Algebras and Languages for Molecular Programming

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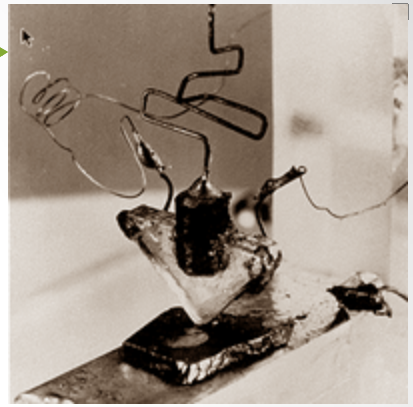
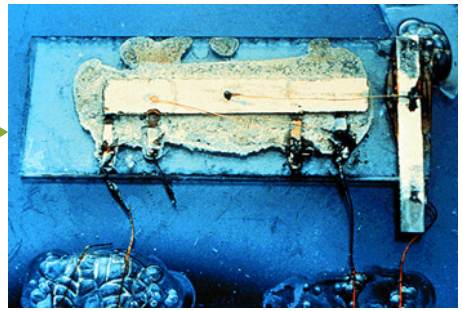
Smaller and Smaller

Dec. 23, 1947. John Bardeen and Walter Brattain show the first working transistor.

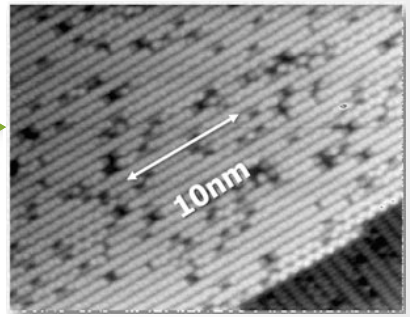


Sep. 1958. Jack Kilby builds the first integrated circuit.



50 years later

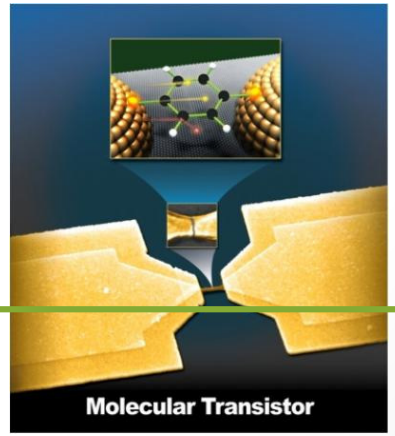
Jan. 2010. Intel and Micron announce 25nm NAND flash.



Scanning tunneling microscope image of a silicon surface showing 10nm is ~20 atoms across

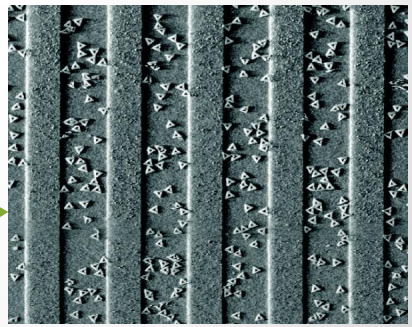
Dec. 24, 2009. Working transistor made of a single molecule.

Observation of molecular orbital gating. *Nature*, 2009; 462 (7276): 1039



<10 iterations of Moore's Law left!

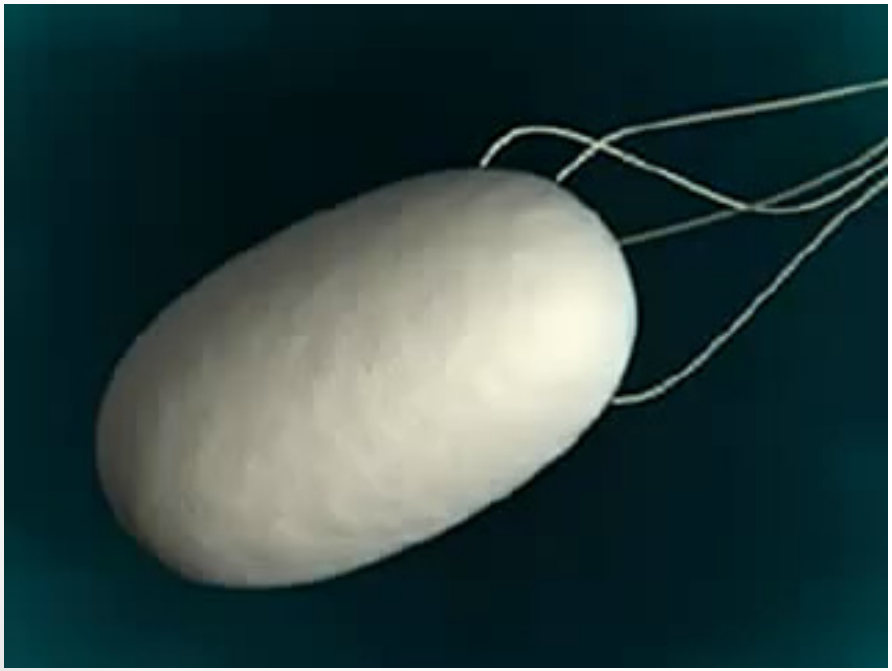
The race is on for *molecular scale integrated circuits*.



Placement and orientation of individual DNA shapes on lithographically patterned surfaces. Nature Nanotechnology 4, 557 – 561 (2009).

