A Theory of Objects

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ECOOP'97 Tutorial

Outline

- Topic of this tutorial: a foundation for object-oriented languages based on object calculi.

- Part 1: Object-oriented features.
- Part 2: Object calculi.
- Part 3: Interpretation of object-oriented languages.

Object-Oriented Features

CLASS-BASED LANGUAGES

- The mainstream.
- We review only common, kernel properties.

Classes and Objects

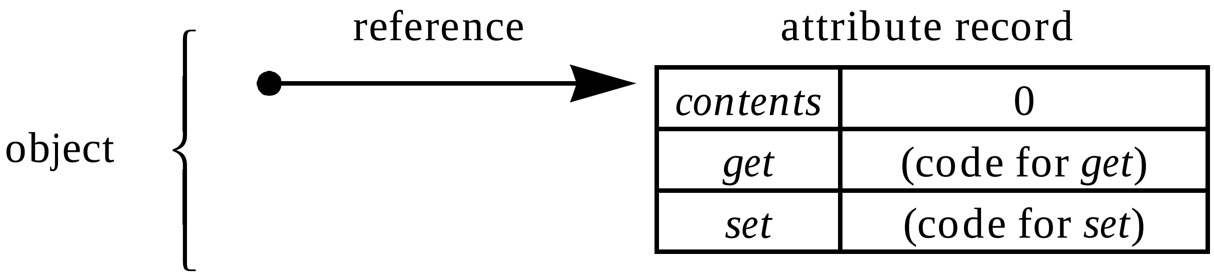
- Classes are descriptions of objects.
- Example: storage cells.

```
class cell is
    var contents: Integer := 0;
    method get(): Integer is
        return self.contents;
    end;
    method set(n: Integer) is
        self.contents := n;
    end;
end;
```

- Classes generate objects.
- Objects can refer to themselves.

Naive Storage Model

- Object = reference to a record of attributes.



object

reference

attribute record

contents	0
get	(code for *get*)
set	(code for *set*)

Naive storage model

Object Operations

- Object creation.
  - ~ *InstanceTypeOf(c)* indicates the type of an object of class *c*.

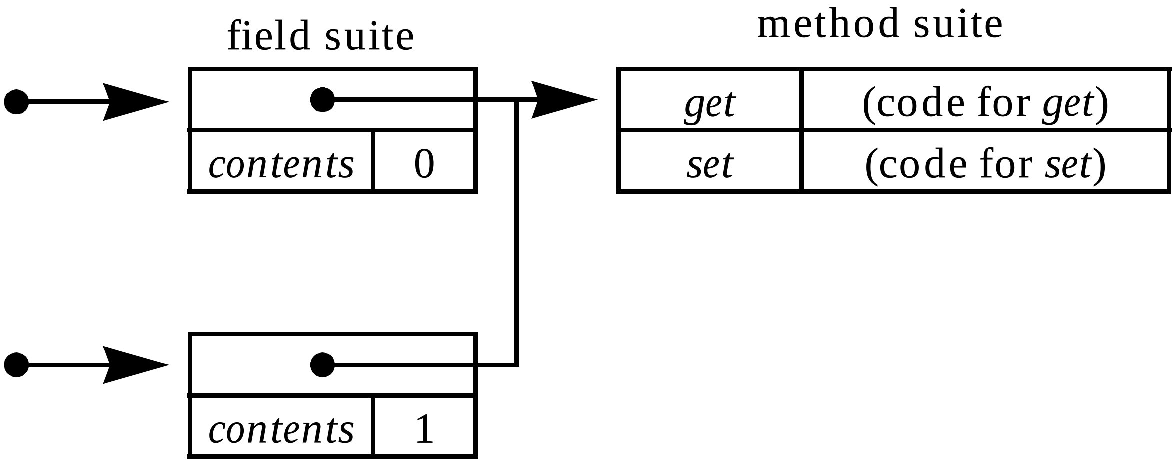
```
var myCell: InstanceTypeOf(cell) := new cell;
```

- Field selection.
- Field update.
- Method invocation.

```
procedure double(aCell: InstanceTypeOf(cell)) is
    aCell.set(2 * aCell.get());
end;
```

The Method-Suites Storage Model

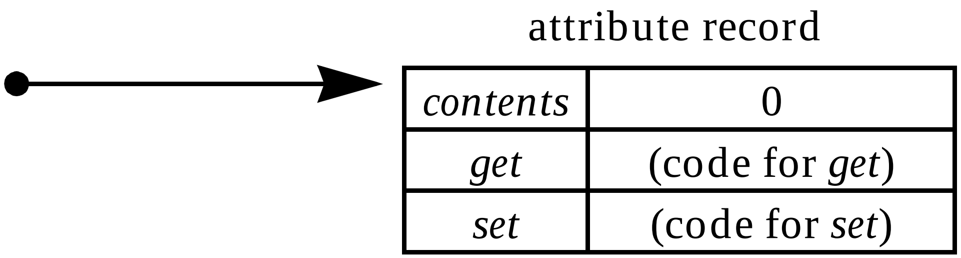
- A more refined storage model for class-based languages.



Method suites

Embedding vs. Delegation

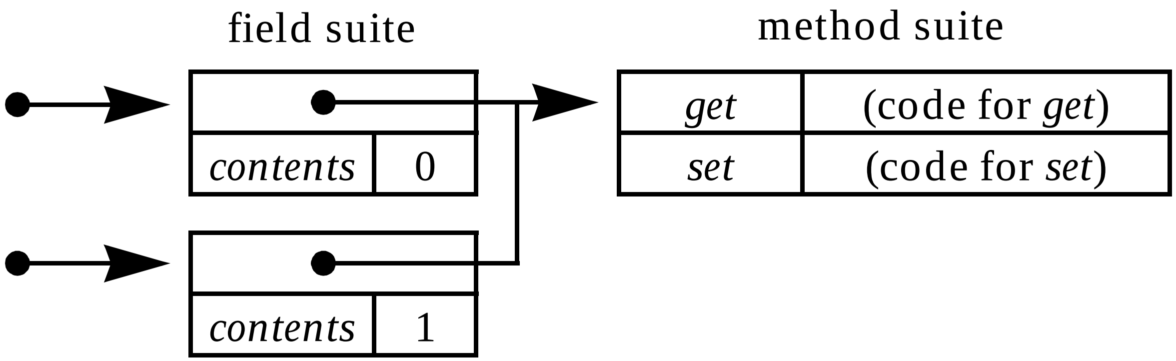
- In the naive storage model, methods are *embedded* in objects.



attribute record

contents	0
get	(code for *get*)
set	(code for *set*)

- In the methods-suites storage model, methods are *delegated* to the method suites.



field suite

contents	0

contents	1

method suite

get	(code for *get*)
set	(code for *set*)

- Naive and method-suites models are semantically equivalent for class-based languages.
- They are not equivalent (as we shall see) in object-based languages, where the difference between embedding and delegation is critical.

Method Lookup

- Method lookup is the process of finding the code to run on a method invocation $o.m(\ldots)$. The details depend on the language and the storage model.
- In class-based languages, method lookup gives the *illusion* that methods are embedded in objects (cf. $o.x$, $o.m(...)$), hiding storage model details.
- Self is always the *receiver*: the object that *appears* to contain the method.
- Features that would distinguish embedding from delegation implementations (e.g., method update) are usually avoided.

Subclasses and Inheritance

- A *subclass* is a differential description of a class.
- The *subclass relation* is the partial order induced by the subclass declarations.
- Example: restorable cells.

```
subclass reCell of cell is
    var backup: Integer := 0;
    override set(n: Integer) is
        self.backup := self.contents;
        super.set(n);
    end;
    method restore() is
        self.contents := self.backup;
    end;
end;
```

Subclasses and Self

- Because of subclasses, the meaning of **self** becomes dynamic.

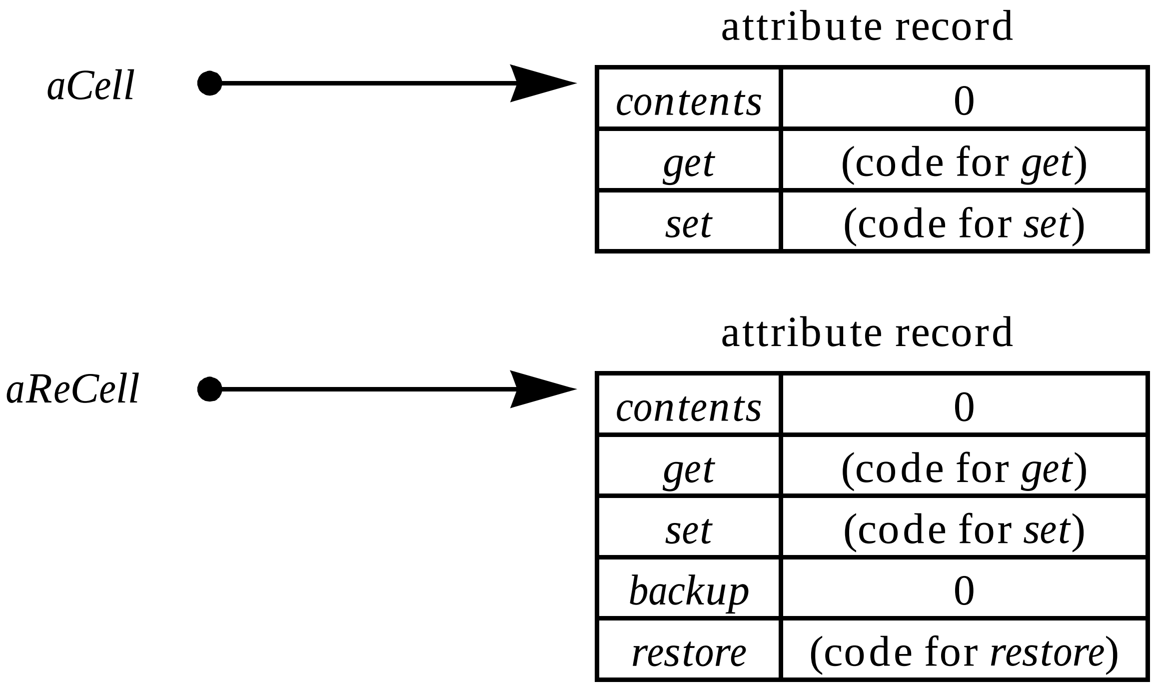
  self.*m*(...)

- Because of subclasses, the concept of **super** becomes useful.

  super.*m*(...)

Subclasses and Naive Storage

- In the naive implementation, the existence of subclasses does not cause any change in the storage model.



aCell

attribute record

contents	0
get	(code for *get*)
set	(code for *set*)

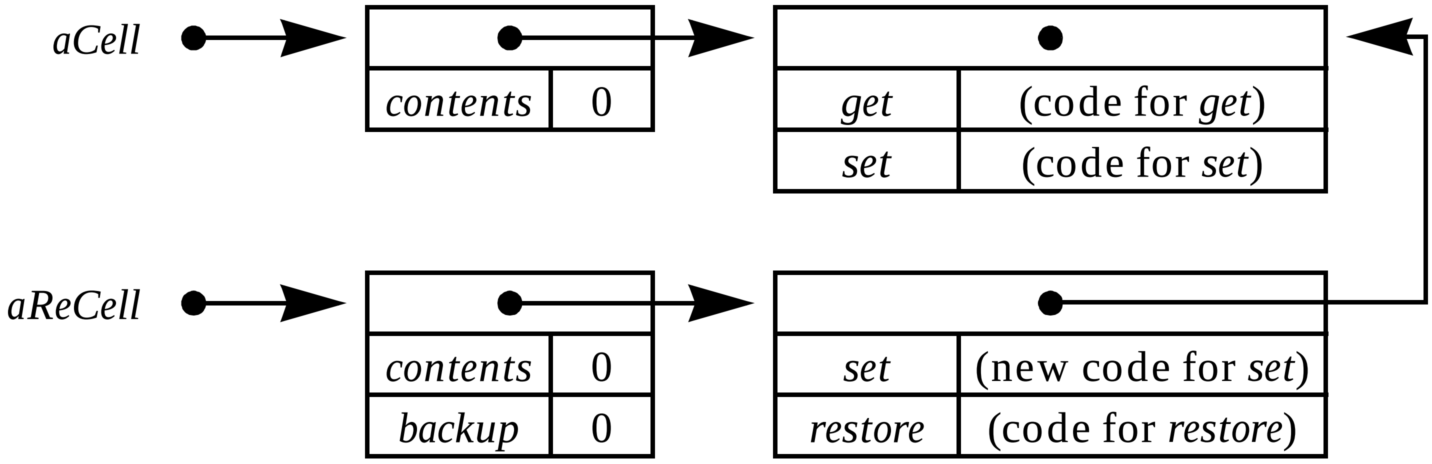
aReCell

attribute record

contents	0
get	(code for *get*)
set	(code for *set*)
backup	0
restore	(code for *restore*)

