u \circ \hat{}v \searrow \qquad\qquad \swarrow u \mid \hat{}a \circ v$$

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two coinitial reductions are causally independent if the corresponding labels have no sublabel in common

(*causal dependency* is the opposite notion)

causal independence/cont.

an example

$$u{:}\overline{a} \mid \hat{}a \bullet v{:}\overline{b} \mid u{:}a \bullet \hat{}v{:}\overline{b}$$

$$\color{red}{u \mid \hat{}a \circ v} \swarrow \qquad\qquad \searrow \color{red}{u \circ \hat{}v}$$

$$u{:}a \bullet \hat{}v{:}\overline{b} \mid u{:}a \bullet \hat{}v{:}\overline{b} \qquad\qquad u{:}\overline{a} \mid \hat{}a \bullet v{:}\overline{b} \mid u{:}a \bullet v{:}\overline{b}\hat{} \mid v{:}\overline{b}$$

$$\color{red}{u \circ \hat{}v} \searrow \qquad\qquad \swarrow \color{red}{u \mid \hat{}a \circ v}$$

$$u{:}a \bullet \hat{}v{:}\overline{b} \mid u{:}a \bullet v{:}\overline{b}\hat{} \mid v{:}\overline{b}$$

two coinitial reductions are causally independent if the corresponding labels have no sublabel in common
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the diamond lemma

Lemma. *Let* $\mathsf{S} \xrightarrow{\mu} \mathsf{S}'$ *and* $\mathsf{S} \xrightarrow{\nu} \mathsf{S}''$ *be such that* μ *and* ν *are causally independent. Then there exists* S''' *such that* $\mathsf{S}' \xrightarrow{\nu} \mathsf{S}'''$ *and* $\mathsf{S}'' \xrightarrow{\mu} \mathsf{S}'''$.

causal independence: issues

$$u{:}\bar{a} \mid u{:}\bar{a} \mid \hat{}\, a \,.\, v{:}\bar{b} \mid w{:}c \,.\, u{:}\bar{a}\hat{} \quad \xrightarrow{u \,|\, \hat{}\, a \circ v} \quad u{:}\bar{a} \mid u{:}a \,.\, \hat{}\, v{:}\bar{b} \mid w{:}c \,.\, u{:}\bar{a}\hat{}$$

$$\xrightarrow{u \,|\, w \circ u\hat{}} \quad u{:}a \,.\, \hat{}\, v{:}\bar{b} \mid w{:}c \,.\, \hat{}\, u{:}\bar{a}$$

cannot be swapped because $u \mid \hat{}\, a \circ v$ and $u \mid w \circ u\hat{}$ have sublabels in common

rationale:

- labels are not expressive enough to catch *multiplicities*
  - a similar anomaly is present in Petri nets (*cf.* Degano, Meseguer, Montanari)
- *in massive concurrent systems, different occurrences of a same molecule cannot be separated*

causal independence: issues

$$u{:}\bar{a} \mid u{:}\bar{a} \mid \hat{}\,a \,\bullet\, v{:}\bar{b} \mid w{:}c \,\bullet\, u{:}\bar{a}\,\hat{} \quad \xrightarrow{u \,\mid\, \hat{}\,a \circ v} \quad u{:}\bar{a} \mid u{:}a \,\bullet\, \hat{}\,v{:}\bar{b} \mid w{:}c \,\bullet\, u{:}\bar{a}\,\hat{}$$

$$\xrightarrow{u \,\mid\, w \circ u\,\hat{}} \quad u{:}a \,\bullet\, \hat{}\,v{:}\bar{b} \mid w{:}c \,\bullet\, \hat{}\,u{:}\bar{a}$$

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$$u{:}\bar{a} \;|\; u{:}\bar{a} \;|\; \hat{}a \,.\, v{:}\bar{b} \;|\; w{:}c \,.\, u{:}\bar{a}\hat{} \quad \xrightarrow{u \,|\, \hat{}a \circ v} \quad u{:}\bar{a} \;|\; u{:}a \,.\, \hat{}v{:}\bar{b} \;|\; w{:}c \,.\, u{:}\bar{a}\hat{}$$

$$\xrightarrow{u \,|\, w \circ u\hat{}} \quad u{:}a \,.\, \hat{}v{:}\bar{b} \;|\; w{:}c \,.\, \hat{}u{:}\bar{a}$$

cannot be swapped because $u \;|\; \hat{}a \circ v$ and $u \;|\; w \circ u\hat{}$ have sublabels in common