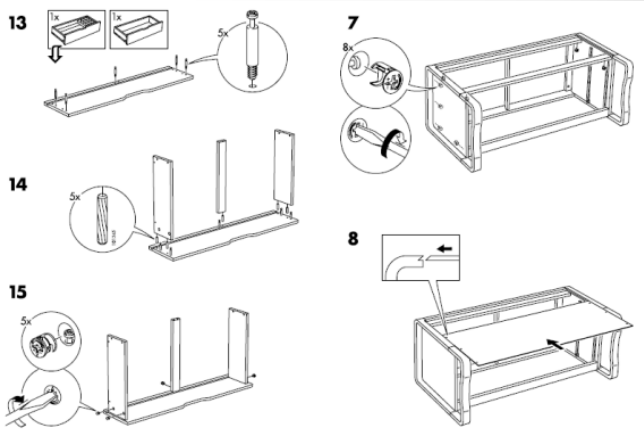
Building The Small*est* Things

- How do we build structures that are by definition smaller than your tools?
- Basic answer: you can't. Structures (and tools) should build themselves!
- By *programmed self-assembly*.

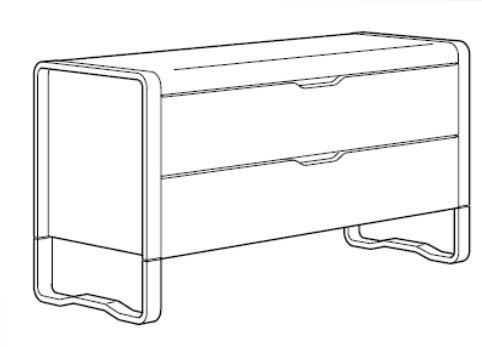


Molecular IKEA

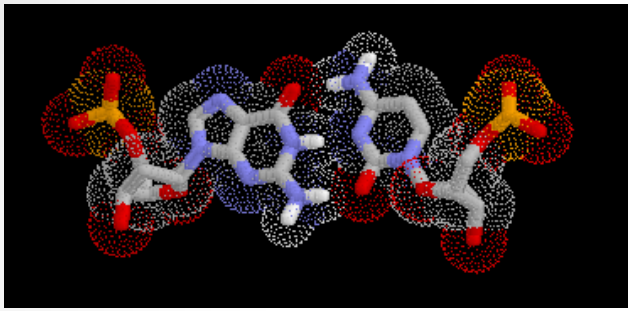
- Nature can self-assemble. Can we?
- *"Dear IKEA, please send me a chest of drawers that assembles itself."*
- We need a magical material where the pieces are pre-programmed to fit into to each other.
- At the molecular scale many such materials exist; let's pick one…



