# Anytime, Anywhere
## Modal Logics for Mobile Ambients

*Luca Cardelli*
*Andy Gordon*

**Microsoft Research**

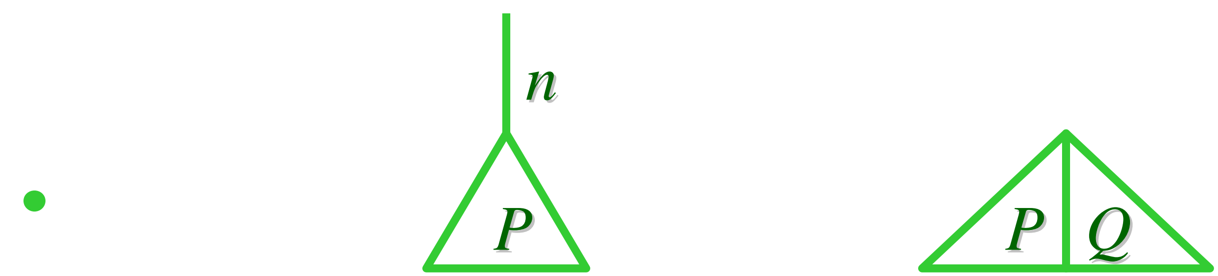
**POPL 2000**

# Introduction

- We want to describe mobile behaviors. The *ambient calculus* provides an operational model, where spatial structures (agents, networks, etc.) are represented by nested locations.
- We also want to specify mobile behaviors. To this end, we devise an *ambient logic* that can talk about spatial structures.

| *Processes* | | *Formulas* | |
|---|---|---|---|
| $\mathbf{0}$ | (void) | $\mathbf{0}$ | (there is nothing here) |
| $n[P]$ | (location) | $n[\mathcal{A}]$ | (there is one thing here) |
| $P \mid Q$ | (composition) | $\mathcal{A} \mid \mathcal{B}$ | (there are two things here) |

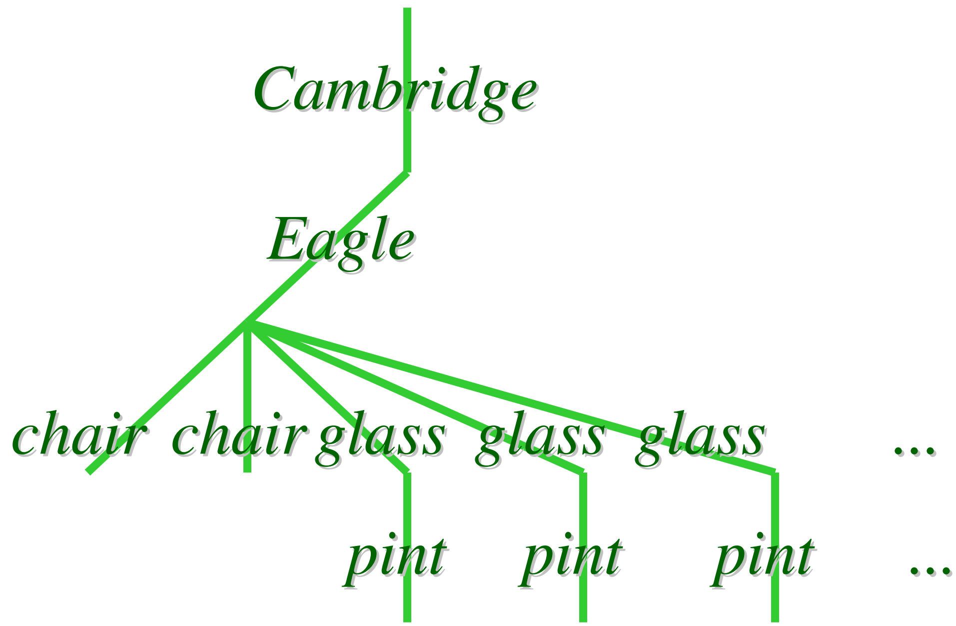
*Trees*




(void) (location) (composition)

# Spatial Structures

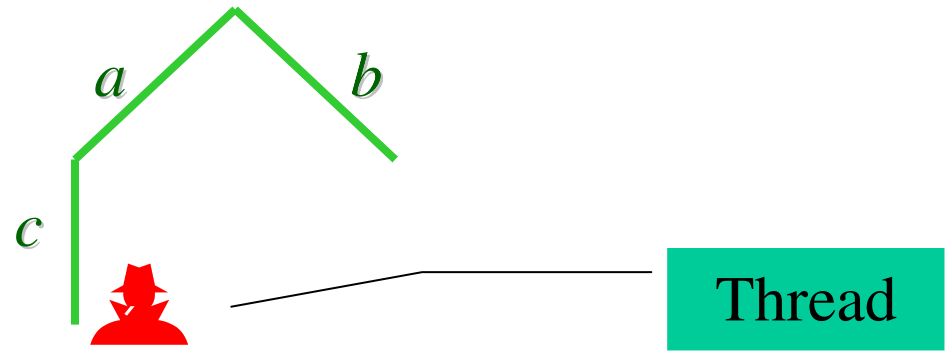
- Our basic model of space is going to be *finite-depth edge-labeled unordered trees*; for short: *spatial trees*, represented by a syntax of *spatial expressions*. Unbounded resources are represented by infinite branching:




$$Cambridge[Eagle[chair[\mathbf{0}] \mid chair[\mathbf{0}] \mid \mathbf{!}glass[pint[\mathbf{0}]]] \mid ...]$$

# Ambient Structures

- Spatial expressions/trees are a subset of ambient expressions/trees, which can represent both the spatial and the dynamic aspects of mobile computation.

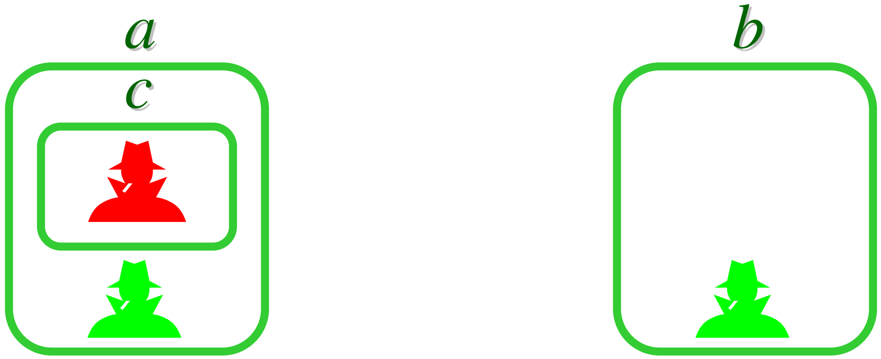


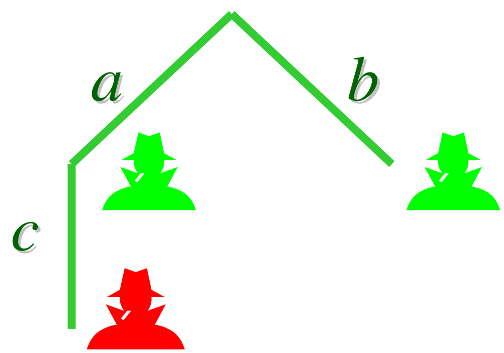

- An ambient tree is a spatial tree with, possibly, threads at each node that can locally change the shape of the tree.

$$a[c[\mathit{out}\ a.\ \mathit{in}\ b.\ P]]\ |\ b[\mathbf{0}]$$

# Mobility

- *Mobility* is change of spatial structures over time.