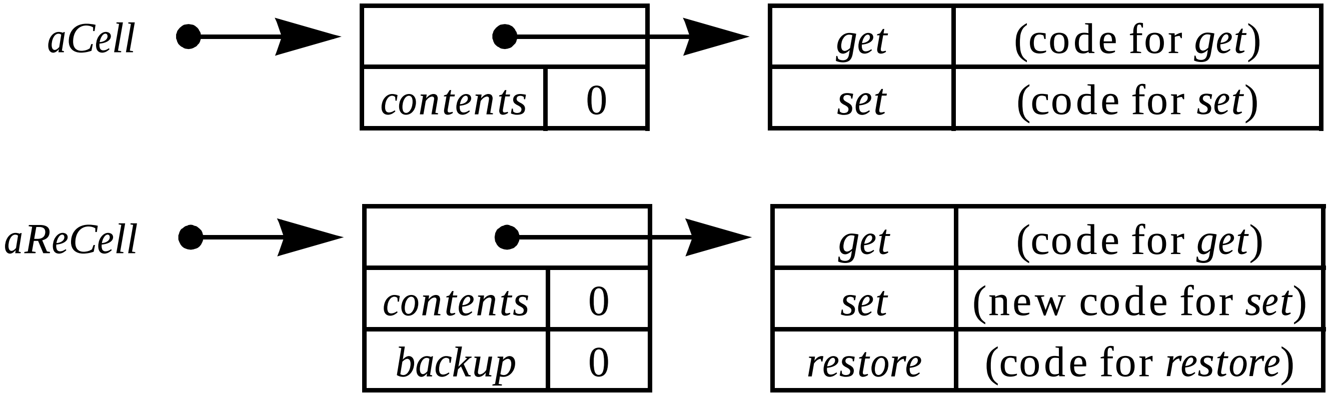
Subclasses and Method Suites

- Because of subclasses, the method-suites model has to be reconsidered. In dynamically-typed class-based languages, method suites are chained:



Hierarchical method suites

- In statically-typed class-based languages, however, the method-suites model can be maintained in its original form.



aCell

contents	0

get	(code for *get*)
set	(code for *set*)

aReCell

contents	0
backup	0

get	(code for *get*)
set	(new code for *set*)
restore	(code for *restore*)

Collapsed method suites

Embedding/Delegation View of Class Hierarchies

- Hierarchical method suites: *delegation* (of objects to suites) combined with *delegation* (of sub-suites to super-suites).
- Collapsed method suites: *delegation* (of objects to suites) combined with *embedding* (of super-suites in sub-suites).

Class-Based Summary

- In analyzing the meaning and implementation of class-based languages we end up inventing and analyzing sub-structures of objects and classes.
- These substructures are independently interesting: they have their own semantics, and can be combined in useful ways.
- What if these substructures were directly available to programmers?

OBJECT-BASED LANGUAGES

- Slow to emerge.
- Simple and flexible.
- Usually untyped.

- Just objects and dynamic dispatch.
- When typed, just object types and subtyping.
- Direct object-to-object inheritance.

An Object, All by Itself

- Classes are replaced by object constructors.
- Object types are immediately useful.

```
ObjectType Cell is
    var contents: Integer;
    method get(): Integer;
    method set(n: Integer);
end;

object cell: Cell is
    var contents: Integer := 0;
    method get(): Integer is return self.contents end;
    method set(n: Integer) is self.contents := n end;
end;
```

An Object Generator

- Procedures as object generators.

```
procedure newCell(m: Integer): Cell is
    object cell: Cell is
        var contents: Integer := m;
        method get(): Integer is return self.contents end;
        method set(n: Integer) is self.contents := n end;
    end;
    return cell;
end;

var cellInstance: Cell := newCell(0);
```

- Quite similar to classes!

Decomposing Class-Based Features

- General idea: decompose class-based notions and orthogonally recombine them.

- We have seen how to decompose simple classes into objects and procedures.
- We will now investigate how to decompose inheritance.
  - ~ Object generation by parameterization.
  - ~ Vs. object generation by cloning and mutation.

Prototypes and Clones

- Classes describe objects.
- Prototypes describe objects and *are* objects.

- Regular objects are clones of prototypes.

  var *cellClone*: *Cell* := **clone** *cellInstance*;

- **clone** is a bit like **new**, but operates on objects instead of classes.

Mutation of Clones

- Clones are customized by mutation (e.g., update).
- Field update.

```
cellClone.contents := 3;
```

- Method update.

```
cellClone.get :=
    method (): Integer is
        if self.contents < 0 then return 0 else return self.contents end;
    end;
```

- Self-mutation possible.

Object-Based Inheritance

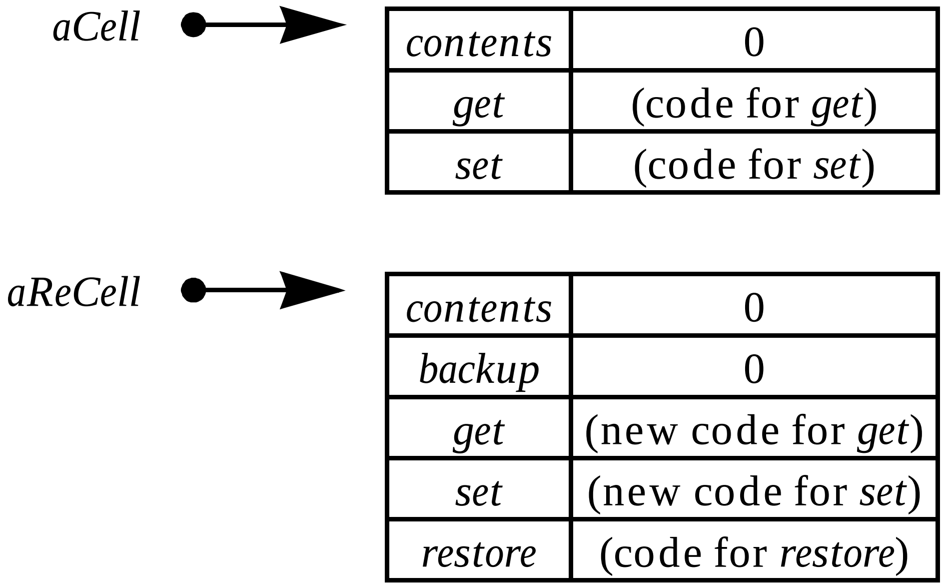
- Object generation can be obtained by procedures, but with no real notion of inheritance.
- Object inheritance can be achieved by cloning (reuse) and update (override), but with no shape change.
- How can one inherit with a change of shape?
- An option is object extension. But:
  - ~ Not easy to typecheck.
  - ~ Not easy to implement efficiently.
  - ~ Provided rarely or restrictively.

Donors and Hosts

- General object-based inheritance: building new objects by “reusing” attributes of existing objects.
- Two orthogonal aspects:
    - obtaining the attributes of a ***donor*** object, and
    - incorporating those attributes into a new ***host*** object.
- Four categories of object-based inheritance:
    - The attributes of a donor may be obtained ***implicitly*** or ***explicitly***.
    - Orthogonally, those attributes may be either ***embedded*** into a host, or ***delegated*** to a donor.

Embedding

- Host objects contain copies of the attributes of donor objects.



contents	0
get	(code for *get*)
set	(code for *set*)

contents	0
backup	0
get	(new code for *get*)
set	(new code for *set*)
restore	(code for *restore*)

Embedding

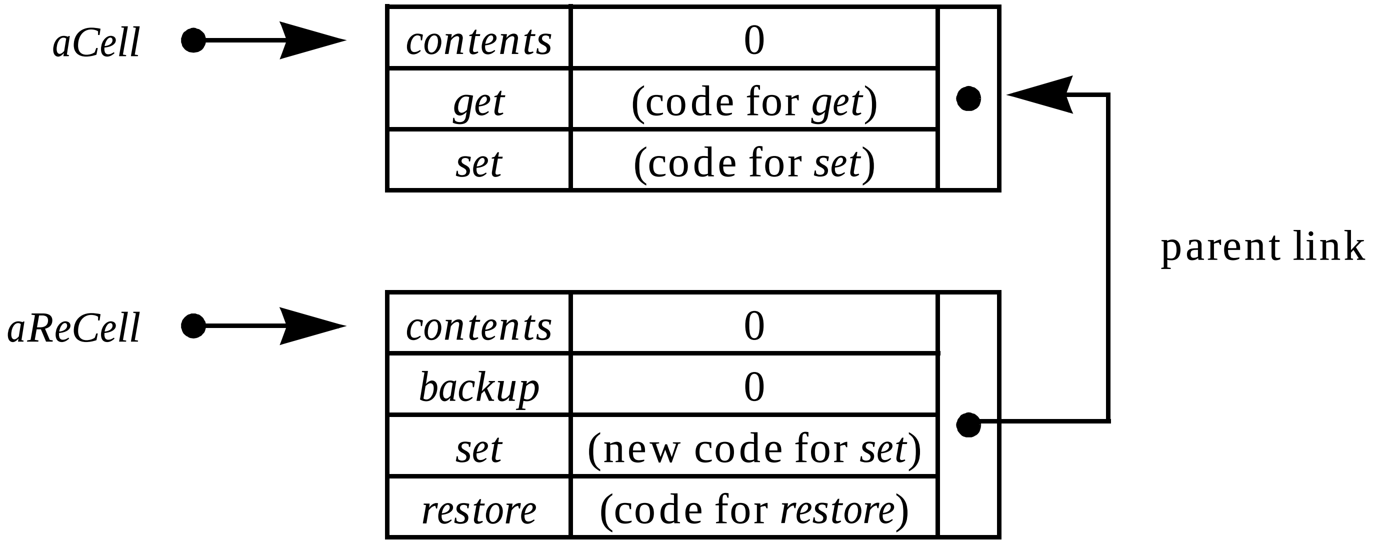
Embedding-Based Languages

- Embedding provides the simplest explanation of the standard semantics of **self** as the receiver.
- Embedding was described by Borning as part of one of the first proposals for prototype-based languages.
- Recently, it has been adopted by languages like Kevo and Obliq. We call these languages *embedding-based* (*concatenation-based*, in Kevo terminology).

Delegation

- Host objects contain *links* to the attributes of donor objects.
- Prototype-based languages that permit the sharing of attributes across objects are called *delegation-based*.
- Operationally, delegation is the redirection of field access and method invocation from an object or prototype to another, <u>in such a way that an object can be seen as an extension of another</u>.
- A crucial aspect of delegation inheritance is the interaction of donor links with the binding of **self**.

Delegation Inheritance



(Single-parent) Delegation

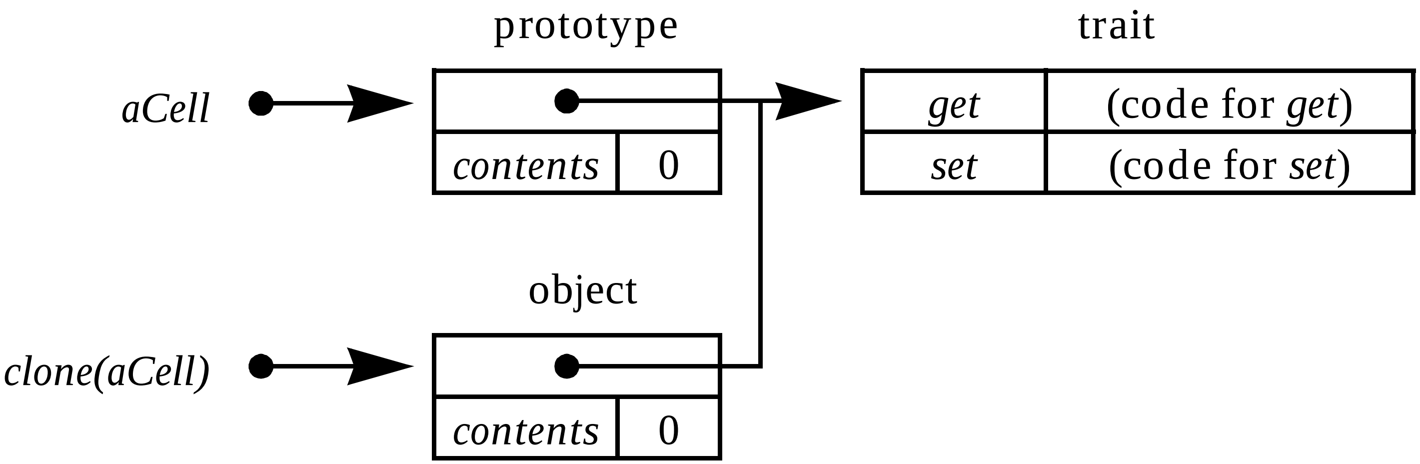
- Note: similar to hierarchical method suites.