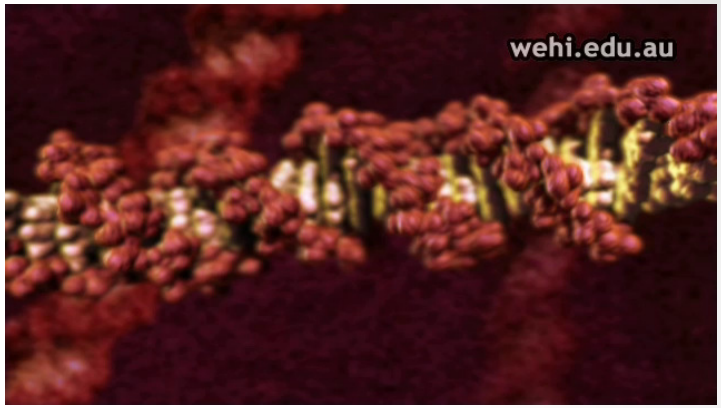
Add water

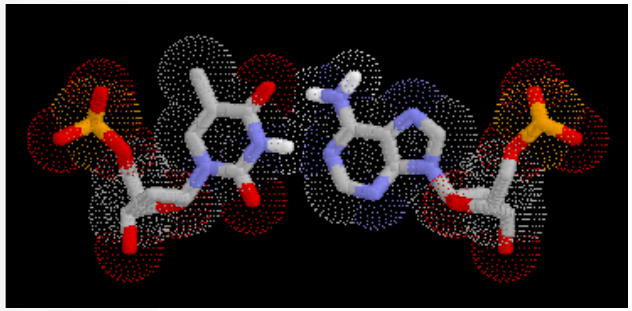


DNA



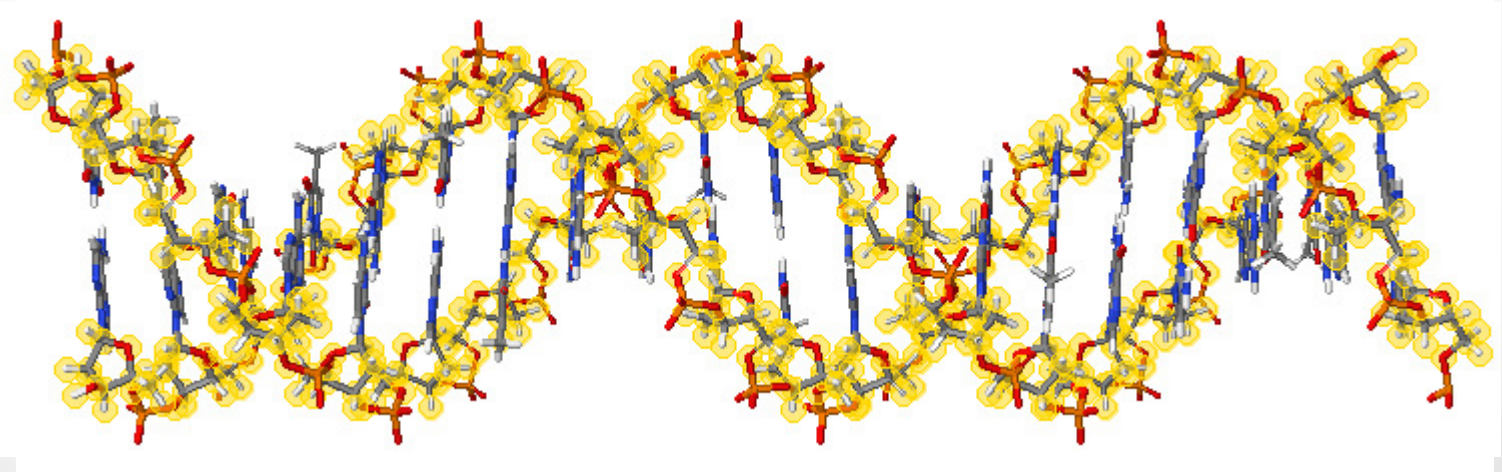
GC Base Pair
Guanine-Cytosine





TA Base Pair
Thymine-Adenine

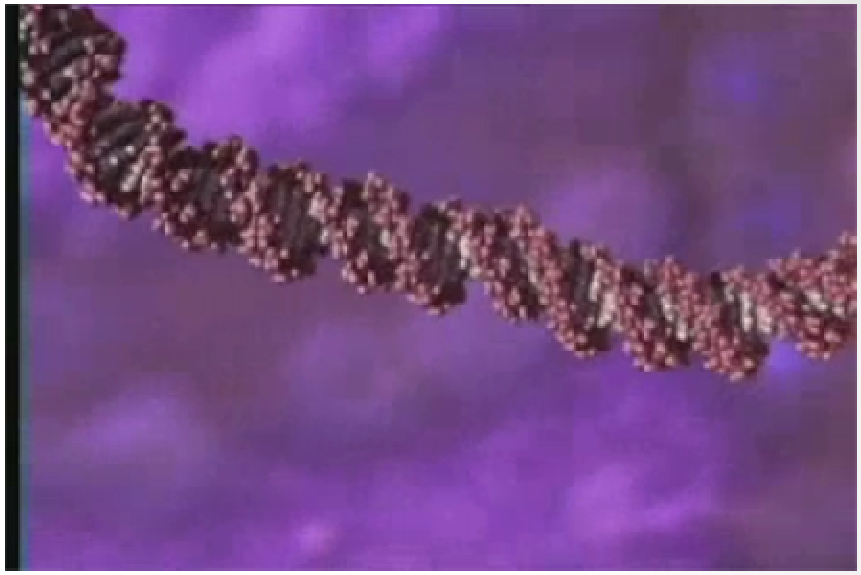
Interactive DNA Tutorial
(http://www.biosciences.bham.ac.uk/labs/minchin/tutorials/dna.html)



Sequence of Base Pairs (GACT alphabet)

Robust, and *Long*

- DNA in each human cell:
  - 3 billion base pairs
  - 2 meters long, 2nm thick
  - folded into a 6μm ball
  - 750 MegaBytes
- A huge amount for a cell
  - Every time a cell replicates it has to copy *2 meters of DNA* reliably.
  - To get a feeling for the scale disparity, compute:
- DNA in human body
  - 10 trillion cells
  - 133 Astronomical Units long
  - 7.5 OctaBytes
- DNA in human population
  - 20 million light years long

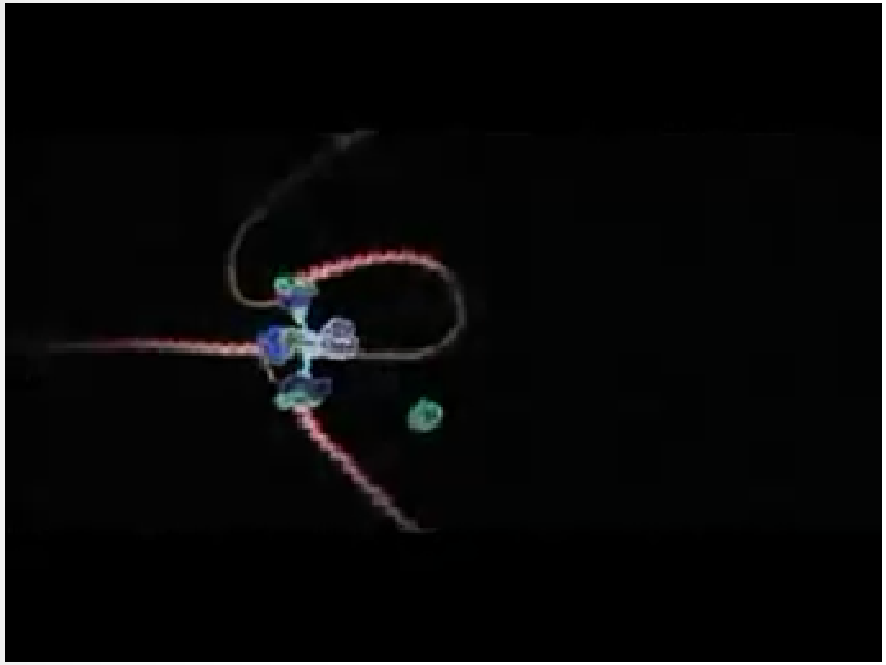


DNA wrapping into chromosomes

Andromeda Galaxy
2.5 million light years

Zipping Along

- DNA can support structural and computational complexity.