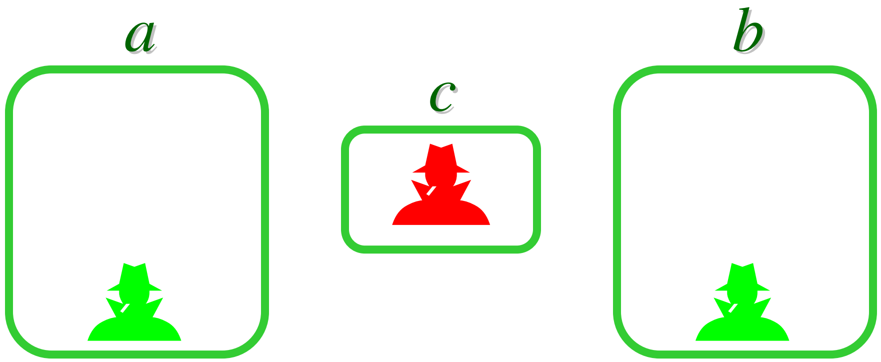


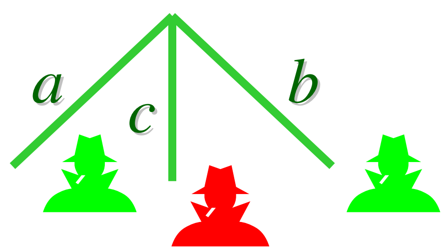




$a[Q \mid c[\mathit{out}\ a.\ \mathit{in}\ b.\ P]] \quad \mid b[R]$

# Mobility

- *Mobility* is change of spatial structures over time.

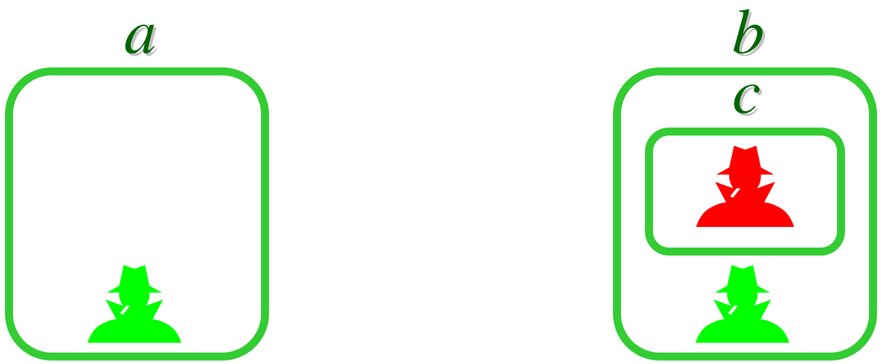


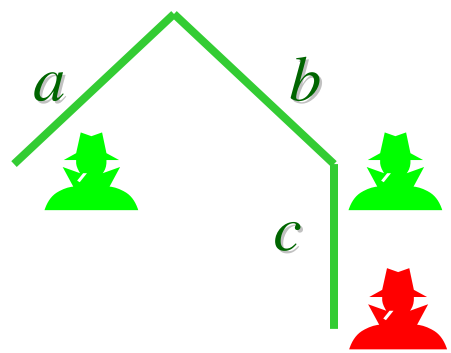




$$a[Q] \quad \mid c[in\ b.\ P] \mid b[R]$$

# Mobility

- *Mobility* is change of spatial structures over time.




$a[Q] \mid b[R \mid c[P]]$

# Restriction-Free Ambient Calculus

| | | | |
|---|---|---|---|
| $P \in \Pi ::=$ | Processes | $M ::=$ | Messages |
| $\mathbf{0}$ | inactivity | $n$ | name |
| $P \mid P'$ | parallel | $in\ M$ | entry capability |
| $!P$ | replication | $out\ M$ | exit capability |
| $M[P]$ | ambient | $open\ M$ | open capability |
| $M.P$ | exercise a capability | $\varepsilon$ | empty path |
| $(n).P$ | input locally, bind to $n$ | $M.M'$ | composite path |
| $\langle M \rangle$ | output locally (async) | | |

$n[] \triangleq n[\mathbf{0}]$

$M \triangleq M.\mathbf{0}$ (where appropriate)

# Reduction Semantics

- A structural congruence relation $P \equiv Q$:
  - On spatial expressions, $P \equiv Q$ iff $P$ and $Q$ denote the same tree.
  - On full ambient expressions, $P \equiv Q$ if in addition the respective threads are "trivially equivalent".
  - Prominent in the definition of the logic.

- A reduction relation $P \longrightarrow^* Q$:
  - Defining the meaning of mobility and communication actions.
  - Closed up to structural congruence:

$$P \equiv P',\ P' \longrightarrow^* Q',\ Q' \equiv Q \quad \Rightarrow \quad P \longrightarrow^* Q$$

# Space-Time Modalities

- In a modal logic, the truth of a formula is relative to a state (called a *world*).
- In our case, the truth of a space-time modal formula is relative to the *here and now* of a process.
  - The formula $n[\mathbf{0}]$ is read:

    there is here and now an empty location called $n$
  - The operator $n[\mathcal{A}]$ is a single step in space (akin to the temporal next), which allows us talk about that place one step down into $n$.
  - Other modal operators can be used to talk about undetermined times (in the future) and undetermined places (in the location tree).

# Logical Formulas

| | | |
|---|---|---|
| $\mathcal{A} \in \Phi$ ::= | Formulas | ($\eta$ is a name or a variable) |
| $\mathbf{T}$ | true | |
| $\neg\mathcal{A}$ | negation | |
| $\mathcal{A} \vee \mathcal{A}'$ | disjunction | |
| $\mathbf{0}$ | void | |
| $\eta[\mathcal{A}]$ | location | |
| $\mathcal{A} \mid \mathcal{A}'$ | composition | |
| $✧\mathcal{A}$ | somewhere modality | |
| $\Diamond\mathcal{A}$ | sometime modality | |
| $\mathcal{A}@\eta$ | location adjunct | |
| $\mathcal{A} \triangleright \mathcal{A}'$ | composition adjunct | |
| $\forall x.\mathcal{A}$ | universal quantification over names | |