Traits: from Prototypes back to Classes?

- Prototypes were initially intended to replace classes.
- Several prototype-based languages, however, seem to be moving towards a more traditional approach based on class-like structures.
- Prototypes-based languages like Omega, Self, and Cecil have evolved usage-based distinctions between objects.

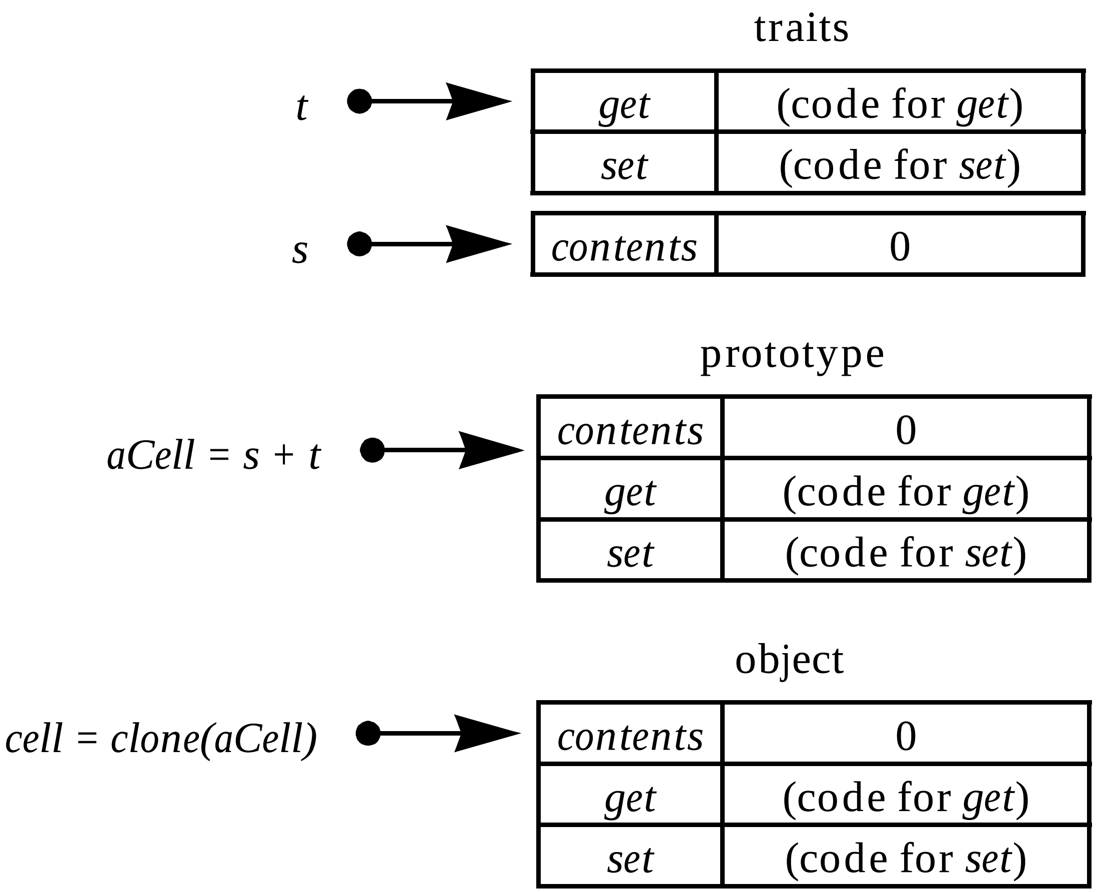
Different Kinds of Objects

- Trait objects.
- Prototype objects.
- Normal objects.



Traits

Embedding-Style Traits



traits

t →

get	(code for *get*)
set	(code for *set*)

s →

contents	0

prototype

aCell = *s* + *t* →

contents	0
get	(code for *get*)
set	(code for *set*)

object

cell = *clone*(*aCell*) →

contents	0
get	(code for *get*)
set	(code for *set*)

Traits

Traits are not Prototypes

- This separation of roles violates the original spirit of prototype-based languages: traits objects cannot function on their own. They typically lack instance variables.
- With the separation between traits and other objects, we seem to have come full circle back to class-based languages and to the separation between classes and instances.
- Trait-based techniques looks exactly like implementation techniques for classes.

Contributions of the Object-Based Approach

- The achievement of object-based languages is to make clear that classes are just one of the possible ways of generating objects with common properties.
- Objects are more primitive than classes, and they should be understood and explained before classes.
- Different class-like constructions can be used for different purposes; hopefully, more flexibly than in strict class-based languages.

Going Further

- Language analysis:
  - Class-based langs. → Object-based langs. → Object calculi
- Language synthesis:
  - Object calculi → Object-based langs. → Class-based langs.

Our Approach to Modeling

- We have identified embedding and delegation as underlying many object-oriented features.
- In our object calculi, we choose embedding over delegation as the principal object-oriented paradigm.
- The resulting calculi can model classes well, although they are not class-based (since classes are not built-in).
- They can model delegation-style traits just as well, but not “true” delegation. (Object calculi for delegation exist but are more complex.)

SUMMARY

- Class-based: various implementation techniques based on embedding and / or delegation. Self is the receiver.
- Object-based: various language mechanisms based on embedding and / or delegation. Self is the receiver.
- Object-based can emulate class-based. (By traits, or by otherwise reproducing the implementations techniques of class-based languages.)

Foundations

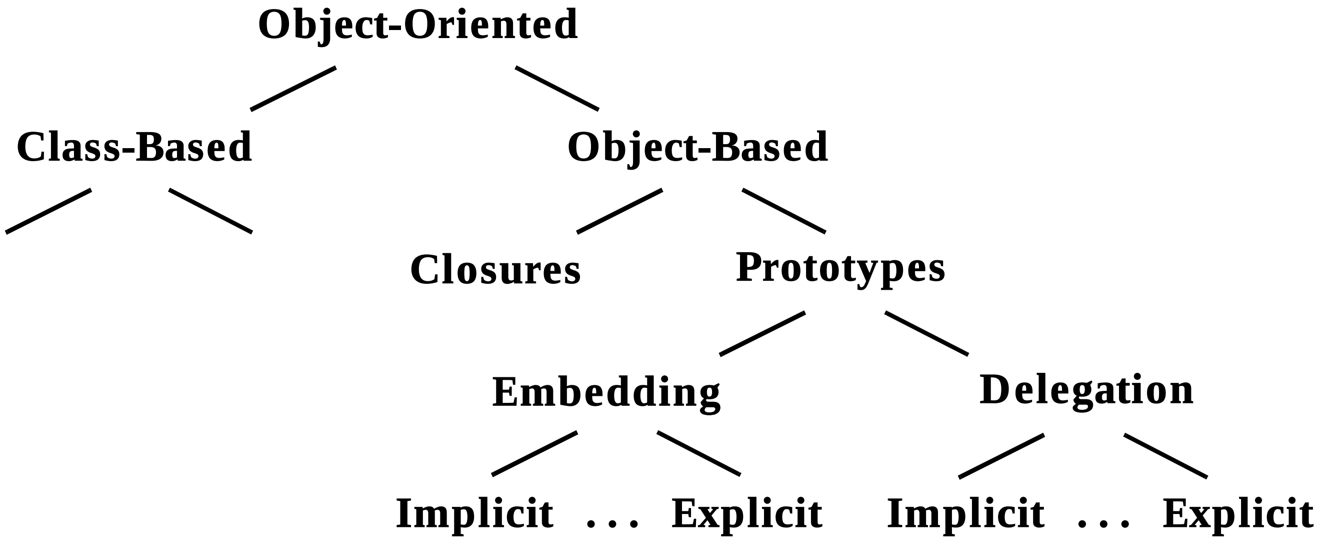
- Objects can emulate classes (by traits) and procedures (by “stack frame objects”).
- *Everything* can indeed be an object.

Future Directions

- I look forward to the continued development of typed object-based languages.
  - The notion of object type arise more naturally in object-based languages.
  - Traits, method update, and mode switching are typable (general reparenting is not easily typable).
- No need for dichotomy: object-based and class-based features can be merged within a single language, based on the common object-based semantics (Beta, O–1, O–2, O–3).

- Embedding-based languages seem to be a natural fit for distributed-objects situations. E.g. COM vs. CORBA.
  - Objects are self-contained and are therefore *localized*.
  - For this reason, Obliq was designed as an embedding-based language.

A New Hierarchy



Object-Oriented

Class-Based

Object-Based

Closures

Prototypes

Embedding

Delegation

Implicit . . . Explicit

Implicit . . . Explicit

Object Calculi

Understanding Objects

- Many characteristics of object-oriented languages are different presentations of a few general ideas.
- The situation is analogous in procedural programming.

  The λ-calculus has provided a basic, flexible model, and a better understanding of actual languages.

From Functions to Objects

- We develop a calculus of objects, analogous to the λ-calculus but independent.
  - ~ It is entirely based on objects, not on functions.
  - ~ We go in this direction because object types are not easily, or at all, definable in most standard formalisms.
- The calculus of objects is intended as a paradigm and a foundation for object-oriented languages.

- We have, in fact, a family of object calculi:
  - ~ functional and imperative;
  - ~ untyped, first-order, and higher-order.

Untyped and first-order object calculi

Calculus:	ς	$\mathbf{Ob}_{1}$	$\mathbf{Ob}_{1<:}$	*nn*	$\mathbf{Ob}_{1\mu}$	$\mathbf{Ob}_{1<:\mu}$	*nn*	$\mathbf{imp}\varsigma$	*nn*
objects	•	•	•	•	•	•	•	•	•
object types		•	•	•	•	•	•		•
subtyping			•	•		•	•		•
variance				•					
recursive types					•	•	•		
dynamic types							•		
side-effects								•	•

Higher-order object calculi

Calculus:	**Ob**	$\mathbf{Ob}_{\mu}$	$\mathbf{Ob}_{<:}$	$\mathbf{Ob}_{<:\mu}$	**ςOb**	**S**	$\mathbf{S}_{\forall}$	*nn*	$\boldsymbol{Ob}_{\omega<:\mu}$
objects	●	●	●	●	●	●	●	●	●
object types	●	●	●	●	●	●	●	●	●
subtyping			●	●	●	●	●	●	●
variance			○	○		●	●	●	●
recursive types		●		●					●
dynamic types									
side-effects								●	
quantified types	●	●	●	●			●	●	●
Self types				○	●	●	●	●	○
structural rules						●	●	●	●
type operators									●

There are several other calculi (*e.g.*, Castagna’s, Fisher&Mitchell’s).

Object Calculi

- As in λ-calculi, we have:
  - ~ operational semantics,
  - ~ denotational semantics,
  - ~ type systems,
  - ~ type inference algorithms (due to J. Palsberg),
  - ~ equational theories,
  - ~ a theory of bisimilarity (due to A. Gordon and G. Rees),
  - ~ examples,
  - ~ (small) language translations,
  - ~ guidance for language design.

The Role of “Functional” Object Calculi

- Functional object calculi are object calculi without side-effects (with or without syntax for functions).
- We have developed both functional and imperative object calculi.
- Functional object calculi have simpler operational semantics.

- “Functional object calculus” sounds odd: objects are supposed to encapsulate state!
- However, many of the techniques developed in the context of functional calculi carry over to imperative calculi.

- Sometimes the same code works functionally and imperatively. Often, imperative versions require just a little more care.
- All transparencies make sense functionally, except those that say “imperative” explicitly.

An Untyped Object Calculus: Syntax

An object is a collection of methods. (Their order does not matter.)

Each method has:

- a bound variable for self (which denotes the object itself),
- a body that produces a result.

The only operations on objects are:

- method invocation,
- method update.

Syntax of the ς-calculus

$a,b ::=$	terms
x	variable
$[l_i{=}\varsigma(x_i)b_i\ ^{i\in 1..n}]$	object (l_i distinct)
$a.l$	method invocation
$a.l{\Leftarrow}\varsigma(x)b$	method update

First Examples

An object *o* with two methods, *l* and *m*:

$$
\begin{aligned}
o \;&\triangleq \\
&[l = \varsigma(x)\,[\,],\\
&\;m = \varsigma(x)\,x.l]
\end{aligned}
$$

- *l* returns an empty object.
- *m* invokes *l* through self.