DNA replication in *real time*

In Humans: 50 nucleotides/second
Whole genome in a few hours (with parallel processing)

In Bacteria: 1000 nucleotides/second
(higher error rate)

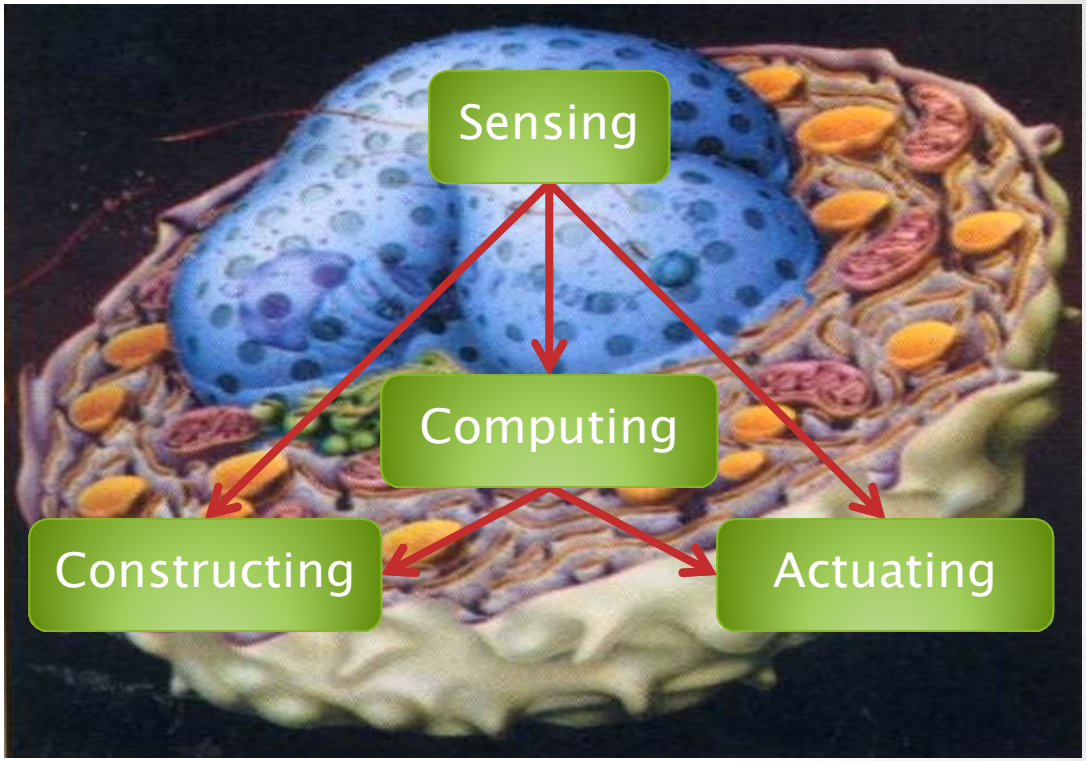
DNA transcription in *real time*

RNA polymerase II: 15-30 base/second

Drew Berry
http://www.wehi.edu.au/wehi-tv

Nanoscale Engineering

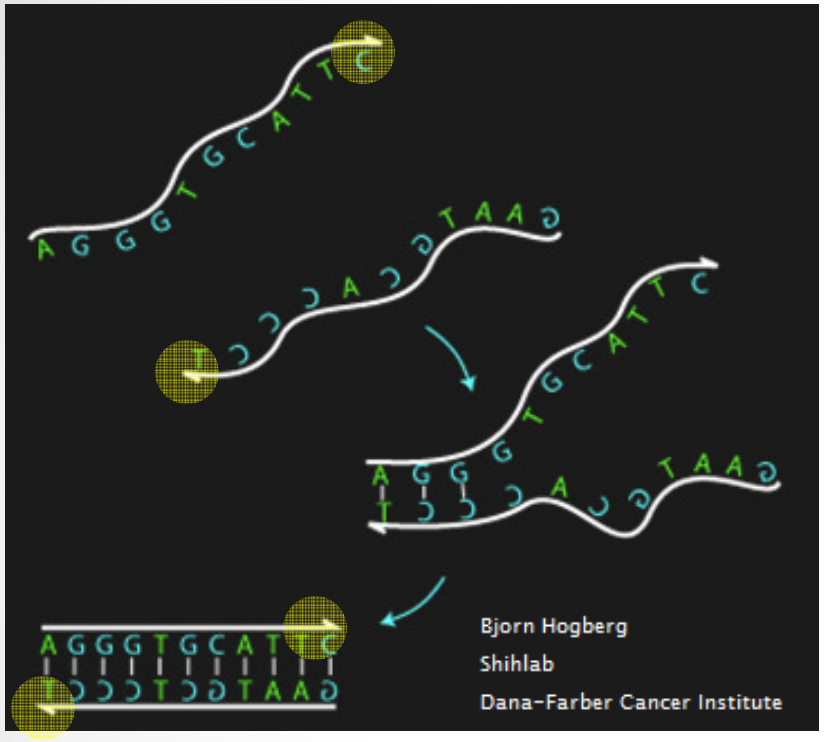
- Sensing
  - Reacting to forces
  - Binding to molecules
- Actuating
  - Releasing molecules
  - Producing forces
- Constructing
  - Chassis
  - Growth
- Computing
  - Signal Processing
  - Decision Making