# Satisfaction Relation

| | | |
|---|---|---|
| $P \vDash \mathbf{T}$ | | |
| $P \vDash \neg\mathcal{A}$ | $\triangleq$ | $\neg\, P \vDash \mathcal{A}$ |
| $P \vDash \mathcal{A} \vee \mathcal{B}$ | $\triangleq$ | $P \vDash \mathcal{A} \vee P \vDash \mathcal{B}$ |
| $P \vDash \mathbf{0}$ | $\triangleq$ | $P \equiv \mathbf{0}$ |
| $P \vDash n[\mathcal{A}]$ | $\triangleq$ | $\exists P' \in \Pi.\ P \equiv n[P'] \wedge P' \vDash \mathcal{A}$ |
| $P \vDash \mathcal{A} \mid \mathcal{B}$ | $\triangleq$ | $\exists P', P'' \in \Pi.\ P \equiv P' \mid P'' \wedge P' \vDash \mathcal{A} \wedge P'' \vDash \mathcal{B}$ |
| $P \vDash ✧\mathcal{A}$ | $\triangleq$ | $\exists P' \in \Pi.\ P \downarrow^{*} P' \wedge P' \vDash \mathcal{A}$ |
| $P \vDash \Diamond\mathcal{A}$ | $\triangleq$ | $\exists P' \in \Pi.\ P \longrightarrow^{*} P' \wedge P' \vDash \mathcal{A}$ |
| $P \vDash \mathcal{A}@n$ | $\triangleq$ | $n[P] \vDash \mathcal{A}$ |
| $P \vDash \mathcal{A} \triangleright \mathcal{B}$ | $\triangleq$ | $\forall P' \in \Pi.\ P' \vDash \mathcal{A} \Rightarrow P \mid P' \vDash \mathcal{B}$ |
| $P \vDash \forall x.\mathcal{A}$ | $\triangleq$ | $\forall m \in \Lambda.\ P \vDash \mathcal{A}\{x \leftarrow m\}$ |

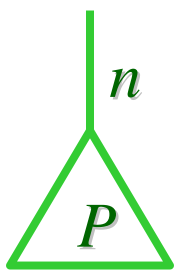
$P \downarrow P'$ iff $\exists n, P''.\ P \equiv n[P'] \mid P''$

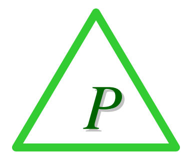
$\downarrow^{*}$ is the reflexive and transitive closure of $\downarrow$

# Satisfaction Relation for Trees

- $\models \mathbf{0}$

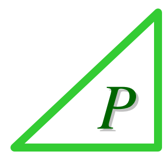
---

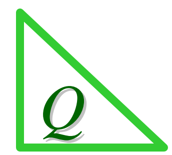


$\models n[\mathcal{A}]$ if 

$\models \mathcal{A}$

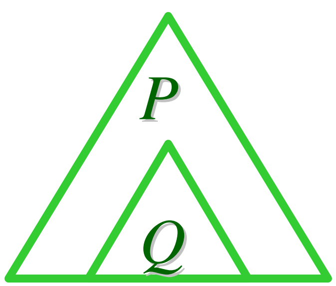
---

$\models \mathcal{A} \mid \mathcal{B}$ if 

$\models \mathcal{A}$ and 

$\models \mathcal{B}$

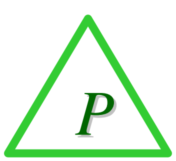
---

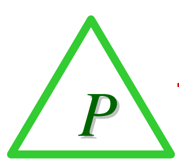


$\models \diamond\!\!\!\!\!\cdot\,\mathcal{A}$ if 

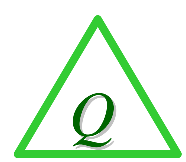
$\models \mathcal{A}$

---

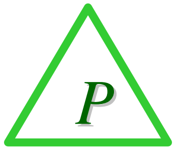


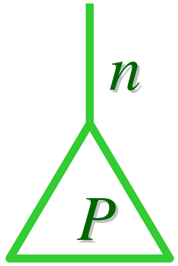
$\models \Diamond\mathcal{A}$ if 

$\rightarrow^*$ 

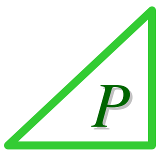
and 

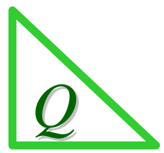
$\models \mathcal{A}$



$\models \mathcal{A}@n$ if


$\models \mathcal{A}$



$\models \mathcal{A} \triangleright \mathcal{B}$ if for all


$\models \mathcal{A}$ we have


$\models \mathcal{B}$

- Basic Fact: satisfaction is invariant under structural congruence:

$$P \models \mathcal{A},\ P \equiv P' \ \Rightarrow\ P' \models \mathcal{A}$$

I.e.: $\{P \in \Pi \mid P \models \mathcal{A}\}$ is closed under $\equiv$.

Hence, formulas describe only congruence-invariant properties.

# Some Derived Connectives

| | | |
|---|---|---|
| $\mathbf{F}$ | $\triangleq \neg \mathbf{T}$ | false |
| $\mathcal{A} \wedge \mathcal{B}$ | $\triangleq \neg(\neg\mathcal{A} \vee \neg\mathcal{B})$ | conjunction |
| $\mathcal{A} \Rightarrow \mathcal{B}$ | $\triangleq \neg\mathcal{A} \vee \mathcal{B}$ | implication |
| $\Box\mathcal{A}$ | $\triangleq \neg\Diamond\neg\mathcal{A}$ | everytime modality |
| $\text{¤}\mathcal{A}$ | $\triangleq \neg\text{✧}\neg\mathcal{A}$ | everywhere modality |
| $\exists x.\mathcal{A}$ | $\triangleq \neg\forall x.\neg\mathcal{A}$ | existential quantification |
| $\mathcal{A} \propto \mathcal{B}$ | $\triangleq \neg(\mathcal{B} \triangleright \neg\mathcal{A})$ | fusion |
| $\mathcal{A} \mid\!\Rightarrow \mathcal{B}$ | $\triangleq \neg(\mathcal{A} \mid \neg\mathcal{B})$ | fusion adjunct |
| $\mathcal{A} \parallel \mathcal{B}$ | $\triangleq \neg(\neg\mathcal{A} \mid \neg\mathcal{B})$ | decomposition |
| $\mathcal{A}^{\forall}$ | $\triangleq \mathcal{A} \parallel \mathbf{F}$ | every component satisfies $\mathcal{A}$ |
| $\mathcal{A}^{\exists}$ | $\triangleq \mathcal{A} \mid \mathbf{T}$ | some component satisfies $\mathcal{A}$ |
| $\mathcal{A}^{\mathbf{F}}$ | $\triangleq \mathcal{A} \triangleright \mathbf{F}$ | $\mathcal{A}$ is unsatisfiable |

# Claims

- The satisfaction relation looks natural (to us):
  - The definitions of $\mathbf{0}$, $n[\mathcal{A}]$, and $\mathcal{A} \mid \mathcal{B}$ seem inevitable, once we accept that formulas should be able to talk about the tree structure of locations (up to $\equiv$).
  - The connectives $\mathcal{A}@n$ and $\mathcal{A} \triangleright \mathcal{B}$ have security motivations.
  - The modalities $\Diamond\mathcal{A}$ and ✧$\mathcal{A}$ talk about process evolution and structure in an undetermined way (good for specs).
  - The fragment $\mathbf{T}$, $\neg\mathcal{A}$, $\mathcal{A} \vee \mathcal{B}$, $\forall x.\mathcal{A}$, is classical: why not?
- The logic is induced by the satisfaction relation.
  - We did not have any preconceptions about what kind of logic this ought to be. We didn't invent this logic, we discovered it!