A storage cell with two methods, *contents* and *set*:

$$
\begin{aligned}
cell \;&\triangleq \\
&[contents = \varsigma(x)\,0,\\
&\;set = \varsigma(x)\,\lambda(n)\,x.contents \Lleftarrow \varsigma(y)\,n]
\end{aligned}
$$

- *contents* returns 0.
- *set* updates *contents* through self.

An Untyped Object Calculus: Reduction

- The notation $b \leadsto c$ means that b reduces to c.
- The substitution of a term c for the free occurrences of a variable x in a term b is written $b\{\!\{x{\leftarrow}c\}\!\}$, or $b\{\!\{c\}\!\}$ when x is clear from context.

$$\text{Let } o \equiv [l_i{=}\varsigma(x_i)b_i\ ^{i\in 1..n}] \qquad (l_i \text{ distinct})$$

$$\begin{array}{lcll} o.l_j & \leadsto & b_j\{\!\{x_j{\leftarrow}o\}\!\} & (j\in 1..n) \\ o.l_j{\Leftarrow}\varsigma(y)b & \leadsto & [l_j{=}\varsigma(y)b,\ l_i{=}\varsigma(x_i)b_i\ ^{i\in(1..n)-\{j\}}] & (j\in 1..n) \end{array}$$

We are dealing with a calculus of objects, not of functions.

The semantics is deterministic (Church-Rosser).
It is not imperative or concurrent.

Some Example Reductions

Let $o \triangleq [l=\varsigma(x)x.l]$ divergent method
then $o.l \leadsto x.l\{\!\{x\leftarrow o\}\!\} \equiv o.l \leadsto \ldots$

Let $o' \triangleq [l = \varsigma(x)x]$ self-returning method
then $o'.l \leadsto x\{\!\{x\leftarrow o'\}\!\} \equiv o'$

Let $o'' \triangleq [l = \varsigma(y)\,(y.l \Leftarrow \varsigma(x)x)]$ self-modifying method
then $o''.l \leadsto (o''.l \Leftarrow \varsigma(x)x) \leadsto o'$

An Imperative Untyped Object Calculus

- An object is still a collection of methods.
- Method update works by side-effect (“in-place”).
- Some new operations make sense:
  - ~ let (for controlling execution order),
  - ~ object cloning.

Syntax of the impς-calculus

a,b ::=	programs
...	(as before)
$let\ x = a\ in\ b$	let
$clone(a)$	cloning

- The semantics is given in terms of stacks and stores.

Expressiveness

- Our calculus is based entirely on methods;
  fields can be seen as methods that do not use their self parameter:

$$[\ldots, l{=}b, \ldots] \triangleq [\ldots, l{=}\varsigma(y)b, \ldots] \qquad \text{for an unused } y$$

$$o.l{:=}b \triangleq o.l \Leftarrow \varsigma(y)b \qquad \text{for an unused } y$$

- In addition, we can represent:

  ~ basic data types,

  ~ functions,

  ~ classes and subclasses.

- Method update is the most exotic construct, but:

  ~ it leads to simpler rules, and

  ~ it corresponds to features of several languages.

Some Examples

These examples are:

- easy to write in the untyped calculus,
- patently object-oriented (in a variety of styles),
- sometimes hard to type.

A Cell

Let $\quad cell \triangleq$

$$[contents = 0,$$
$$set = \varsigma(x)\ \lambda(n)\ x.contents := n]$$

Then $\quad cell.set(3)$

$$\rightsquigarrow (\lambda(n)[contents = 0,\ set = \varsigma(x)\ \lambda(n)\ x.contents := n]$$
$$.contents{:=}n)(3)$$
$$\rightsquigarrow [contents = 0,\ set = \varsigma(x)\lambda(n)\ x.contents := n]$$
$$.contents{:=}3$$
$$\rightsquigarrow [contents = 3,\ set = \varsigma(x)\ \lambda(n)\ x.contents := n]$$

and $\quad cell.set(3).contents$

$$\rightsquigarrow \ldots$$
$$\rightsquigarrow 3$$

A Cell with an Accessor

$$\text{Let} \quad gcell \triangleq$$
$$[contents = 0,$$
$$set = \varsigma(x)\,\lambda(n)\,x.contents := n,$$
$$get = \varsigma(x)\,x.contents]$$

- The *get* method fetches *contents*.
- A user of the cell may not even know about *contents*.

A Cell with Undo

$$
\begin{array}{ll}
\text{Let} & uncell \triangleq \\
& \quad [contents = 0, \\
& \quad\; set = \varsigma(x)\, \lambda(n)\, (x.undo := x).contents := n, \\
& \quad\; undo = \varsigma(x)\, x]
\end{array}
$$

- The *undo* method returns the cell before the latest call to *set*.
- The *set* method updates the *undo* method, keeping it up to date.

The code above works only if update has a functional semantics. An imperative version is:

$$
\begin{array}{ll}
\mathit{uncell} & \triangleq \\
& [\mathit{contents} = 0, \\
& \ \mathit{set} = \varsigma(x)\ \lambda(n) \\
& \qquad \mathit{let}\ y = \mathit{clone}(x)\ \mathit{in} \\
& \qquad (x.\mathit{undo} := y).\mathit{contents} := n, \\
& \ \mathit{undo} = \varsigma(x)\ x]
\end{array}
$$

Object-Oriented Booleans

true and *false* are objects with methods *if*, *then*, and *else*. Initially, *then* and *else* are set to diverge when invoked.

$$\begin{aligned} true &\triangleq [if = \varsigma(x)\, x.then,\ then = \varsigma(x)\, x.then,\ else = \varsigma(x)\, x.else] \\ false &\triangleq [if = \varsigma(x)\, x.else,\ then = \varsigma(x)\, x.then,\ else = \varsigma(x)\, x.else] \end{aligned}$$

then and *else* are updated in the conditional expression:

$$cond(b,c,d) \triangleq ((b.then{:=}c).else{:=}d).if$$

So:

$$\begin{aligned} &cond(true, false, true) \equiv ((true.then{:=}false).else{:=}true).if \\ &\rightsquigarrow ([if = \varsigma(x)\, x.then,\ then = false,\ else = \varsigma(x)\, x.else].else{:=}true).if \\ &\rightsquigarrow [if = \varsigma(x)\, x.then,\ then = false,\ else = true].if \\ &\rightsquigarrow [if = \varsigma(x)\, x.then,\ then = false,\ else = true].then \\ &\rightsquigarrow false \end{aligned}$$

Object-Oriented Natural Numbers

- Each numeral has a *case* field that contains either $\lambda(z)\lambda(s)z$ for zero, or $\lambda(z)\lambda(s)s(x)$ for non-zero, where x is the predecessor (self).

  Informally: $n.case(z)(s) = \text{if } n \text{ is } zero \text{ then } z \text{ else } s(n\text{-}1)$

- Each numeral has a *succ* method that can modify the *case* field to the non-zero version.

zero is a prototype for the other numerals:

$$\begin{aligned} zero \triangleq\; & [case = \lambda(z)\,\lambda(s)\,z, \\ & \;succ = \varsigma(x)\,x.case := \lambda(z)\,\lambda(s)\,s(x)\,] \end{aligned}$$

So:

$$\begin{array}{llll} zero & & & \equiv\; [case = \lambda(z)\,\lambda(s)\,z,\;\; succ = \ldots\,] \\ one & \triangleq & zero.succ & \equiv\; [case = \lambda(z)\,\lambda(s)\,s(zero),\;\; succ = \ldots\,] \\ pred & \triangleq & \lambda(n)\,n.case(zero)(\lambda(p)p) & \end{array}$$

A Calculator

The calculator uses method update for storing pending operations.

$$
\begin{aligned}
&\mathit{calculator} \triangleq \\
&\quad [\mathit{arg} = 0.0, \\
&\quad\ \mathit{acc} = 0.0, \\
&\quad\ \mathit{enter} = \varsigma(s)\ \lambda(n)\ s.\mathit{arg} := n, \\
&\quad\ \mathit{add} = \varsigma(s)\ (s.\mathit{acc} := s.\mathit{equals}).\mathit{equals} \Leftarrow \varsigma(s')\ s'.\mathit{acc}+s'.\mathit{arg}, \\
&\quad\ \mathit{sub} = \varsigma(s)\ (s.\mathit{acc} := s.\mathit{equals}).\mathit{equals} \Leftarrow \varsigma(s')\ s'.\mathit{acc}-s'.\mathit{arg}, \\
&\quad\ \mathit{equals} = \varsigma(s)\ s.\mathit{arg}]
\end{aligned}
$$

We obtain the following calculator-style behavior:

$$
\begin{aligned}
&\mathit{calculator}\ .\mathit{enter}(5.0)\ .\mathit{equals} = 5.0 \\
&\mathit{calculator}\ .\mathit{enter}(5.0)\ .\mathit{sub}\ .\mathit{enter}(3.5)\ .\mathit{equals} = 1.5 \\
&\mathit{calculator}\ .\mathit{enter}(5.0)\ .\mathit{add}\ .\mathit{add}\ .\mathit{equals} = 15.0
\end{aligned}
$$

Functions as Objects

A function is an object with two slots:

- ~ one for the argument (initially undefined),
- ~ one for the function code.

Translation of the untyped λ-calculus

$$\langle\!\langle x \rangle\!\rangle \triangleq x$$

$$\langle\!\langle \lambda(x)b \rangle\!\rangle \triangleq$$

$$[arg = \varsigma(x)\ x.arg,$$

$$val = \varsigma(x)\ \langle\!\langle b \rangle\!\rangle\{\!\{x \leftarrow x.arg\}\!\}]$$

$$\langle\!\langle b(a) \rangle\!\rangle \triangleq (\langle\!\langle b \rangle\!\rangle.arg := \langle\!\langle a \rangle\!\rangle).val$$

Self variables get statically nested. A keyword **self** would not suffice.

The translation validates the β rule:

$$\langle\!\langle(\lambda(x)b)(a)\rangle\!\rangle \;\;\rightsquigarrow\;\; \langle\!\langle b\{\!\{x \leftarrow a\}\!\}\rangle\!\rangle$$

For example:

$$\begin{aligned}
\langle\!\langle(\lambda(x)x)(y)\rangle\!\rangle \;\;&\triangleq\;\; ([arg = \varsigma(x)\ x.arg,\ val = \varsigma(x)\ x.arg].arg := y).val \\
&\rightsquigarrow [arg = \varsigma(x)\ y,\ val = \varsigma(x)\ x.arg].val \\
&\rightsquigarrow [arg = \varsigma(x)\ y,\ val = \varsigma(x)\ x.arg].arg \\
&\rightsquigarrow y \\
&\triangleq\;\; \langle\!\langle y\rangle\!\rangle
\end{aligned}$$

The translation has typed and imperative variants.

Procedures as Imperative Objects

Translation of an imperative λ-calculus

$$
\begin{aligned}
&\langle\!\langle x \rangle\!\rangle \ \triangleq\ x\\
&\langle\!\langle x := a \rangle\!\rangle \ \triangleq\\
&\qquad let\ y = \langle\!\langle a \rangle\!\rangle\\
&\qquad in\ x.arg := y\\
&\langle\!\langle \lambda(x)b \rangle\!\rangle \ \triangleq\\
&\qquad [arg = \varsigma(x)\ x.arg,\\
&\qquad\ val = \varsigma(x)\ \langle\!\langle b \rangle\!\rangle\{\!\{x \leftarrow x.arg\}\!\}]\\
&\langle\!\langle b(a) \rangle\!\rangle \ \triangleq\\
&\qquad let\ f = clone(\langle\!\langle b \rangle\!\rangle)\\
&\qquad in\ let\ y = \langle\!\langle a \rangle\!\rangle\\
&\qquad in\ (f.arg := y).val
\end{aligned}
$$

Cloning on application corresponds to allocating a new stack frame.

Classes

A class is an object with:

- a *new* method, for generating new objects,
- code for methods for the objects generated from the class.

For generating the object:

$$o \triangleq [l_i = \varsigma(x_i)\ b_i\ ^{i \in 1..n}]$$

we use the class:

$$\begin{aligned} c \triangleq &\\ &[new = \varsigma(z)\ [l_i = \varsigma(x)\ z.l_i(x)\ ^{i \in 1..n}],\\ &\ l_i = \lambda(x_i)\ b_i\ ^{i \in 1..n}] \end{aligned}$$

The method *new* is a **generator**. The call *c*.*new* yields *o*.

Each field l_i is a **pre-method**.