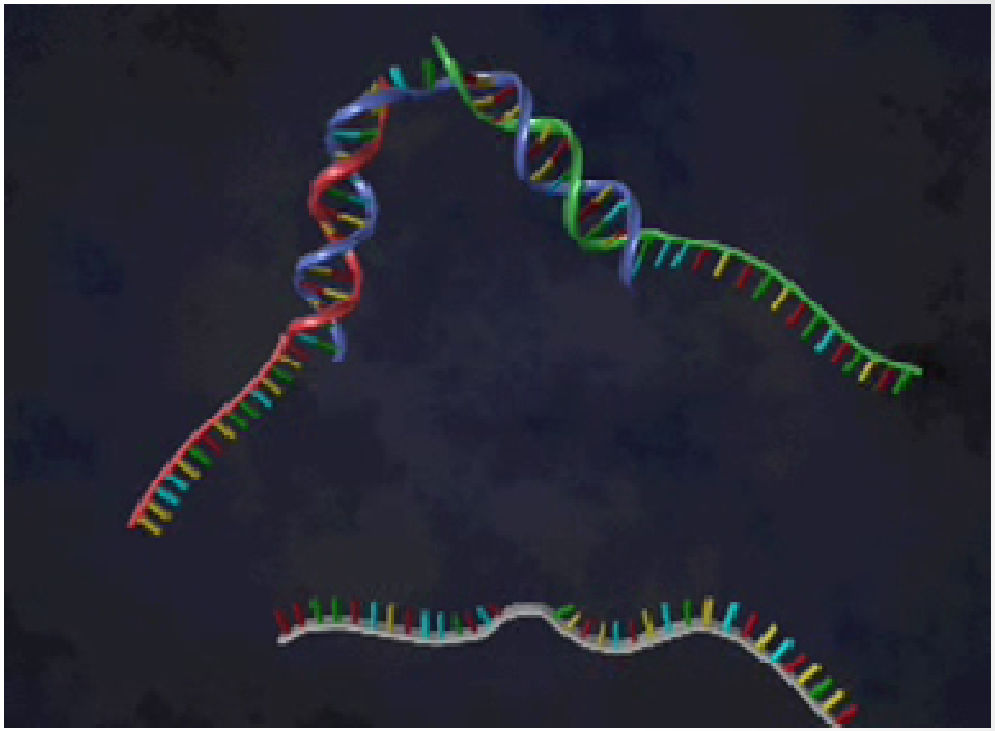


Nucleic Acids can do all this.

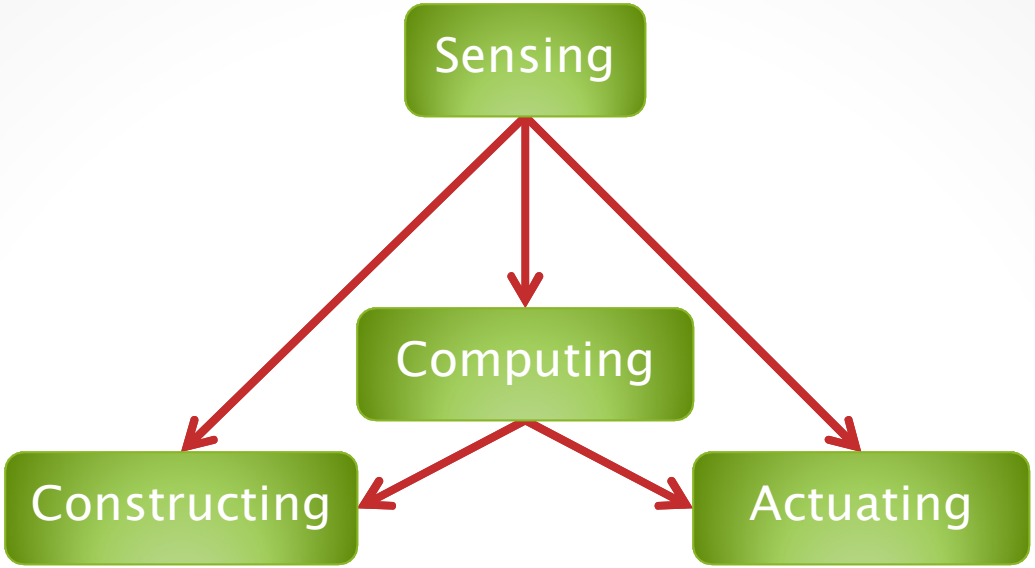
And interface to biology.

Hybridization

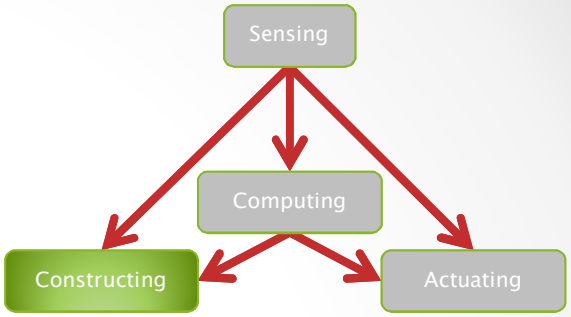




- Strands with **opposite orientation** and **complementary base pairs** stick to each other (Watson–Crick duality).
- This is all we are going to use
  - We are not going to exploit DNA replication, transcription, translation, **restriction and ligation enzymes**, etc., which enable other classes of tricks.