# From Satisfaction to (Propositional) Logic

- Propositional validity

$$\textbf{\textit{vld}}\ \mathcal{A} \ \triangleq\ \forall P\in\Pi.\ P \vDash \mathcal{A} \qquad\qquad \mathcal{A}\text{ (closed) is valid}$$

- Sequents

$$\mathcal{A} \vdash \mathcal{B} \ \triangleq\ \forall P\in\Pi.\ P \vDash \mathcal{A} \Rightarrow P \vDash \mathcal{B}$$

- Rules

$$\mathcal{A}_1 \vdash \mathcal{B}_1; \ldots; \mathcal{A}_n \vdash \mathcal{B}_n \;\}\; \mathcal{A} \vdash \mathcal{B} \ \triangleq \qquad\qquad (n\geq 0)$$

$$\mathcal{A}_1 \vdash \mathcal{B}_1 \wedge \ldots \wedge \mathcal{A}_n \vdash \mathcal{B}_n \Rightarrow \mathcal{A} \vdash \mathcal{B}$$

(N.B.: all the rules shown later are validated accordingly.)

- Conventions:
  - $\dashv\vdash$ means $\vdash$ in both directions

    $\{\}$ means $\}$ in both directions

# Logical Adjunctions

- This is a logic with multiple logical adjunctions (3 of them!):
  - $\wedge$ / $\Rightarrow$ (classical)

    $\mathcal{A} \wedge \mathcal{C} \vdash \mathcal{B}$ iff $\mathcal{A} \vdash \mathcal{C} \Rightarrow \mathcal{B}$
  - $|$ / $\triangleright$ (linear, $\otimes$ / $\multimap$)

    $\mathcal{A} \mid \mathcal{C} \vdash \mathcal{B}$ iff $\mathcal{A} \vdash \mathcal{C} \triangleright \mathcal{B}$
  - $n[\text{-}]$ / $\text{-}@n$

    $n[\mathcal{A}] \vdash \mathcal{B}$ iff $\mathcal{A} \vdash \mathcal{B}@n$
- Which one should be taken as *the* logical adjunction for sequents? I.e., what should "," mean in a sequent?

# "Neutral" Sequents

- Our logic is formulated with single-premise, single-conclusion sequents. We don't pre-judge ",".
  - By taking $\wedge$ on the left and $\vee$ on the right of $\vdash$ as structural operators, we can derive all the standard rules of sequent and natural deduction systems with multiple premises/conclusions.
  - By taking $|$ on the left of $\vdash$ as a structural operator, we can derive all the rules of intuitionistic linear logic (by appropriate mappings of the ILL connectives).
  - By taking nestings of $\wedge$ and $|$ on the left of $\vdash$ as structural "bunches", we obtain a bunched logic, with its two associated implications, $\Rightarrow$ and $\triangleright$.
- This is convenient. We do not know much, however, about the meta-theory of this presentation style.

# Rules: Propositional Calculus

| | |
|---|---|
| (A-L) | $\mathcal{A}\wedge(\mathcal{C}\wedge\mathcal{D}) \vdash \mathcal{B}$ $\{\}$ $(\mathcal{A}\wedge\mathcal{C})\wedge\mathcal{D} \vdash \mathcal{B}$ |
| (A-R) | $\mathcal{A} \vdash (\mathcal{C}\vee\mathcal{D})\vee\mathcal{B}$ $\{\}$ $\mathcal{A} \vdash \mathcal{C}\vee(\mathcal{D}\vee\mathcal{B})$ |
| (X-L) | $\mathcal{A}\wedge\mathcal{C} \vdash \mathcal{B}$ $\}$ $\mathcal{C}\wedge\mathcal{A} \vdash \mathcal{B}$ |
| (X-R) | $\mathcal{A} \vdash \mathcal{C}\vee\mathcal{B}$ $\}$ $\mathcal{A} \vdash \mathcal{B}\vee\mathcal{C}$ |
| (C-L) | $\mathcal{A}\wedge\mathcal{A} \vdash \mathcal{B}$ $\}$ $\mathcal{A} \vdash \mathcal{B}$ |
| (C-R) | $\mathcal{A} \vdash \mathcal{B}\vee\mathcal{B}$ $\}$ $\mathcal{A} \vdash \mathcal{B}$ |
| (W-L) | $\mathcal{A} \vdash \mathcal{B}$ $\}$ $\mathcal{A}\wedge\mathcal{C} \vdash \mathcal{B}$ |
| (W-R) | $\mathcal{A} \vdash \mathcal{B}$ $\}$ $\mathcal{A} \vdash \mathcal{C}\vee\mathcal{B}$ |
| (Id) | $\}$ $\mathcal{A} \vdash \mathcal{A}$ |
| (Cut) | $\mathcal{A} \vdash \mathcal{C}\vee\mathcal{B}$; $\mathcal{A}'\wedge\mathcal{C} \vdash \mathcal{B}'$ $\}$ $\mathcal{A}\wedge\mathcal{A}' \vdash \mathcal{B}\vee\mathcal{B}'$ |
| (**T**) | $\mathcal{A}\wedge\mathbf{T} \vdash \mathcal{B}$ $\}$ $\mathcal{A} \vdash \mathcal{B}$ |
| (**F**) | $\mathcal{A} \vdash \mathbf{F}\vee\mathcal{B}$ $\}$ $\mathcal{A} \vdash \mathcal{B}$ |
| ($\neg$-L) | $\mathcal{A} \vdash \mathcal{C}\vee\mathcal{B}$ $\}$ $\mathcal{A}\wedge\neg\mathcal{C} \vdash \mathcal{B}$ |
| ($\neg$-R) | $\mathcal{A}\wedge\mathcal{C} \vdash \mathcal{B}$ $\}$ $\mathcal{A} \vdash \neg\mathcal{C}\vee\mathcal{B}$ |