A Class for Cells

$$\begin{aligned}
&\mathit{cellClass} \triangleq \\
&\quad [\mathit{new} = \varsigma(z) \\
&\qquad [\mathit{contents} = \varsigma(x)\ z.\mathit{contents}(x),\ \mathit{set} = \varsigma(x)\ z.\mathit{set}(x)], \\
&\quad\ \mathit{contents} = \lambda(x)\ 0, \\
&\quad\ \mathit{set} = \lambda(x)\ \lambda(n)\ x.\mathit{contents} := n]
\end{aligned}$$

Writing the *new* method is tedious but straightforward.

Writing the pre-methods is like writing the corresponding methods.

$\mathit{cellClass}.\mathit{new}$ yields a standard cell:

$$[\mathit{contents} = 0,\ \mathit{set} = \varsigma(x)\ \lambda(n)\ x.\mathit{contents} := n]$$

Inheritance

Inheritance is the reuse of pre-methods.

Given a class c with pre-methods $c.l_i\ ^{i\in 1..n}$
we may define a new class c':

$$c' \triangleq [new=\ldots,\ l_i=c.l_i\ ^{i\in 1..n},\ l_j=\ldots\ ^{j\in n+1..m}]$$

We may say that c' is a subclass of c.

Inheritance for Cells

$$
\begin{aligned}
&\mathit{cellClass} \triangleq \\
&\qquad [\mathit{new} = \varsigma(z) \\
&\qquad\qquad [\mathit{contents} = \varsigma(x)\ z.\mathit{contents}(x),\ \mathit{set} = \varsigma(x)\ z.\mathit{set}(x)], \\
&\qquad\ \mathit{contents} = \lambda(x)\ 0, \\
&\qquad\ \mathit{set} = \lambda(x)\ \lambda(n)\ x.\mathit{contents} := n]
\end{aligned}
$$

$$
\begin{aligned}
&\mathit{uncellClass} \triangleq \\
&\qquad [\mathit{new} = \varsigma(z)\ [\ldots], \\
&\qquad\ \mathit{contents} = \mathit{cellClass}.\mathit{contents}, \\
&\qquad\ \mathit{set} = \lambda(x)\ \mathit{cellClass}.\mathit{set}(x.\mathit{undo} := x), \\
&\qquad\ \mathit{undo} = \lambda(x)\ x]
\end{aligned}
$$

- The pre-method *contents* is inherited.
- The pre-method *set* is overridden, though using a call to **super**.
- The pre-method *undo* is added.

Object Types and Subtyping

An **object type** is a set of method names and of result types:

$$[l_i{:}B_i \; ^{i\in 1..n}]$$

An object has type $[l_i{:}B_i \; ^{i\in 1..n}]$ if it has at least the methods $l_i^{i\in 1..n}$, with a self parameter of some type $A <: [l_i{:}B_i \; ^{i\in 1..n}]$ and a result of type B_i, e.g., $[]$ and $[l_1 : [], l_2 : []]$.

An object type with more methods is a **subtype** of one with fewer:

$$[l_i{:}B_i \; ^{i\in 1..n+m}] \quad <: \quad [l_i{:}B_i \; ^{i\in 1..n}]$$

A longer object can be used instead of a shorter one by **subsumption**:

$$a{:}A \quad \wedge \quad A{<:}B \quad \Rightarrow \quad a{:}B$$

A First-Order Calculus

Environments:

$$E \equiv x_i{:}A_i \; {}^{i \in 1..n}$$

Judgments:

$E \vdash \diamond$	environment E is well-formed
$E \vdash A$	A is a type in E
$E \vdash A <: B$	A is a subtype of B in E
$E \vdash a : A$	a has type A in E

Types:

$A,B \; ::= \; Top$	the biggest type
$\quad [l_i{:}B_i \; {}^{i \in 1..n}]$	object type

Terms: as for the untyped calculus (but with types for variables).

First-order type rules for the ς-calculus: rules for objects

(Type Object) (l_i distinct)

$$\frac{E \vdash B_i \qquad \forall i \in 1..n}{E \vdash [l_i{:}B_i\ ^{i\in 1..n}]}$$

(Sub Object) (l_i distinct)

$$\frac{E \vdash B_i \qquad \forall i \in 1..n{+}m}{E \vdash [l_i{:}B_i\ ^{i\in 1..n+m}] <: [l_i{:}B_i\ ^{i\in 1..n}]}$$

(Val Object) (where $A \equiv [l_i{:}B_i\ ^{i\in 1..n}]$)

$$\frac{E, x_i{:}A \vdash b_i : B_i \qquad \forall i \in 1..n}{E \vdash [l_i{=}\varsigma(x_i{:}A)b_i\ ^{i\in 1..n}] : A}$$

(Val Select)

$$\frac{E \vdash a : [l_i{:}B_i\ ^{i\in 1..n}] \qquad j \in 1..n}{E \vdash a.l_j : B_j}$$

(Val Update) (where $A \equiv [l_i{:}B_i\ ^{i\in 1..n}]$)

$$\frac{E \vdash a : A \qquad E, x{:}A \vdash b : B_j \qquad j \in 1..n}{E \vdash a.l_j \Leftarrow \varsigma(x{:}A)b : A}$$

(Val Clone) (where $A \equiv [l_i{:}B_i\ ^{i\in 1..n}]$)

$$\frac{E \vdash a : A}{E \vdash \mathit{clone}(a) : A}$$

First-order type rules for the ς-calculus: standard rules

(Env ∅)

$$\frac{}{\emptyset \vdash \diamond}$$

(Env *x*)

$$\frac{E \vdash A \qquad x \notin dom(E)}{E,x{:}A \vdash \diamond}$$

(Val *x*)

$$\frac{E',x{:}A,E'' \vdash \diamond}{E',x{:}A,E'' \vdash x{:}A}$$

(Sub Refl)

$$\frac{E \vdash A}{E \vdash A <: A}$$

(Sub Trans)

$$\frac{E \vdash A <: B \qquad E \vdash B <: C}{E \vdash A <: C}$$

(Val Subsumption)

$$\frac{E \vdash a : A \qquad E \vdash A <: B}{E \vdash a : B}$$

(Type Top)

$$\frac{E \vdash \diamond}{E \vdash Top}$$

(Sub Top)

$$\frac{E \vdash A}{E \vdash A <: Top}$$

(Val Let)

$$\frac{E \vdash a : A \qquad E,\ x{:}A \vdash b : B}{E \vdash let\ x{=}a\ in\ b : B}$$

Some Results (for the Functional Calculus)

Each well-typed term has a minimum type:

Theorem (Minimum types)

If $E \vdash a : A$ then there exists B such that $E \vdash a : B$ and,
for any A', if $E \vdash a : A'$ then $E \vdash B <: A'$.

The type system is sound for the operational semantics:

Theorem (Subject reduction)

If $\emptyset \vdash a : C$
and a reduces to v
then $\emptyset \vdash v : C$.

Unsoundness of Covariance

Object types are **invariant** (not co/contravariant) in components.

$U \triangleq []$ The unit object type.

$L \triangleq [l{:}U]$ An object type with just l.

$L <: U$

$P \triangleq [x{:}U, f{:}U]$

$Q \triangleq [x{:}L, f{:}U]$

Assume $Q <: P$ by an (erroneous) covariant rule.

$q : Q \triangleq [x = [l{=}[]], f = \varsigma(s{:}Q)\, s.x.l]$

then $q : P$ by subsumption with $Q <: P$

hence $q.x{:=}[] : P$ that is $[x = [], f = \varsigma(s{:}Q)\, s.x.l] : P$

But $(q.x{:=}[]).f$ fails!

Typed Cells

- We assume an imperative semantics (in order to postpone the use of recursive types).
- If *set* works by side-effect, its result type can be uninformative. (We can write $x.set(3)$; $x.contents$ instead of $x.set(3).contents$.)

Assuming a type *Nat* and function types, we let:

$$\begin{aligned} Cell &\triangleq [contents : Nat,\ set : Nat \to [\,]] \\ GCell &\triangleq [contents : Nat,\ set : Nat \to [\,],\ get : Nat] \end{aligned}$$

We get:

$$GCell <: Cell$$

$$cell \triangleq [contents = 0,\ set = \varsigma(x{:}Cell)\ \lambda(n{:}Nat)\ x.contents := n]$$

has type *Cell*

$$gcell \triangleq [\ldots,\ get = \varsigma(x{:}GCell)\ x.contents]$$

has types *GCell* and *Cell*

Classes, with Types

If $A \equiv [l_i{:}B_i\ ^{i\in 1..n}]$ is an object type,
then $Class(A)$ is the type of the classes for objects of type A:

$$Class(A) \triangleq [new{:}A,\ l_i{:}A\rightarrow B_i\ ^{i\in 1..n}]$$

$new{:}A$ is a **generator** for objects of type A.

$l_i{:}A\rightarrow B_i$ is a **pre-method** for objects of type A.

$$c : Class(A) \triangleq$$
$$[new = \varsigma(z{:}Class(A))\,[l_i = \varsigma(x{:}A)\ z.l_i(x)\ ^{i\in 1..n}],$$
$$l_i = \lambda(x_i{:}A)\ b_i\{x_i\}\ ^{i\in 1..n}]$$
$$c.new : A$$

- Types are distinct from classes.
- More than one class may generate objects of a type.

Inheritance, with Types

Let $A \equiv [l_i{:}B_i{}^{i\in 1..n}]$ and $A' \equiv [l_i{:}B_i{}^{i\in 1..n}, l_j{:}B_j{}^{j\in n+1..m}]$, with $A' <: A$.

Note that $Class(A)$ and $Class(A')$ are not related by subtyping.

Let c: $Class(A)$, then for $i \in 1..n$

$$c.l_i : A \rightarrow B_i <: A' \rightarrow B_i.$$

Hence $c.l_i$ is a good pre-method for a class of type $Class(A')$.

We may define a subclass c' of c:

$$c' : Class(A') \quad \triangleq \quad [new = \ldots, l_i = c.l_i{}^{i\in 1..n}, l_j = \ldots{}^{j\in n+1..m}]$$

where class c' inherits the methods l_i from class c.

So inheritance typechecks:

If $A' <: A$ then a class for A' may inherit from a class for A.

Class Types for Cells

$$
\begin{aligned}
&\mathit{Class}(\mathit{Cell}) \quad \triangleq \\
&\qquad [\mathit{new} : \mathit{Cell}, \\
&\qquad\ \mathit{contents} : \mathit{Cell} \rightarrow \mathit{Nat}, \\
&\qquad\ \mathit{set} : \mathit{Cell} \rightarrow \mathit{Nat} \rightarrow [\,]]
\end{aligned}
$$

$$
\begin{aligned}
&\mathit{Class}(\mathit{GCell}) \quad \triangleq \\
&\qquad [\mathit{new} : \mathit{GCell}, \\
&\qquad\ \mathit{contents} : \mathit{GCell} \rightarrow \mathit{Nat}, \\
&\qquad\ \mathit{set} : \mathit{GCell} \rightarrow \mathit{Nat} \rightarrow [\,], \\
&\qquad\ \mathit{get} : \mathit{GCell} \rightarrow \mathit{Nat}]
\end{aligned}
$$

$\mathit{Class}(\mathit{Cell})$ and $\mathit{Class}(\mathit{GCell})$ are not related by subtyping, but inheritance is possible.

Variance Annotations

In order to gain expressiveness within a first-order setting, we extend the syntax of object types with variance annotations:

$$[l_i \upsilon_i{:}B_i\ ^{i \in 1..n}]$$

Each υ_i is a variance annotation; it is one of three symbols ^{o}, $^+$, and $^-$.

Intuitively,

- $^+$ means read-only: it prevents update, but allows covariant component subtyping;
- $^-$ means write-only: it prevents invocation, but allows contravariant component subtyping;
- ^{o} means read-write: it allows both invocation and update, but requires exact matching in subtyping.

By convention, any omitted annotations are taken to be equal to ^{o}.

Subtyping with Variance Annotations

$[\ldots\ l^{o}{:}B\ \ldots] <: [\ldots\ l^{o}{:}B'\ \ldots]$ if $B \equiv B'$	invariant (read-write)
$[\ldots\ l^{+}{:}B\ \ldots] <: [\ldots\ l^{+}{:}B'\ \ldots]$ if $B <: B'$	covariant (read-only)
$[\ldots\ l^{-}{:}B\ \ldots] <: [\ldots\ l^{-}{:}B'\ \ldots]$ if $B' <: B$	contravariant (write-only)
$[\ldots\ l^{o}{:}B\ \ldots] <: [\ldots\ l^{+}{:}B'\ \ldots]$ if $B <: B'$ $[\ldots\ l^{o}{:}B\ \ldots] <: [\ldots\ l^{-}{:}B'\ \ldots]$ if $B' <: B$	invariant <: variant

Protection by Subtyping

- Variance annotations can provide protection against updates from the outside.
- In addition, object components can be hidden by subsumption.