Hybridization Tricks



Constructing

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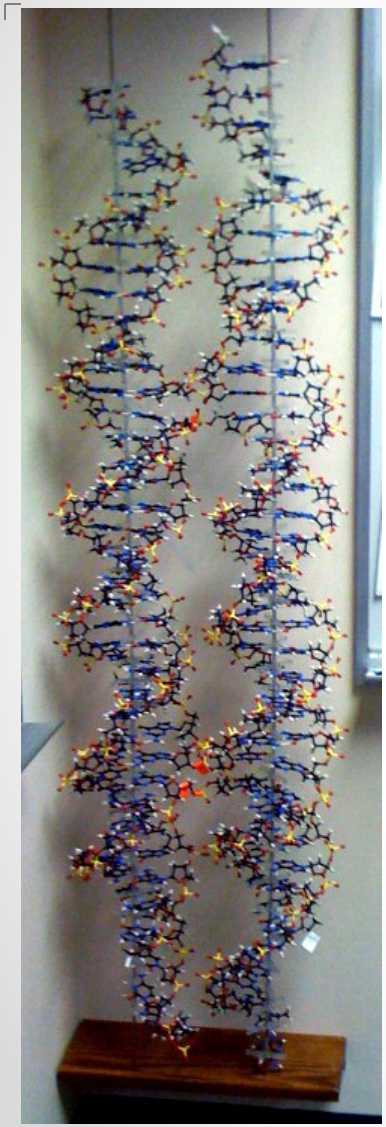
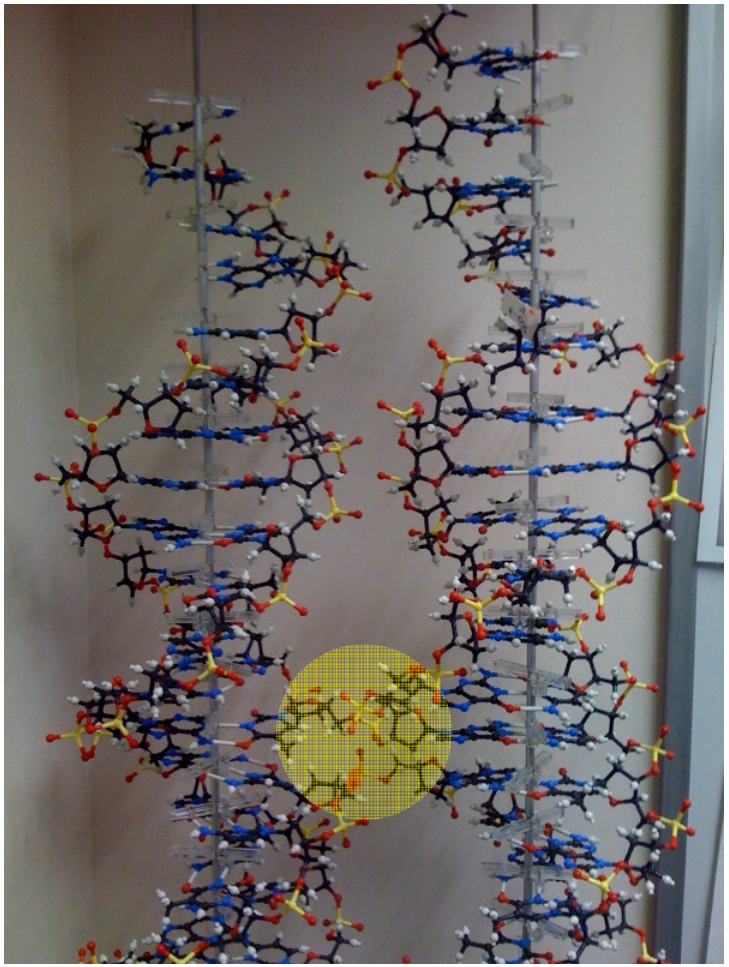
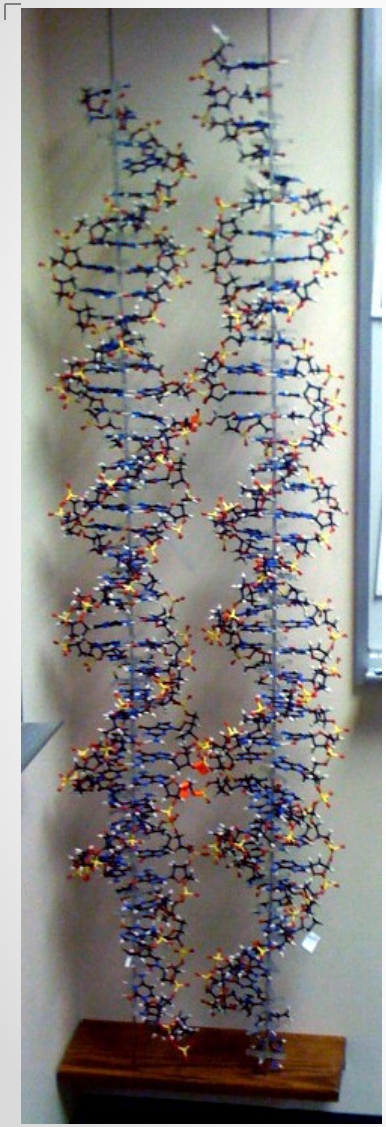
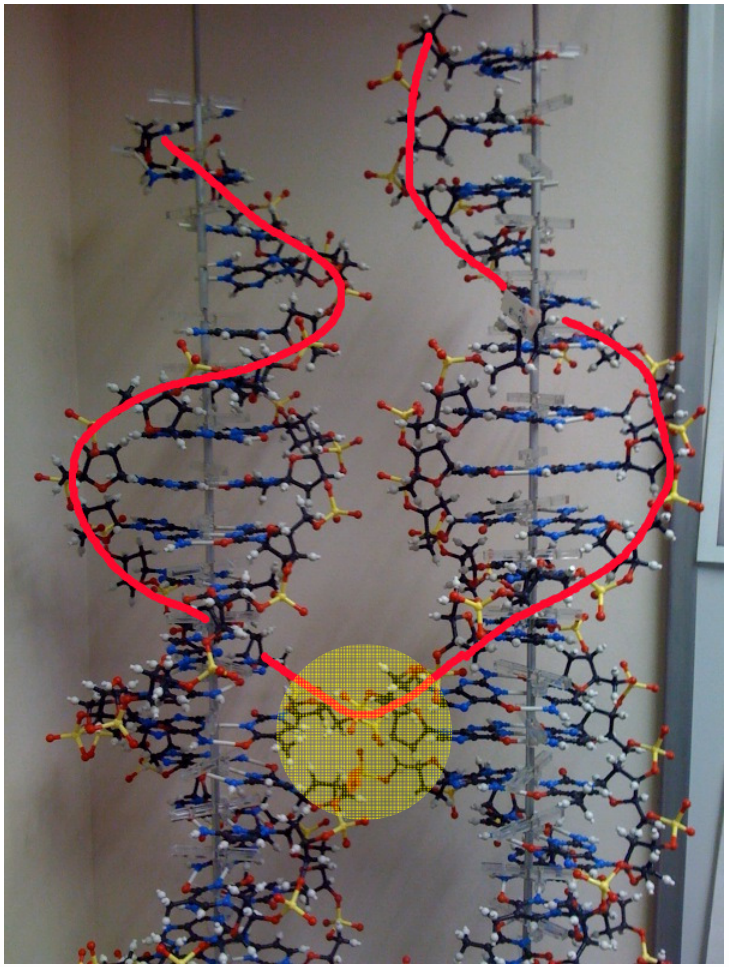
Crosslinking