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permutation equivalence

notation:

let $\mu_1; \cdots; \mu_n$ be the computation $S_1 \xrightarrow{\mu_1} \cdots \xrightarrow{\mu_n} S_{n+1}$

let $[\mu]^+$ be the *reverse label* of μ defined as

$$\begin{array}{rcl} [u \mid v\hat{}a_\circ w]^+ & = & \widetilde{v}u\hat{}_\circ\widetilde{w} \\ [vu\hat{}_\circ w]^+ & = & u \mid v\hat{}a_\circ w \\ [vu_\circ\hat{}w]^+ & = & u \mid vu_\circ w\hat{} \\ [u \mid vu_\circ w\hat{}]^+ & = & vu_\circ\hat{}w \end{array}$$

permutation equivalence $\sim$ is the least equivalence relation between computations closed under composition and such that:

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permutation equivalence: examples

- $v{:}\bar{a} \mid w{:}\bar{a} \mid \hat{}a \,.\, u{:}\bar{b} \xrightarrow{v \mid \hat{} a \circ u} w{:}\bar{a} \mid v{:}a \,.\, \hat{}u{:}\bar{b} \xrightarrow{v \hat{} \circ u} v{:}\bar{a} \mid w{:}\bar{a} \mid \hat{}a \,.\, u{:}\bar{b}$

  is computationally equivalent to ε

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the standardization theorem

let $\mu_1 ; \cdots ; \mu_n$ be a computation of a weak coherent structure *such that μ_n is the converse of μ_1*

- there is a shorter computation that is permutation equivalent to $\mu_1 ; \cdots ; \mu_n$

the evolution of a gate in a computation without converse labels (normal) is unidirectional

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coherence

a solution must contain exactly one molecule of every species

a weak-coherent structure is *coherent* whenever

- different gates have types with no id in common – *the type of a gate is the sequence of ids in the output part*
- ids occur at most twice: one occurrence is positive and the other is negative

$$u_1{:}a_1 . \cdots u_m{:}a_m \,.\, \underbrace{v_1{:}\overline{b_1} \,.\, \cdots \,.\, v_k{:}\overline{b_k}}_{\text{negative occurrences}} \,.\, \hat{}\,v_{k+1}{:}\overline{b_{k+1}} \,.\, \cdots \,.\, v_n{:}\overline{b_n}$$

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consequences of coherence

theorem: *two coinitial computations of a coherent structure are permutation equivalent if and only if they are cofinal*

(false in weak-coherent structures)

theorem: *the reachability problem in coherent structure has a computational complexity of* $O(n^2)$*, where* n *is the number of gates in the structure*

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expressive power of coherent structures

coherent structures encode in a causally consistent way
asynchronous Reversible CCS

asynchronous Reversible CCS in a nutshell

memories	m	$::=$	$\langle\,\rangle \mid \langle i \rangle_n \bullet m \mid \langle m, \alpha, Q \rangle \bullet m$
processes	P	$::=$	$\mathbf{0} \mid \sum_{i \in I} a_i.P_i + \sum_{j \in J} \overline{a_j} \mid \prod_{i \in I} P_i \mid (\mathtt{new}\ a)P$
r-t processes	R	$::=$	$m \triangleright P \mid R \mid R \mid (\mathtt{new}\ a)R$

transitions $m \triangleright (a.P + Q) \mid m' \triangleright (\overline{a} + R) \longleftrightarrow \langle m', a, Q \rangle \bullet m \triangleright P \mid \langle m, \overline{a}, R \rangle \bullet m' \triangleright \mathbf{0}$

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the encoding of mixed choice in *asynchronous RCCS*

$a.P + \overline{a}$ is encoded as $[\![a.P + \overline{a}]\!]_c =$

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conclusions

possible research directions

- coherence is very hard to achieve in nature
  + biology prompts a thorough study of reversible concurrent calculi where processes have multiplicities and the causal dependencies between copies may be exchanged

- reversible structures may be extended with irreversible combinators (that may be implemented in DNA)
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