For example:

$$\begin{array}{ll}
\text{Let} & GCell \triangleq [contents : Nat, set : Nat \rightarrow [\,], get : Nat] \\
 & PGCell \triangleq [set : Nat \rightarrow [\,], get : Nat] \\
 & ProtectedGCell \triangleq [set^{+} : Nat \rightarrow [\,], get^{+} : Nat] \\
 & gcell : GCell \\
\text{then} & GCell \mathrel{<:} PGCell \mathrel{<:} ProtectedGCell \\
\text{so} & gcell : ProtectedGCell.
\end{array}$$

Given a *ProtectedGCell*, one cannot access its *contents* directly.

From the inside, *set* and *get* can still update and read *contents*.

Encoding Function Types

An invariant translation of function types:

$$\langle\!\langle A \rightarrow B \rangle\!\rangle \quad \triangleq \quad [\mathit{arg} : \langle\!\langle A \rangle\!\rangle,\ \mathit{val} : \langle\!\langle B \rangle\!\rangle]$$

A covariant / contravariant translation, using annotations:

$$\langle\!\langle A \rightarrow B \rangle\!\rangle \quad \triangleq \quad [\mathit{arg}^{-} : \langle\!\langle A \rangle\!\rangle,\ \mathit{val}^{+} : \langle\!\langle B \rangle\!\rangle]$$

A covariant / contravariant translation, using quantifiers:

$$\langle\!\langle A \rightarrow B \rangle\!\rangle \quad \triangleq \quad \forall (X <: \langle\!\langle A \rangle\!\rangle)\, \exists (Y <: \langle\!\langle B \rangle\!\rangle)\, [\mathit{arg} : X,\ \mathit{val} : Y]$$

where $\forall$ is for polymorphism and $\exists$ is for data abstraction.

Recursive Types

Informally, we may want to define a recursive type as in:

$$Cell \quad \triangleq \quad [contents : Nat,\ set : Nat \to Cell]$$

Formally, we write instead:

$$Cell \quad \triangleq \quad \mu(X)[contents : Nat,\ set : Nat \to X]$$

Intuitively, $\mu(X)A\{X\}$ is the solution for the equation $X = A\{X\}$.

Subtyping Recursive Types

The basic subtyping rule for recursive types is:

$\mu(X)A\{X\} <: \mu(X)B\{X\}$
if
either $A\{X\}$ and $B\{X\}$ are equal for all X
or $A\{X\} <: B\{\!\{Y\}\!\}$ for all X and Y such that $X <: Y$

There are variants, for example:

$\mu(X)A\{X\} <: \mu(X)B\{X\}$
if
either $A\{X\}$ and $B\{X\}$ are equal for all X
or $A\{X\} <: B\{\!\{\mu(X)B\{X\}\}\!\}$ for all X such that $X <: \mu(X)B\{X\}$

But $A\{X\} <: B\{X\}$ does not imply $\mu(X)A\{X\} <: \mu(X)B\{X\}$.

Cells (with Recursive Types)

Let $\quad Cell \triangleq [contents : Nat, set : Nat \rightarrow Cell]$

$cell : Cell \triangleq$

$\quad [contents = 0,$

$\quad\ set = \varsigma(x{:}Cell)\ \lambda(n{:}Nat)\ x.contents := n]$

The type *Cell* is a recursive type.

Now we can typecheck $cell.set(3).contents$.

Because of the recursion, we do not get interesting subtypings.

Let $\quad GCell \triangleq [contents : Nat, set : Nat \rightarrow GCell, get : Nat]$

then $\quad$ *GCell* is not a subtype of *Cell*.

The fact that *GCell* is not a subtype of *Cell* is unacceptable, but necessary for soundness.

Consider the following correct but somewhat strange *GCell*:

$$
\begin{aligned}
&gcell' : GCell \triangleq \\
&\qquad [contents = \varsigma(x{:}Cell)\ x.set(x.get).get, \\
&\qquad\ set = \varsigma(x{:}Cell)\ \lambda(n{:}Nat)\ x.get := n, \\
&\qquad\ get = 0]
\end{aligned}
$$

If *GCell* were a subtype of *Cell* then we would have:

$$
\begin{aligned}
&gcell' : Cell \\
&gcell'' : Cell \triangleq (gcell'.set := \lambda(n{:}Nat)\ cell)
\end{aligned}
$$

where *cell* is a fixed element of *Cell*, without a *get* method.
Then we can write:

$$m : Nat \triangleq gcell''.contents$$

But the computation of *m* yields a “message not understood” error.

Five Solutions (Overview)

- Avoid methods specialization, redefining *GCell*:

$$
\begin{aligned}
Cell &\triangleq [contents : Nat,\ set : Nat \rightarrow Cell] \\
GCell &\triangleq [contents : Nat,\ set : Nat \rightarrow Cell,\ get : Nat]
\end{aligned}
$$

~ This is a frequent approach in common languages.

~ It requires dynamic type tests after calls to the *set* method. *E.g.*,

typecase *gcell.set*(3)
when (*x*:*GCell*) *x.get*
else ...

- Add variance annotations:

$$\begin{aligned} Cell &\triangleq [contents : Nat,\ set^{+} : Nat \to Cell] \\ GCell &\triangleq [contents : Nat,\ set^{+} : Nat \to GCell,\ get : Nat] \end{aligned}$$

~ This approach yields the desired subtypings.

~ But it forbids even sound updates of the *set* method.

~ It would require reconsidering the treatment of classes in order to support inheritance of the *set* method.

- Go back to an imperative framework, where the typing problem disappears because the result type of *set* is [].

$$\begin{aligned} Cell &\triangleq [contents : Nat, set : Nat \to []] \\ GCell &\triangleq [contents : Nat, set : Nat \to [], get : Nat] \end{aligned}$$

  ~ This works sometimes.

  ~ But methods that allocate a new object of the type of self still call for the use of recursive types:

$$UnCell \triangleq [contents : Nat, set : Nat \to [], undo : UnCell]$$

- Axiomatize some notion of Self types, and write:

$$\begin{aligned} Cell &\triangleq [contents : Nat, set : Nat \to Self] \\ GCell &\triangleq [contents : Nat, set : Nat \to Self, get : Nat] \end{aligned}$$

~ But the rules for Self types are not trivial or obvious.

- Move up to higher-order calculi, and see what can be done there.

$$Cell \triangleq \exists(Y{<:}Cell)\,[contents : Nat,\ set : Nat \to Y]$$
$$GCell \triangleq \exists(Y{<:}GCell)\,[contents : Nat,\ set : Nat \to Y,\ get : Nat]$$

  - ~ The existential quantifiers yield covariance, so *GCell* <: *Cell*.
  - ~ Intuitively, the existentially quantified type is the type of self: the Self type.
  - ~ This technique is general, and suggests sound rules for primitive Self types.

We obtain:

- ~ subtyping with methods that return self,
- ~ inheritance for methods that return self or that take arguments of the type of self (“binary methods”), but without subtyping.

Typed Reasoning

In addition to a type theory, we have a simple typed proof system.

There are some subtleties in reasoning about objects.

Consider:

$$\begin{aligned} A &\triangleq [x : Nat, f : Nat] \\ a : A &\triangleq [x = 1, f = 1] \\ b : A &\triangleq [x = 1, f = \varsigma(s{:}A)\, s.x] \end{aligned}$$

Informally, we may say that $a.x = b.x : Nat$ and $a.f = b.f : Nat$.

So, do we have $a = b$?

It would follow that $(a.x{:=}2).f = (b.x{:=}2).f$

and then $1 = 2$.

Hence:

a ¦ $b : A$

Still, as objects of $[x : Nat]$, a and b are indistinguishable from $[x = 1]$.

Hence:

$$a = b : [x : Nat]$$

Finally, we may ask:

$$a \overset{?}{=} b : [f : Nat]$$

This is sound; it can be proved via bisimilarity.

In summary, there is a notion of typed equality that may support some interesting transformations (inlining of methods).

(Work in progress:
specification and verification for a typed object-oriented language.)

Conclusions

Object calculi are both simple and expressive.

- Functions vs. objects:
  - ~ Functions can be translated into objects.
    Therefore, pure object-based languages are at least as expressive as procedural languages.
    (Despite all the Smalltalk philosophy, to our knowledge nobody had proved that one can build functions from objects.)
  - ~ Conversely, using sophisticated type systems, it is possible to translate objects into functions.
    (But this translation is difficult and not practical.)

- Classes vs. objects:
  - ~ Classes can be encoded in object calculi, easily and faithfully. Therefore, object-based languages are just as expressive as class-based ones.
    (To our knowledge, nobody had shown that one can build type-correct classes out of objects.)
  - ~ Method update, a distinctly object-based construct, is tractable and can be useful.

Interpretation of Object-Oriented Languages

A FIRST-ORDER LANGUAGE

- Let’s assess the contributions that object calculi bring to the task of modeling programming language constructs.
- For this purpose, we study a simple object-oriented language named O–1.
- We have studied more advanced languages that include Self types and matching.

Features of O–1

- Both class-based and object-based constructs.
- First-order object types with subtyping and variance annotations.
- Classes with single inheritance; method overridding and specialization.
- Recursion.
- Typecase.
- Separation interfaces from implementations, and inheritance from subtyping.

Syntax

Syntax of O–1 types

$A,B ::=$	types
X	type variable
$\mathbf{Top}$	the biggest type
$\mathbf{Object}(X)[l_i\upsilon_i{:}B_i\ {}^{i\in 1..n}]$	object type (l_i distinct)
$\mathbf{Class}(A)$	class type

Syntax of O–1 terms

a,b,c ::=	terms
x	variable
object$(x{:}A)\ l_i{=}b_i\ ^{i\in 1..n}$ **end**	direct object construction
$a.l$	field selection / method invocation
$a.l := b$	update with a term
$a.l :=$ **method**$(x{:}A)\ b$ **end**	update with a method
new c	object construction from a class
root	root class
subclass of $c{:}C$ **with**$(x{:}A)$	subclass
$l_i{=}b_i\ ^{i\in n+1..n+m}$	additional attributes
override $l_i{=}b_i\ ^{i\in Ovr\subseteq 1..n}$ **end**	overridden attributes
$c\wedge l(a)$	class selection
typecase a **when** $(x{:}A)b_1$ **else** b_2 **end**	typecase

- We could drop the object-based constructs (object construction and method update). The result would be a language expressive enough for traditional class-based programming.
- Alternatively, we could drop the class-based construct (root class, subclass, new, and class selection), obtaining an object-based language.
- Classes, as well as objects, are first-class values. A parametric class can be obtained as a function that returns a class.

Abbreviations

Root $\triangleq$
Class(**Object**(X)[])

class with$(x{:}A)$ $l_i{=}b_i\ ^{i\in 1..n}$ **end** $\triangleq$
subclass of root:**Root with**$(x{:}A)$ $l_i{=}b_i\ ^{i\in 1..n}$ **override end**

subclass of $c{:}C$ **with** $(x{:}A)$ … **super**.l … **end** $\triangleq$
subclass of $c{:}C$ **with** $(x{:}A)$ … $c\wedge l(x)$ … **end**

object$(x{:}A)$ … l **copied from** c … **end** $\triangleq$
object$(x{:}A)$ … $l{=}c\wedge l(x)$ … **end**

Examples

- We assume basic types (*Bool*, *Int*) and function types ($A \rightarrow B$, contravariant in A and covariant in B).

$$Point \triangleq \mathbf{Object}(X)[x\colon Int,\ eq^{+}\colon X \rightarrow Bool,\ mv^{+}\colon Int \rightarrow X]$$

$$CPoint \triangleq \mathbf{Object}(X)[x\colon Int,\ c\colon Color,\ eq^{+}\colon Point \rightarrow Bool,\ mv^{+}\colon Int \rightarrow Point]$$

- $CPoint <: Point$
- The type of *mv* in *CPoint* is $Int \rightarrow Point$.
  One can explore the effect of changing it to $Int \rightarrow X$.
- The type of *eq* in *CPoint* is $Point \rightarrow Bool$.
  If we were to change it to $X \rightarrow Bool$ we would lose the subtyping $CPoint <: Point$.