# Rules: Composition

| | | |
|---|---|---|
| $(\mid \mathbf{0})$ | $\} \ \mathcal{A} \mid \mathbf{0} \dashv\vdash \mathcal{A}$ | $\mathbf{0}$ is nothing |
| $(\mid \neg\mathbf{0})$ | $\} \ \mathcal{A} \mid \neg\mathbf{0} \vdash \neg\mathbf{0}$ | if a part is non-$\mathbf{0}$, so is the whole |
| $(\mathrm{A}\mid)$ | $\} \ \mathcal{A} \mid (\mathcal{B} \mid \mathcal{C}) \dashv\vdash (\mathcal{A} \mid \mathcal{B}) \mid \mathcal{C}$ | $\mid$ associativity |
| $(\mathrm{X}\mid)$ | $\} \ \mathcal{A} \mid \mathcal{B} \vdash \mathcal{B} \mid \mathcal{A}$ | $\mid$ commutativity |
| $(\mid \vdash)$ | $\mathcal{A}' \vdash \mathcal{B}'; \ \mathcal{A}'' \vdash \mathcal{B}'' \ \} \ \mathcal{A}' \mid \mathcal{A}'' \vdash \mathcal{B}' \mid \mathcal{B}''$ | $\mid$ congruence |
| $(\mid \vee)$ | $\} \ (\mathcal{A} \vee \mathcal{B}) \mid \mathcal{C} \vdash \mathcal{A} \mid \mathcal{C} \vee \mathcal{B} \mid \mathcal{C}$ | $\mid$-$\vee$ distribution |
| $(\mid\mid\mid)$ | $\} \ \mathcal{A}' \mid \mathcal{A}'' \vdash \mathcal{A}' \mid \mathcal{B}'' \vee \mathcal{B}' \mid \mathcal{A}'' \vee \neg\mathcal{B}' \mid \neg\mathcal{B}''$ | decomposition |
| $(\mid \triangleright)$ | $\mathcal{A} \mid \mathcal{C} \vdash \mathcal{B} \ \{\} \ \mathcal{A} \vdash \mathcal{C} \triangleright \mathcal{B}$ | $\mid$-$\triangleright$ adjunction |
| $(\triangleright\mathbf{F}\,\neg)$ | $\} \ \mathcal{A}^{\mathbf{F}} \vdash \mathcal{A}^{\neg}$ | if $\mathcal{A}$ is unsatisfiable then $\mathcal{A}$ is false |
| $(\neg\,\triangleright\mathbf{F})$ | $\} \ \mathcal{A}^{\mathbf{F}\neg} \vdash \mathcal{A}^{\mathbf{FF}}$ | if $\mathcal{A}$ is satisfiable then $\mathcal{A}^{\mathbf{F}}$ is unsatisfiable |

where $\mathcal{A}^{\neg} \triangleq \neg\mathcal{A}$ and $\mathcal{A}^{\mathbf{F}} \triangleq \mathcal{A} \triangleright \mathbf{F}$

# The Decomposition Operator

- Consider the De Morgan dual of $|$ :

| | | |
|---|---|---|
| $\mathcal{A} \parallel \mathcal{B}$ | $\triangleq \neg(\neg\mathcal{A} \mid \neg\mathcal{B})$ | $P \vDash$ - iff $\forall P', P'' \in \Pi.\ P \equiv P' \mid P'' \Rightarrow$ $P' \vDash \mathcal{A} \vee P'' \vDash \mathcal{B}$ |
| $\mathcal{A}^{\forall}$ | $\triangleq \mathcal{A} \parallel \mathbf{F}$ | $P \vDash$ - iff $\forall P', P'' \in \Pi.\ P \equiv P' \mid P'' \Rightarrow P' \vDash \mathcal{A}$ |
| $\mathcal{A}^{\exists}$ | $\triangleq \mathcal{A} \mid \mathbf{T}$ | $P \vDash$ - iff $\exists P', P'' \in \Pi.\ P \equiv P' \mid P'' \wedge P' \vDash \mathcal{A}$ |

| | |
|---|---|
| $\mathcal{A} \parallel \mathcal{B}$ | for every partition, one piece satisfies $\mathcal{A}$ or the other piece satisfies $\mathcal{B}$ |
| $\mathcal{A}^{\forall} \Leftrightarrow \neg((\neg\mathcal{A})^{\exists})$ | every component satisfies $\mathcal{A}$ |
| $\mathcal{A}^{\exists} \Leftrightarrow \neg((\neg\mathcal{A})^{\forall})$ | some component satisfies $\mathcal{A}$ |

Examples:

| | |
|---|---|
| $(p[\mathbf{T}] \Rightarrow p[q[\mathbf{T}]^{\exists}])^{\forall}$ | every $p$ has a $q$ child |
| $(p[\mathbf{T}] \Rightarrow p[q[\mathbf{T}] \mid (\neg q[\mathbf{T}])^{\forall}])^{\forall}$ | every $p$ has a unique $q$ child |

# The Decomposition Axiom

( | || ) } $(\mathcal{A}' \mid \mathcal{A}'') \vdash (\mathcal{A}' \mid \mathcal{B}'') \vee (\mathcal{B}' \mid \mathcal{A}'') \vee (\neg\mathcal{B}' \mid \neg\mathcal{B}'')$

- Alternative formulations and special cases:
  - $(\mathcal{A}' \mid \mathcal{A}'') \wedge (\mathcal{B}' \parallel \mathcal{B}'') \vdash (\mathcal{A}' \mid \mathcal{B}'') \vee (\mathcal{B}' \mid \mathcal{A}'')$

    "If $P$ has a partition into pieces that satisfy $\mathcal{A}'$ and $\mathcal{A}''$, and every partition has one piece that satisfies $\mathcal{B}'$ or the other that satisfies $\mathcal{B}''$, then either $P$ has a partition into pieces that satisfy $\mathcal{A}'$ and $\mathcal{B}''$, or it has a partition into pieces that satisfy $\mathcal{B}'$ and $\mathcal{A}''$."
  - $\neg(\mathcal{A} \mid \mathcal{B}) \vdash (\mathcal{A} \mid \mathbf{T}) \Rightarrow (\mathbf{T} \mid \neg\mathcal{B})$

    "If $P$ has no partition into pieces that satisfy $\mathcal{A}$ and $\mathcal{B}$, but $P$ has a piece that satisfies $\mathcal{A}$, then $P$ has a piece that does not satisfy $\mathcal{B}$."
  - $\neg(\mathbf{T} \mid \mathcal{B}) \vdash \mathbf{T} \mid \neg\mathcal{B}$
  - $\neg(\mathcal{A} \mid \mathcal{B}) \vdash (\neg\mathcal{A} \mid \mathbf{T}) \vee (\mathbf{T} \mid \neg\mathcal{B})$

# The Composition Adjunct

$(\mid\triangleright)\quad \mathcal{A} \mid \mathcal{C} \vdash \mathcal{B} \;\}\{\; \mathcal{A} \vdash \mathcal{C} \triangleright \mathcal{B}$

"Assume that every process that has a partition into pieces that satisfy $\mathcal{A}$ and $\mathcal{C}$, also satisfies $\mathcal{B}$. Then, every process that satisfies $\mathcal{A}$, together with any process that satisfies $\mathcal{C}$, satisfies $\mathcal{B}$. (And vice versa.)" (*c.f.* $(\multimap \mathrm{R})$)

- Interpretations of $\mathcal{A} \triangleright \mathcal{B}$:
  - $P$ provides $\mathcal{B}$ in any context that provides $\mathcal{A}$
  - $P$ ensures $\mathcal{B}$ under any attack that ensures $\mathcal{A}$

That is, $P \vDash \mathcal{A} \triangleright \mathcal{B}$ is a context-system spec (a concurrent version of a pre-post spec).