Crosslinking





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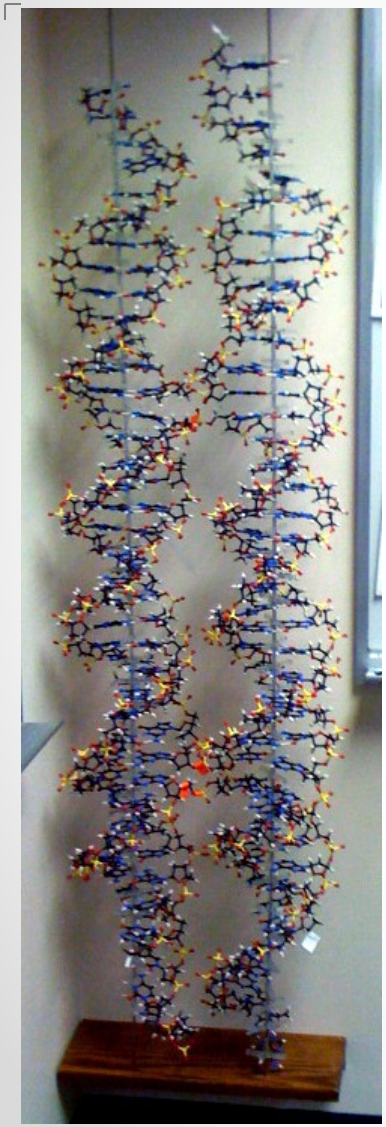

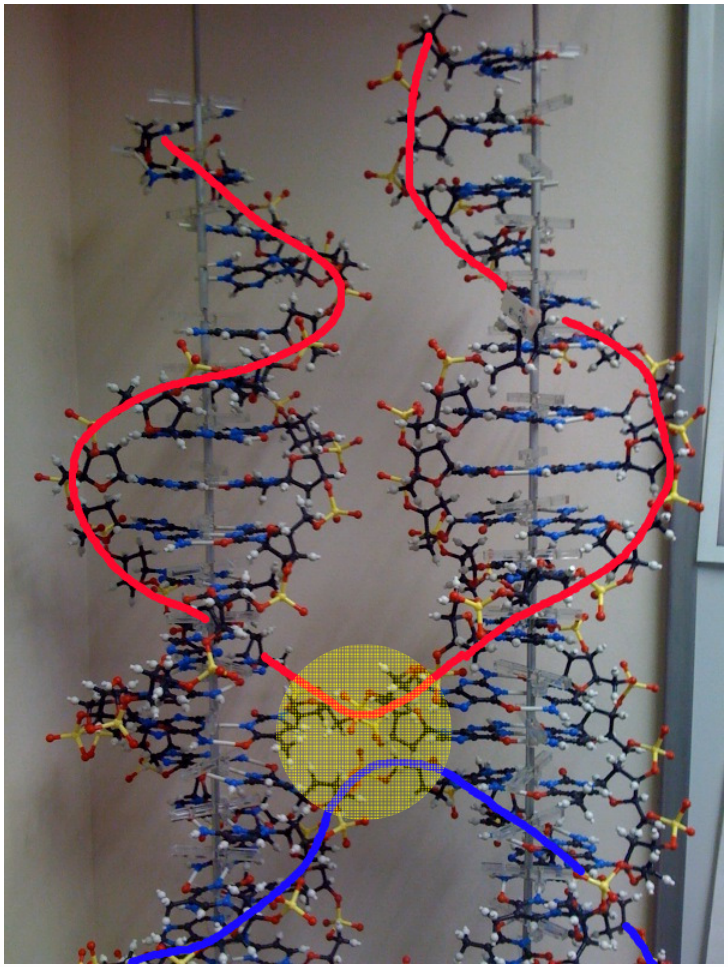

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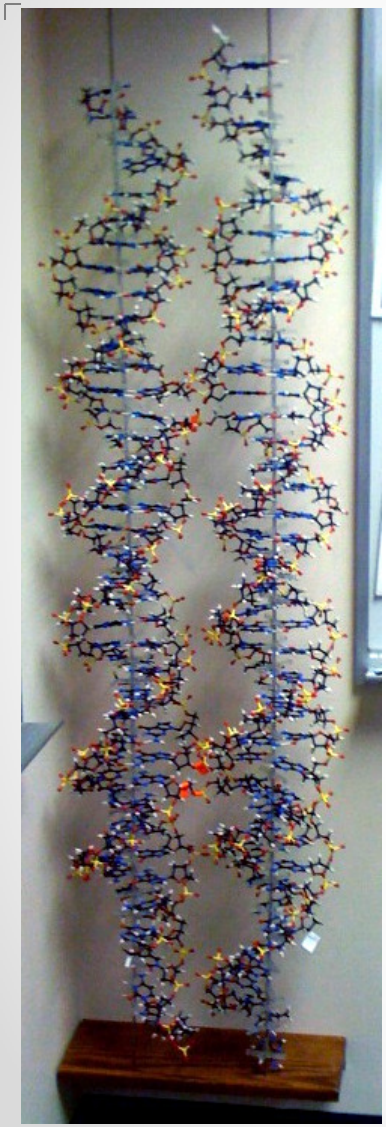