Class(Point)

```
pointClass : Class(Point)  ≜
    class with (self: Point)
        x = 0,
        eq = fun(other: Point) self.x = other.x end,
        mv = fun(dx: Int) self.x := self.x+dx end
    end
```

Class(CPoint)

```
cPointClass : Class(CPoint)  ≜
    subclass of pointClass: Class(Point)
    with (self: CPoint)
        c = black
    override
        eq = fun(other: Point)
                typecase other
                when (other’: CPoint) super.eq(other’) and self.c = other’.c
                else false
                end
            end
    end
```

Comments

- The class *cPointClass* inherits *x* and *mv* from its superclass *pointClass*.
- Although it could inherit *eq* as well, *cPointClass* overrides this method as follows.
  - ~ The definition of *Point* requires that *eq* work with any argument *other* of type *Point*.
  - ~ In the *eq* code for *cPointClass*, the typecase on *other* determines whether *other* has a color.
  - ~ If so, *eq* works as in *pointClass* and in addition tests the color of *other*.
  - ~ If not, *eq* returns *false*.

- We can use *cPointClass* to create color points of type *CPoint*:

$$cPoint : CPoint \triangleq \textbf{new}\ cPointClass$$

- Calls to *mv* lose the color information.
- In order to access the color of a point after it has been moved, a typecase is necessary:

```
movedColor : Color  ≜
    typecase cPoint.mv(1)
    when (cp: CPoint) cp.c
    else black
    end
```

Typing

- The rules of O–1 are based on the following judgments:

Judgments

$E \vdash \diamond$	environment E is well-formed
$E \vdash A$	A is a well-formed type in E
$E \vdash A <: B$	A is a subtype of B in E
$E \vdash \upsilon A <: \upsilon' B$	A is a subtype of B in E, with variance annotations υ and υ'
$E \vdash a : A$	a has type A in E

- The rules for environments are standard:

Environments

(Env $\emptyset$)

$$\frac{}{\emptyset \vdash \diamond}$$

(Env X<:)

$$\frac{E \vdash A \qquad X \notin dom(E)}{E, X{<:}A \vdash \diamond}$$

(Env x)

$$\frac{E \vdash A \qquad x \notin dom(E)}{E, x{:}A \vdash \diamond}$$

Type Formation Rules

Types

(Type X)

$$\frac{E', X{<:}A, E'' \vdash \diamond}{E', X{<:}A, E'' \vdash X}$$

(Type Top)

$$\frac{E \vdash \diamond}{E \vdash \mathbf{Top}}$$

(Type Object) (l_i distinct, $\upsilon_i \in \{^{o}, ^{-}, ^{+}\}$)

$$\frac{E, X{<:}\mathbf{Top} \vdash B_i \qquad \forall i \in 1..n}{E \vdash \mathbf{Object}(X)[l_i \upsilon_i{:}B_i \ ^{i \in 1..n}]}$$

(Type Class) (where $A \equiv \mathbf{Object}(X)[l_i \upsilon_i{:}B_i\{X\} \ ^{i \in 1..n}]$)

$$\frac{E \vdash A}{E \vdash \mathbf{Class}(A)}$$

Subtyping Rules

- Note that there is no rule for subtyping class types.

Subtyping

(Sub Refl)

$$\frac{E \vdash A}{E \vdash A <: A}$$

(Sub Trans)

$$\frac{E \vdash A <: B \qquad E \vdash B <: C}{E \vdash A <: C}$$

(Sub X)

$$\frac{E', X{<:}A, E'' \vdash \diamond}{E', X{<:}A, E'' \vdash X <: A}$$

(Sub Top)

$$\frac{E \vdash A}{E \vdash A <: \mathbf{Top}}$$

(Sub Object) (where $A \equiv \mathbf{Object}(X)[l_i \upsilon_i{:}B_i\{X\}^{\,i \in 1..n+m}]$, $A' \equiv \mathbf{Object}(X')[l_i \upsilon_i'{:}B_i'\{X'\}^{\,i \in 1..n}]$)

$$\frac{E \vdash A \qquad E \vdash A' \qquad E, X{<:}A' \vdash \upsilon_i\, B_i\{X\} <: \upsilon_i'\, B_i'\{\!\{A'\}\!\} \qquad \forall\, i \in 1..n}{E \vdash A <: A'}$$

(Sub Invariant)

$$\frac{E \vdash B}{E \vdash {}^{o}B <: {}^{o}B}$$

(Sub Covariant)

$$\frac{E \vdash B <: B' \qquad \upsilon \in \{{}^{o}, {}^{+}\}}{E \vdash \upsilon\, B <: {}^{+}B'}$$

(Sub Contravariant)

$$\frac{E \vdash B' <: B \qquad \upsilon \in \{{}^{o}, {}^{-}\}}{E \vdash \upsilon\, B <: {}^{-}B'}$$

Term Typing Rules

Terms

(Val Subsumption)

$$\frac{E \vdash a : A \qquad E \vdash A <: B}{E \vdash a : B}$$

(Val x)

$$\frac{E', x{:}A, E'' \vdash \diamond}{E', x{:}A, E'' \vdash x : A}$$

(Val Object) (where $A \equiv \textbf{Object}(X)[l_i \upsilon_i{:}B_i\{X\}^{\,i\in 1..n}]$)

$$\frac{E, x{:}A \vdash b_i : B_i\{\!\{A\}\!\} \qquad \forall i\in 1..n}{E \vdash \textbf{object}(x{:}A)\ l_i{=}b_i^{\,i\in 1..n}\ \textbf{end} : A}$$

(Val Select) (where $A \equiv \mathbf{Object}(X)[l_i \upsilon_i : B_i\{X\}\ ^{i \in 1..n}]$)

$$\frac{E \vdash a : A \qquad \upsilon_j \in \{^{\circ}, ^{+}\} \qquad j \in 1..n}{E \vdash a.l_j : B_j\{\!\!\{A\}\!\!\}}$$

(Val Update) (where $A \equiv \mathbf{Object}(X)[l_i \upsilon_i : B_i\{X\}\ ^{i \in 1..n}]$)

$$\frac{E \vdash a : A \qquad E \vdash b : B_j\{\!\!\{A\}\!\!\} \qquad \upsilon_j \in \{^{\circ}, ^{-}\} \qquad j \in 1..n}{E \vdash a.l_j := b : A}$$

(Val Method Update) (where $A \equiv \mathbf{Object}(X)[l_i \upsilon_i : B_i\{X\}\ ^{i \in 1..n}]$)

$$\frac{E \vdash a : A \qquad E, x{:}A \vdash b : B_j\{\!\!\{A\}\!\!\} \qquad \upsilon_j \in \{^{\circ}, ^{-}\} \qquad j \in 1..n}{E \vdash a.l_j := \mathbf{method}(x{:}A)b\ \mathbf{end} : A}$$

(Val New)

$$\frac{E \vdash c : \mathbf{Class}(A)}{E \vdash \mathbf{new}\ c : A}$$

(Val Root)

$$\frac{E \vdash \diamond}{E \vdash \mathbf{root} : \mathbf{Class}(\mathbf{Object}(X)[\,])}$$

(Val Subclass) (where $A \equiv \mathbf{Object}(X)[l_i \upsilon_i{:}B_i\{X\}^{\,i \in 1..n+m}]$, $A' \equiv \mathbf{Object}(X')[l_i \upsilon_i'{:}B_i'\{X'\}^{\,i \in 1..n}]$, $Ovr \subseteq 1..n$)

$$\frac{\begin{array}{c} E \vdash c' : \mathbf{Class}(A') \qquad E \vdash A <: A' \\ E \vdash B_i'\{\!\{A'\}\!\} <: B_i\{\!\{A\}\!\} \qquad \forall i \in 1..n-Ovr \\ E, x{:}A \vdash b_i : B_i\{\!\{A\}\!\} \qquad \forall i \in Ovr \cup n+1..n+m \end{array}}{\begin{array}{c} E \vdash \mathbf{subclass\ of}\ c'{:}\mathbf{Class}(A')\ \mathbf{with}(x{:}A)\ l_i{=}b_i^{\,i \in n+1..n+m}\ \mathbf{override}\ l_i{=}b_i^{\,i \in Ovr}\ \mathbf{end} \\ : \mathbf{Class}(A) \end{array}}$$

(Val Class Select) (where $A \equiv \mathbf{Object}(X)[l_i \upsilon_i{:}B_i\{X\}^{\,i\in 1..n}]$)

$$\frac{E \vdash a : A \qquad E \vdash c : \mathbf{Class}(A) \qquad j\in 1..n}{E \vdash c\hat{\ }l_j(a) : B_j\{\!\{A\}\!\}}$$

(Val Typecase)

$$\frac{E \vdash a : A' \qquad E, x{:}A \vdash b_1 : D \qquad E \vdash b_2 : D}{E \vdash \mathbf{typecase}\ a\ \mathbf{when}\ (x{:}A)b_1\ \mathbf{else}\ b_2\ \mathbf{end} : D}$$

- These rules are hard to read and understand.
- But they are the ultimate truth about typing in O–1.

Translation

- We give a translation into a functional calculus (with all the features described earlier).
- A similar translation could be given into an appropriate imperative calculus.
- At the level of types, the translation is simple.
  - ~ We write $\langle\!\langle A \rangle\!\rangle$ for the translation of A.
  - ~ We map an object type **Object**$(X)[l_i \upsilon_i{:}B_i\ ^{i\in 1..n}]$ to a recursive object type $\mu(X)[l_i \upsilon_i{:}\langle\!\langle B_i \rangle\!\rangle\ ^{i\in 1..n}]$.
  - ~ We map a class type **Class**(**Object**$(X)[l_i \upsilon_i{:}B_i\{X\}\ ^{i\in 1..n}]$) to an object type that contains components for pre-methods and a *new* component.

Translation of Types

Translation of O–1 types

$$\langle\!\langle X \rangle\!\rangle \;\triangleq\; X$$

$$\langle\!\langle \mathbf{Top} \rangle\!\rangle \;\triangleq\; \mathit{Top}$$

$$\langle\!\langle \mathbf{Object}(X)[l_i \upsilon_i{:}B_i \;^{i\in 1..n}] \rangle\!\rangle \;\triangleq\; \mu(X)[l_i \upsilon_i{:}\langle\!\langle B_i \rangle\!\rangle \;^{i\in 1..n}]$$

$$\langle\!\langle \mathbf{Class}(A) \rangle\!\rangle \;\triangleq\; [\mathit{new}^{+}{:}\langle\!\langle A \rangle\!\rangle,\; l_i^{+}{:}\langle\!\langle A \rangle\!\rangle \rightarrow \langle\!\langle B_i \rangle\!\rangle\{\langle\!\langle A \rangle\!\rangle\} \;^{i\in 1..n}]$$

$$\text{where } A \equiv \mathbf{Object}(X)[l_i \upsilon_i{:}B_i\{X\} \;^{i\in 1..n}]$$

Translation of O–1 environments

$$\langle\!\langle \emptyset \rangle\!\rangle \;\triangleq\; \emptyset$$

$$\langle\!\langle E, X{<:}A \rangle\!\rangle \;\triangleq\; \langle\!\langle E \rangle\!\rangle, X{<:}\langle\!\langle A \rangle\!\rangle$$

$$\langle\!\langle E, x{:}A \rangle\!\rangle \;\triangleq\; \langle\!\langle E \rangle\!\rangle, x{:}\langle\!\langle A \rangle\!\rangle$$

Translation of Terms

- The translation is guided by the type structure.
- The translation maps a class to a collection of pre-methods plus a *new* method.
  - For a class **subclass of** *c'* … **end**, the collection of pre-methods consists of the pre-methods of *c'* that are not overridden, plus all the pre-methods given explicitly.
  - The *new* method assembles the pre-methods into an object; **new** *c* is interpreted as an invocation of the *new* method of $\langle\langle c \rangle\rangle$.
  - The construct $c\hat{}l(a)$ is interpreted as the extraction and the application of a pre-method.