Moreover $\mathcal{A} \triangleright \mathcal{B}$ is, in a precise sense, linear implication: the context that satisfies $\mathcal{A}$ is used exactly once in the system that satisfies $\mathcal{B}$.

# Some Derived Rules

$\} \ (\mathcal{A} \triangleright \mathcal{B}) \mid \mathcal{A} \vdash \mathcal{B}$

"If $P$ provides $\mathcal{B}$ in any context that provides $\mathcal{A}$, and $Q$ provides $\mathcal{A}$, then $P$ and $Q$ together provide $\mathcal{B}$."

- Proof: $\mathcal{A} \triangleright \mathcal{B} \vdash \mathcal{A} \triangleright \mathcal{B} \ \} \ (\mathcal{A} \triangleright \mathcal{B}) \mid \mathcal{A} \vdash \mathcal{B}$ by (Id), ( $\mid \triangleright$ )

$\mathcal{D} \vdash \mathcal{A}; \ \mathcal{B} \vdash \mathcal{C} \ \} \ \mathcal{D} \mid (\mathcal{A} \triangleright \mathcal{B}) \vdash \mathcal{C}$ **(*c.f.* ($\multimap$ L))**

"If anything that satisfies $\mathcal{D}$ satisfies $\mathcal{A}$, and anything that satisfies $\mathcal{B}$ satisfies $\mathcal{C}$, then: anything that has a partition into a piece satisfying $\mathcal{D}$ (and hence $\mathcal{A}$), and another piece satisfying $\mathcal{B}$ in a context that satisfies $\mathcal{A}$, it satisfies ($\mathcal{B}$ and hence) $\mathcal{C}$."

- Proof:

| | |
|---|---|
| $\mathcal{D} \vdash \mathcal{A}; \ \mathcal{A} \triangleright \mathcal{B} \vdash \mathcal{A} \triangleright \mathcal{B} \ \} \ \mathcal{D} \mid \mathcal{A} \triangleright \mathcal{B} \vdash \mathcal{A} \mid \mathcal{A} \triangleright \mathcal{B}$ | assumption, (Id), ( $\mid \vdash$ ) |
| $\mathcal{A} \mid \mathcal{A} \triangleright \mathcal{B} \vdash \mathcal{B}$ | above |
| $\mathcal{B} \vdash \mathcal{C}$ | assumption |

# More Derived Rules

$\}\ \mathcal{A} \vdash \mathbf{T} \mid \mathcal{A}$ — you can always add more pieces (if they are **0**)

$\}\ \mathbf{F} \mid \mathcal{A} \vdash \mathbf{F}$ — if a piece is absurd, so is the whole

$\}\ \mathbf{0} \vdash \neg(\neg\mathbf{0} \mid \neg\mathbf{0})$ — **0** is single-threaded

$\}\ \mathcal{A} \mid \mathcal{B} \wedge \mathbf{0} \vdash \mathcal{A}$ — you can split **0** (but you get **0**). Proof uses ( | || )

$\mathcal{A}' \vdash \mathcal{A};\ \mathcal{B} \vdash \mathcal{B}'\ \}\ \mathcal{A} \triangleright \mathcal{B} \vdash \mathcal{A}' \triangleright \mathcal{B}'$ — $\triangleright$ is contravariant on the left

$\}\ \mathcal{A} \triangleright \mathcal{B} \mid \mathcal{B} \triangleright \mathcal{C} \vdash \mathcal{A} \triangleright \mathcal{C}$ — $\triangleright$ is transitive

$\}\ (\mathcal{A} \mid \mathcal{B}) \triangleright \mathcal{C} \dashv\vdash \mathcal{A} \triangleright (\mathcal{B} \triangleright \mathcal{C})$ — $\triangleright$ curry/uncurry

$\}\ \mathcal{A} \triangleright (\mathcal{B} \triangleright \mathcal{C}) \vdash \mathcal{B} \triangleright (\mathcal{A} \triangleright \mathcal{C})$ — contexts commute

$\}\ \mathbf{T} \dashv\vdash \mathbf{T} \triangleright \mathbf{T}$ — truth can withstand any attack

$\}\ \mathbf{T} \vdash \mathbf{F} \triangleright \mathcal{A}$ — anything goes if you can find an absurd partner

$\}\ \mathbf{T} \triangleright \mathcal{A} \vdash \mathcal{A}$ — if $\mathcal{A}$ resists any attack, then it holds

# Rules: Location

| | | |
|---|---|---|
| $(n[]\ \neg\mathbf{0})$ | $\}\ \ n[\mathcal{A}] \vdash \neg\mathbf{0}$ | locations exist |
| $(n[]\ \neg\mid)$ | $\}\ \ n[\mathcal{A}] \vdash \neg(\neg\mathbf{0} \mid \neg\mathbf{0})$ | are not decomposable |
| | | |
| $(n[]\ \vdash)$ | $\mathcal{A} \vdash \mathcal{B}\ \{\}\ \ n[\mathcal{A}] \vdash n[\mathcal{B}]$ | $n[]$ congruence |
| $(n[]\ \wedge)$ | $\}\ \ n[\mathcal{A}] \wedge n[\mathcal{C}] \vdash n[\mathcal{A} \wedge \mathcal{C}]$ | $n[]$-$\wedge$ distribution |
| $(n[]\ \vee)$ | $\}\ \ n[\mathcal{C} \vee \mathcal{B}] \vdash n[\mathcal{C}] \vee n[\mathcal{B}]$ | $n[]$-$\vee$ distribution |
| | | |
| $(n[]\ @)$ | $n[\mathcal{A}] \vdash \mathcal{B}\ \{\}\ \ \mathcal{A} \vdash \mathcal{B}@n$ | $n[]$-@ adjunction |
| $(\neg\ @)$ | $\}\ \ \mathcal{A}@n \dashv\vdash \neg((\neg\mathcal{A})@n)$ | @ is self-dual |

# Rules: Time and Space Modalities

| | | | |
|---|---|---|---|
| (◇) | } ◇𝒜 ⊣⊢ ¬□¬𝒜 | (✧) | } ✧𝒜 ⊣⊢ ¬¤¬𝒜 |
| (□ K) | } □(𝒜⇒ℬ) ⊢ □𝒜⇒□ℬ | (¤ K) | } ¤(𝒜⇒ℬ) ⊢ ¤𝒜⇒¤ℬ |
| (□ T) | } □𝒜 ⊢ 𝒜 | (¤ T) | } ¤𝒜 ⊢ 𝒜 |
| (□ 4) | } □𝒜 ⊢ □□𝒜 | (¤ 4) | } ¤𝒜 ⊢ ¤¤𝒜 |
| (□ **T**) | } **T** ⊢ □**T** | (¤ **T**) | } **T** ⊢ ¤**T** |
| (□ ⊢) | 𝒜 ⊢ ℬ } □𝒜 ⊢ □ℬ | (¤ ⊢) | 𝒜 ⊢ ℬ } ¤𝒜 ⊢ ¤ℬ |
| (◇*n*[]) | } *n*[◇𝒜] ⊢ ◇*n*[𝒜] | (✧*n*[]) | } *n*[✧𝒜] ⊢ ✧𝒜 |
| (◇ \| ) | } ◇𝒜 \| ◇ℬ ⊢ ◇(𝒜 \| ℬ) | (✧ \| ) | } ✧𝒜 \| ℬ ⊢ ✧(𝒜 \| **T**) |
| (✧◇) | } ✧◇𝒜 ⊢ ◇✧𝒜 | | |