(Simplified) Translation of O–1 terms

$\langle\!\langle x \rangle\!\rangle \triangleq x$

$\langle\!\langle \textbf{object}(x{:}A)\ l_i{=}b_i\ ^{i\in 1..n}\ \textbf{end} \rangle\!\rangle \triangleq [l_i{=}\varsigma(x{:}\langle\!\langle A \rangle\!\rangle)\langle\!\langle b_i \rangle\!\rangle\ ^{i\in 1..n}]$

$\langle\!\langle a.l \rangle\!\rangle \triangleq \langle\!\langle a \rangle\!\rangle.l$

$\langle\!\langle a.l := b \rangle\!\rangle \triangleq \langle\!\langle a \rangle\!\rangle.l{:=}\langle\!\langle b \rangle\!\rangle$

$\langle\!\langle a.l := \textbf{method}(x{:}A)\ b\ \textbf{end} \rangle\!\rangle \triangleq \langle\!\langle a \rangle\!\rangle.l \Leftarrow \varsigma(x{:}\langle\!\langle A \rangle\!\rangle)\langle\!\langle b \rangle\!\rangle$

$$\langle\!\langle \mathbf{new}\ c\rangle\!\rangle \triangleq \langle\!\langle c\rangle\!\rangle.new$$

$$\langle\!\langle \mathbf{root}\rangle\!\rangle \triangleq [new=[\,]]$$

$$\begin{aligned}
&\langle\!\langle \mathbf{subclass\ of}\ c'{:}\mathbf{Class}(A')\ \mathbf{with}(x{:}A)\ l_i=b_i{}^{\,i\in n+1..n+m}\ \mathbf{override}\ l_i=b_i{}^{\,i\in Ovr}\ \mathbf{end}\rangle\!\rangle \triangleq\\
&\qquad [new=\varsigma(z{:}\langle\!\langle \mathbf{Class}(A)\rangle\!\rangle)[l_i=\varsigma(s{:}\langle\!\langle A\rangle\!\rangle)z.l_i(s)^{\,i\in 1..n+m}],\\
&\qquad\ l_i=\langle\!\langle c'\rangle\!\rangle.l_i{}^{\,i\in 1..n-Ovr},\\
&\qquad\ l_i=\lambda(x{:}\langle\!\langle A\rangle\!\rangle)\langle\!\langle b_i\rangle\!\rangle^{\,i\in Ovr\cup n+1..n+m}]
\end{aligned}$$

$$\langle\!\langle c{\wedge}l(a)\rangle\!\rangle \triangleq \langle\!\langle c\rangle\!\rangle.l(\langle\!\langle a\rangle\!\rangle)$$

$$\langle\!\langle \mathbf{typecase}\ a\ \mathbf{when}\ (x{:}A)b_1\ \mathbf{else}\ b_2\ \mathbf{end}\rangle\!\rangle \triangleq typecase\ \langle\!\langle a\rangle\!\rangle \mid (x{:}\langle\!\langle A\rangle\!\rangle)\langle\!\langle b_1\rangle\!\rangle \mid \langle\!\langle b_2\rangle\!\rangle$$

Usefulness of the Translation

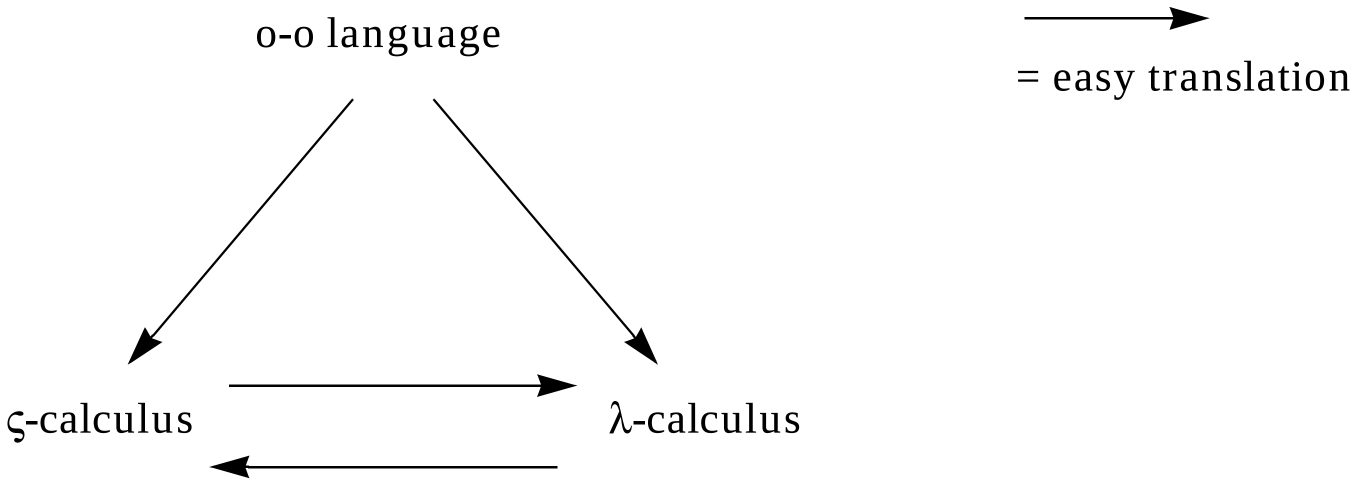
- The translation validates the typing rules of O–1. That is, if $E \vdash J$ is valid in O–1, then $\langle\langle E \vdash J \rangle\rangle$ is valid in the object calculus.
- The translation served as an important guide in finding sound typing rules for O–1, and for “tweaking” them to make them both simpler and more general.
- In particular, typing rules for subclasses are so inherently complex that it is difficult to “guess” them correctly without the aid of some interpretation.
- Thus, we have succeeded in using object calculi as a platform for explaining a relatively rich object-oriented language and for validating its type rules.

TRANSLATIONS

- In order to give insight into type rules for object-oriented languages, translations must be judgment-preserving (in particular, type and subtype preserving).
- Translating object-oriented languages directly to typed λ-calculi is just too hard. Object calculi provide a good stepping stone in this process, or an alternative endpoint.
- Translating object calculi into λ-calculi means, intuitively, “programming in object-oriented style within a procedural language”. This is the hard part.

Untyped Translations

- Give insights into the nature of object-oriented computation.
- Objects = records of functions.



o-o language

= easy translation

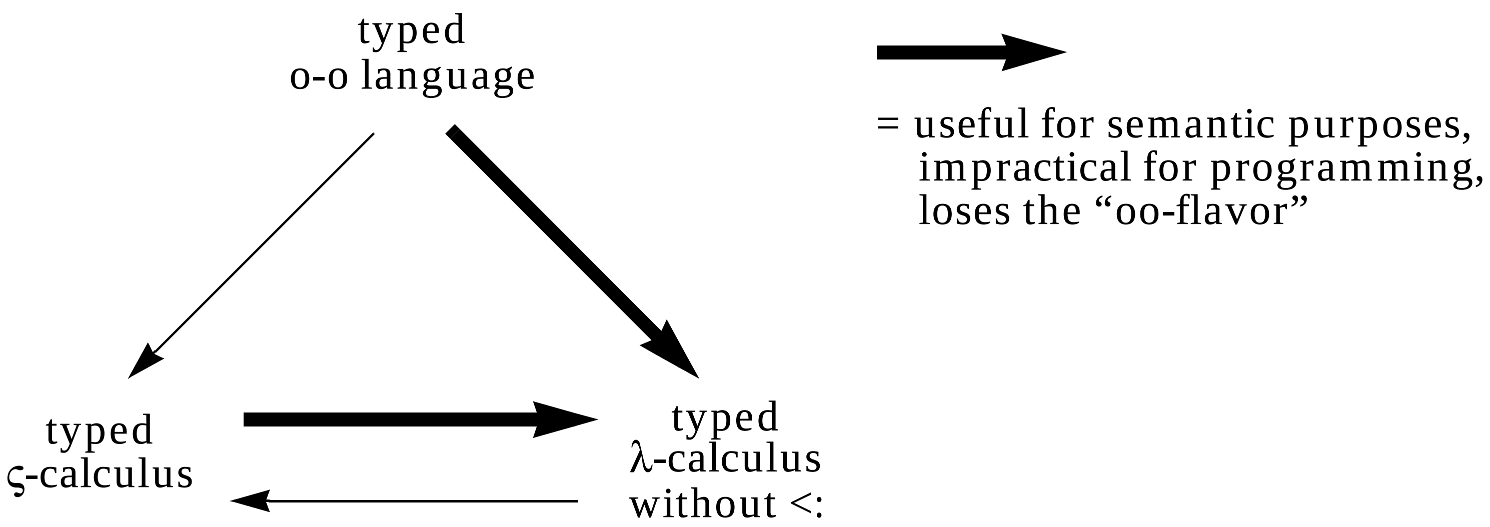
ς-calculus

λ-calculus

Type-Preserving Translations

- Give insights into the nature of object-oriented typing and subsumption / coercion.
- Object types = recursive records-of-functions types.

$$[l_i{:}B_i\ ^{i\in 1..n}] \quad \triangleq \quad \mu(X)\langle l_i{:}X\rightarrow B_i\ ^{i\in 1..n}\rangle$$



typed
o-o language

typed
ς-calculus

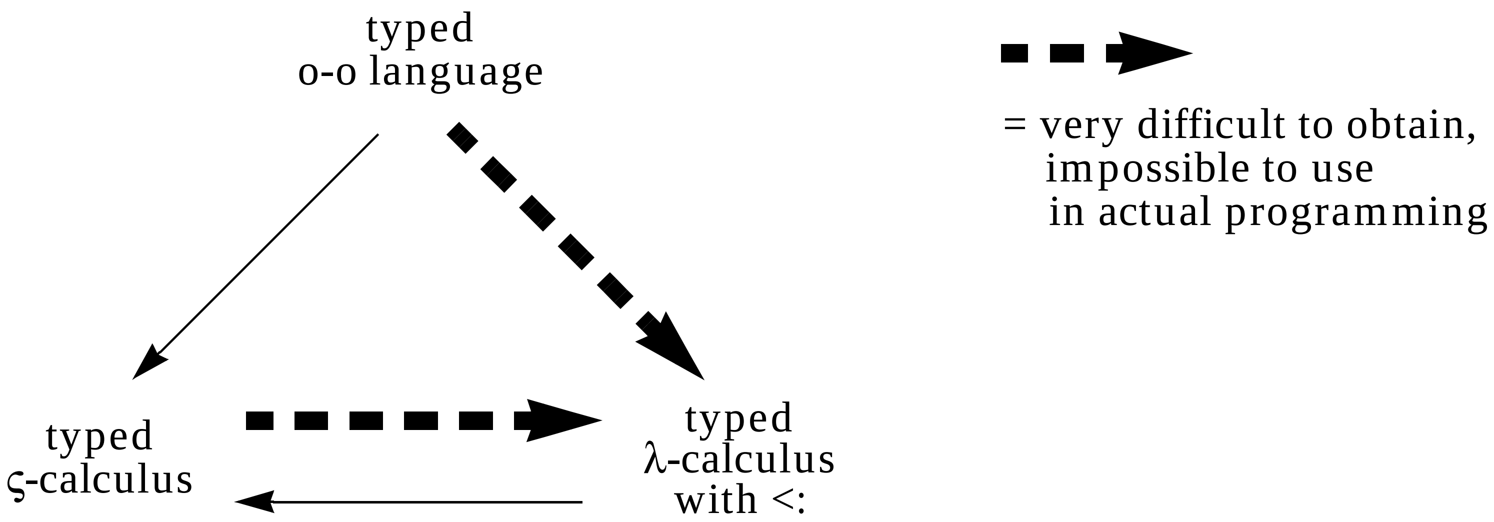
typed
λ-calculus
without <:

= useful for semantic purposes,
impractical for programming,
loses the “oo-flavor”

Subtype-Preserving Translations

- Give insights into the nature of subtyping for objects.
- Object types = recursive bounded existential types (!!).

$$[l_i{:}B_i\ {}^{i\in 1..n}] \triangleq \mu(Y)\exists(X<:Y)\langle r{:}X,\ l_i^{sel}{:}X\rightarrow B_i\ {}^{i\in 1..n},\ l_i^{upd}{:}(X\rightarrow B_i)\rightarrow X\ {}^{i\in 1..n}\rangle$$



= very difficult to obtain,
impossible to use
in actual programming

CONCLUSIONS

- Foundations
  - ~ Subtype-preserving translations of object calculi into λ-calculi are hard.
  - ~ In contrast, subtype-preserving translations of λ-calculi into object-calculi can be easily obtained.
  - ~ In this sense, object calculi are a more convenient foundation for object-oriented programming than λ-calculi.

- Language design
  - Object calculi are a good basis for designing rich object-oriented type systems (including polymorphism, Self types, etc.).
  - Object-oriented languages can be shown sound by fairly direct translations into object calculi.

- Other developments
  - Second-order object types for Self types.
  - Higher-order object types for matching.

- Potential future areas
  - Typed ς-calculi should be a good simple foundation for studying object-oriented specification and verification.
  - They should also give us a formal platform for studying object-oriented concurrent languages (as opposed to “ordinary” concurrent languages).

References

- http://www.research.digital.com/SRC/personal/Luca_Cardelli/TheoryOfObjects.html
- M.Abadi, L.Cardelli: **A Theory of Objects**. Springer, 1996.