S4, but not S5: ¬ ***vld*** ◇𝒜 ⊢ □◇𝒜 ¬ ***vld*** ✧𝒜 ⊢ ¤✧𝒜

(✧◇): if somewhere sometime 𝒜, then sometime somewhere 𝒜

# Some Derived Rules

$\mathcal{A} \vdash \mathcal{B} \ \} \ \mathcal{A}@n \vdash \mathcal{B}@n$ @ congruence

$\} \ n[\mathcal{A}@n] \vdash \mathcal{A}$

$\} \ \mathcal{A} \dashv\vdash n[\mathcal{A}]@n$

$\} \ n[\neg\mathcal{A}] \vdash \neg n[\mathcal{A}]$

$\} \ \neg n[\mathcal{A}] \dashv\vdash \neg n[\mathbf{T}] \vee n[\neg\mathcal{A}]$

# Examples

- $an\ n \triangleq n[\mathbf{T}] \mid \mathbf{T}$ — there is now an $n$ here
- $no\ n \triangleq \neg an\ n$ — there is now no $n$ here
- $one\ n \triangleq n[\mathbf{T}] \mid no\ n$ — there is now exactly one $n$ here
- $\mathcal{A}^{\forall} \triangleq \neg(\neg\mathcal{A} \mid \mathbf{T})$ — everybody here satisfies $\mathcal{A}$
- $(n[\mathbf{T}] \Rightarrow n[\mathcal{A}])^{\forall}$ — every $n$ here satisfies $\mathcal{A}$
- $\boxtimes((n[\mathbf{T}] \Rightarrow n[\mathcal{A}])^{\forall})$ — every $n$ everywhere satisfies $\mathcal{A}$

# Ex: Immovable Object vs. Irresistible Force

$$Im \triangleq \mathbf{T} \triangleright \Box(obj[\mathbf{0}] \mid \mathbf{T})$$

$$Ir \triangleq \mathbf{T} \triangleright \Box\Diamond\neg(obj[\mathbf{0}] \mid \mathbf{T})$$

| | | |
|---|---|---|
| $Im \mid Ir \;=\; (\mathbf{T} \triangleright \Box(obj[\mathbf{0}] \mid \mathbf{T})) \mid Ir$ | | $\mathcal{A} \vdash \mathbf{T}$ |
| $\quad \vdash \; \Box(obj[\mathbf{0}] \mid \mathbf{T})$ | | $(\mathcal{A} \triangleright \mathcal{B}) \mid \mathcal{A} \vdash \mathcal{B}$ |
| $\quad \vdash \; \Diamond P(obj[\mathbf{0}] \mid \mathbf{T})$ | | $\mathcal{A} \vdash \Diamond\mathcal{A}$ |
| $Im \mid Ir \;=\; Im \mid (\mathbf{T} \triangleright \Box\Diamond\neg(obj[\mathbf{0}] \mid \mathbf{T}))$ | | |
| $\quad \vdash \; \Box N\neg(obj[\mathbf{0}] \mid \mathbf{T})$ | | $\Diamond\neg\mathcal{A} \vdash \neg\Box\mathcal{A}$ |
| $\quad \vdash \; \neg\Diamond P(obj[\mathbf{0}] \mid \mathbf{T})$ | | $\Box\neg\mathcal{A} \vdash \neg\Diamond\mathcal{A}$ |
| Hence: $Im \mid Ir \vdash \mathbf{F}$ | | $\mathcal{A} \wedge \neg\mathcal{A} \vdash \mathbf{F}$ |

# Model Checking

- If $P$ is $!$-free and $\mathcal{A}$ is $\triangleright$-free, then $P \vDash \mathcal{A}$ is decidable.
- This provides a way of mechanically checking (certain) assertions about (certain) mobile processes.
- Potential application: checking (the bytecode of) mobile agents against the internal mobility policies of receiving sites. (I.e.: conferring more flexibility than just sandboxing the agent.)

# Connections with Intuitionistic Linear Logic

- Weakening and contraction are not valid rules: principle of *conservation of space*.
- Semantic connection: sets of processes closed under $\equiv$ and ordered by inclusion form a quantale (a model of ILL).
- Multiplicative intuitionistic linear logic (MILL) can be faithfully embedded in our logic:

$$\begin{array}{lcl} \mathbf{1}_{\mathrm{MILL}} & \triangleq & \mathbf{0} \\ \mathcal{A} \otimes_{\mathrm{MILL}} \mathcal{B} & \triangleq & \mathcal{A} \mid \mathcal{B} \\ \mathcal{A} \multimap_{\mathrm{MILL}} \mathcal{B} & \triangleq & \mathcal{A} \triangleright \mathcal{B} \end{array}$$

MILL rules and our rules are interderivable ("our rules" means the rules involving only $\mathbf{0}$, $\mid$, $\triangleright$, plus a derivable cut rule for $\mid$ ).

- Full intuitionistic linear logic (ILL) can be embedded:

| | | | | | |
|---|---|---|---|---|---|
| $\mathbf{1}_{ILL}$ | $\triangleq$ | $\mathbf{0}$ | $\mathcal{A} \oplus \mathcal{B}$ | $\triangleq$ | $\mathcal{A} \vee \mathcal{B}$ |
| $\perp_{ILL}$ | $\triangleq$ | $\mathbf{F}$ | $\mathcal{A} \,\&\, \mathcal{B}$ | $\triangleq$ | $\mathcal{A} \wedge \mathcal{B}$ |
| $\top_{ILL}$ | $\triangleq$ | $\mathbf{T}$ | $\mathcal{A} \otimes \mathcal{B}$ | $\triangleq$ | $\mathcal{A} \mid \mathcal{B}$ |
| $\mathbf{0}_{ILL}$ | $\triangleq$ | $\mathbf{F}$ | $\mathcal{A} \multimap \mathcal{B}$ | $\triangleq$ | $\mathcal{A} \triangleright \mathcal{B}$ |
| | | | $!\mathcal{A}$ | $\triangleq$ | $\mathbf{0} \wedge (\mathbf{0} \Rightarrow \mathcal{A})^{\neg\mathbf{F}}$ |

- The rules of ILL can be logically derived from these definitions. (E.g.: the proof of $!\mathcal{A} \vdash\ !\mathcal{A} \otimes\ !\mathcal{A}$ uses the decomposition axiom.)
- So, $\mathcal{A}_1, ..., \mathcal{A}_n \vdash_{ILL} \mathcal{B}$ implies $\mathcal{A}_1 \mid ... \mid \mathcal{A}_n \vdash \mathcal{B}$.
- Some discrepancies: $\perp_{ILL} = \mathbf{0}_{ILL}$; the additives distribute; $!\mathcal{A}$ is not "replication"; $!\mathcal{A} \multimap \mathcal{B}$ is not so interesting; $\mathcal{A}^{\perp}/\mathcal{A}^{\mathbf{0}}$ is unusually interesting.

# Connection with Relevant Logic

- (Noted after the fact [O'Hearn, Pym].) The definition of the satisfaction relation is very similar to Urquhart's semantics of relevant logic. In particular $\mathcal{A} \mid \mathcal{B}$ is defined just like *intensional conjunction*, and $\mathcal{A} \triangleright \mathcal{B}$ is defined just like *relevant implication* in that semantics.
- Except:
  - We do not have contraction. This does not make sense in process calculi, because $P \mid P \neq P$. Urquhart semantics without contraction does not seem to have been studied.
  - We use an equivalence $\equiv$, instead of a Kripke-style partial order $\not\subset$ as in Urquhart's general case. (We may have a need for a partial order in more sophisticated versions of our logic.)

# Connections with Bunched Logic

- Peter O'Hearn and David Pym study *bunched logics*, where sequents have two structural combinators, instead of the standard single "," combinator (usually meaning $\wedge$ or $\otimes$ on the left) found in most presentations of logic. Thus, sequents are *bunches* of formulas, instead of lists of formulas. Correspondingly, there are two implications that arise as the adjuncts of the two structural combinators.

- The situation is very similar to our combinators $\mid$ and $\wedge$, which can combine to irreducible bunches of formulas in sequents, and to our two implications $\Rightarrow$ and $\triangleright$. However, we have a classical and a linear implication, while bunched logics have so far had an intuitionistic and a linear implication.

# Conclusions

- The novel aspects of our logic lie in its explicit treatment of *space* and of the evolution of space over time (*mobility*). The logic has a linear flavor in the sense that space cannot be instantly created or deleted, although it can be transformed over time.
- These ideas can be applied to any process calculus that embodies a distinction between topological and dynamic operators.
- Our logical rules arise from a particular model. This approach makes the logic very concrete, but raises questions of logical completeness, which are being investigated.
- We are now working on generalizing the logic to the full ambient calculus (including restriction), in order to talk about properties of hidden/secret locations.

# Ambient Calculus: Example

$$\overbrace{\underbrace{a[msg[\langle M\rangle \mid out\ a.\ in\ b.\ P]]}_{\textit{send } M \textit{ from } a \textit{ to } b}}^{\textit{location } a} \mid \overbrace{\underbrace{b[open\ msg.\ (n).\ P]}_{\textit{receive } n;\ \textit{do } P}}^{\textit{location } b}$$

The packet *msg* moves from *a* to *b*, mediated by the capabilities *out a* (to exit *a*), *in b* (to enter *b*), and *open msg* (to open the *msg* envelope).

$$
\begin{array}{lllll}
 & & a[msg[\langle M\rangle \mid out\ a.\ in\ b.\ P]] & & \mid b[open\ msg.\ (n).\ P] \\
\text{(exit)} & \longrightarrow & a[] & \mid msg[\langle M\rangle \mid in\ b.\ P] & \mid b[open\ msg.\ (n).\ P] \\
\text{(enter)} & \longrightarrow & a[] & & \mid b[msg[\langle M\rangle] \mid open\ msg.\ (n).\ P] \\
\text{(open)} & \longrightarrow & a[] & & \mid b[\langle M\rangle \mid (n).\ P] \\
\text{(read)} & \longrightarrow & a[] & & \mid b[P\{n\leftarrow M\}]
\end{array}
$$

# Reduction

- Four basic reductions plus propagation, rearrangement (composition with structural congruence), and transitivity.

| | | |
|---|---|---|
| $n[in\ m.\ P \mid Q] \mid m[R]$ | $\longrightarrow m[n[P \mid Q] \mid R]$ | (Red In) |
| $m[n[out\ m.\ P \mid Q] \mid R]$ | $\longrightarrow n[P \mid Q] \mid m[R]$ | (Red Out) |
| $open\ m.\ P \mid m[Q]$ | $\longrightarrow P \mid Q$ | (Red Open) |
| $(n).P \mid \langle M \rangle$ | $\longrightarrow P\{n \leftarrow M\}$ | (Red Comm) |

$P \longrightarrow Q \;\Rightarrow\; n[P] \longrightarrow n[Q]$ (Red Amb)

$P \longrightarrow Q \;\Rightarrow\; P \mid R \longrightarrow Q \mid R$ (Red Par)

$P' \equiv P,\ P \longrightarrow Q,\ Q \equiv Q' \;\Rightarrow\; P' \longrightarrow Q'$ (Red $\equiv$)

$\longrightarrow^*$ is the reflexive-transitive closure of $\longrightarrow$

# Structural Congruence

- Routine definition, but used heavily in the logic and semantics.

| | |
|---|---|
| $P \equiv P$ | (Struct Refl) |
| $P \equiv Q \;\Rightarrow\; Q \equiv P$ | (Struct Symm) |
| $P \equiv Q, Q \equiv R \;\Rightarrow\; P \equiv R$ | (Struct Trans) |
| $P \equiv Q \;\Rightarrow\; P \mid R \equiv Q \mid R$ | (Struct Par) |
| $P \equiv Q \;\Rightarrow\; !P \equiv !Q$ | (Struct Repl) |
| $P \equiv Q \;\Rightarrow\; M[P] \equiv M[Q]$ | (Struct Amb) |
| $P \equiv Q \;\Rightarrow\; M.P \equiv M.Q$ | (Struct Action) |
| $P \equiv Q \;\Rightarrow\; (n).P \equiv (n).Q$ | (Struct Input) |
| $\varepsilon.P \equiv P$ | (Struct ε) |
| $(M.M').P \equiv M.M'.P$ | (Struct .) |

| | |
|---|---|
| $P \mid Q \equiv Q \mid P$ | (Struct Par Comm) |
| $(P \mid Q) \mid R \equiv P \mid (Q \mid R)$ | (Struct Par Assoc) |
| $P \mid \mathbf{0} \equiv P$ | (Struct Par Zero) |
| $!(P \mid Q) \equiv\ !P \mid\ !Q$ | (Struct Repl Par) |
| $!\mathbf{0} \equiv \mathbf{0}$ | (Struct Repl Zero) |
| $!P \equiv P \mid\ !P$ | (Struct Repl Copy) |
| $!P \equiv\ !!P$ | (Struct Repl Repl) |

- These axioms (particularly the ones for **!**) are sound and complete with respect to equality of spatial trees: edge-labeled finite-depth unordered trees, with infinite-branching but finitely many distinct labels under each node.