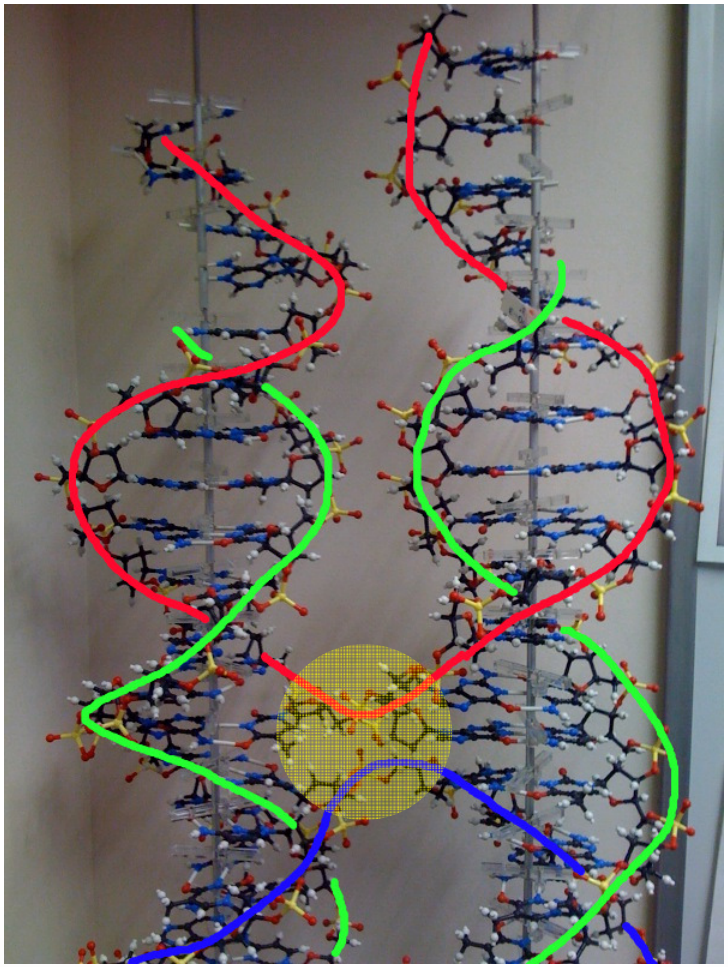

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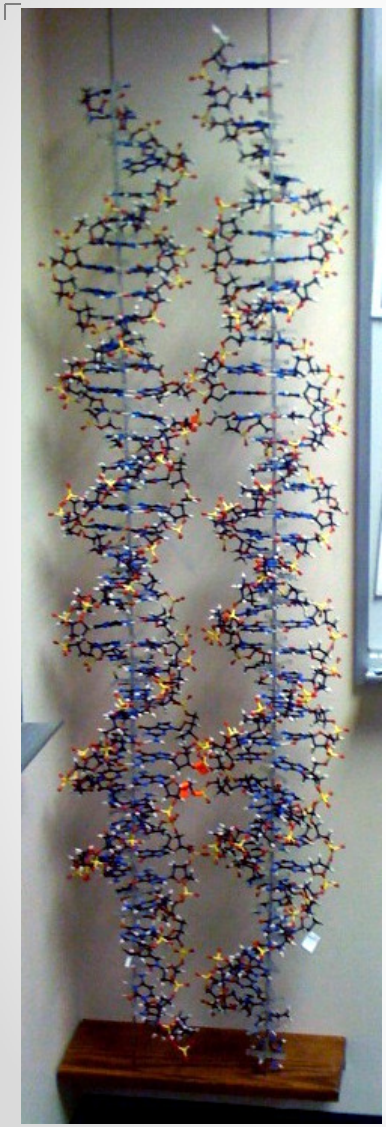
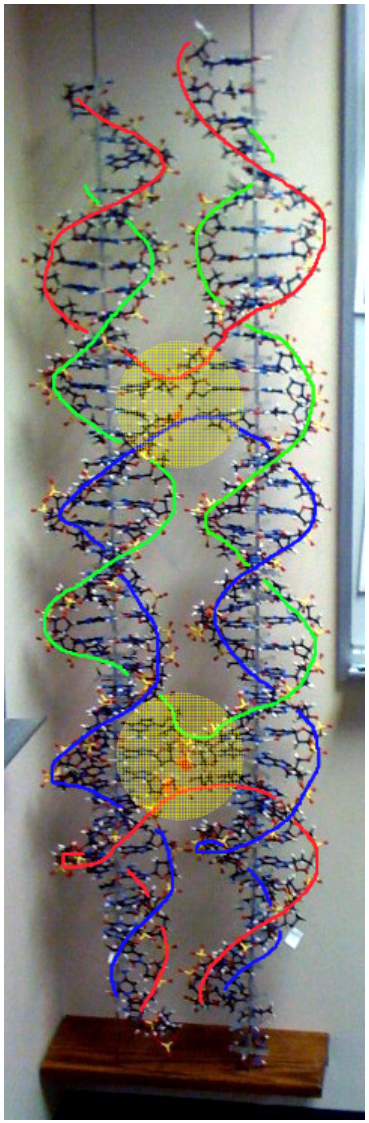
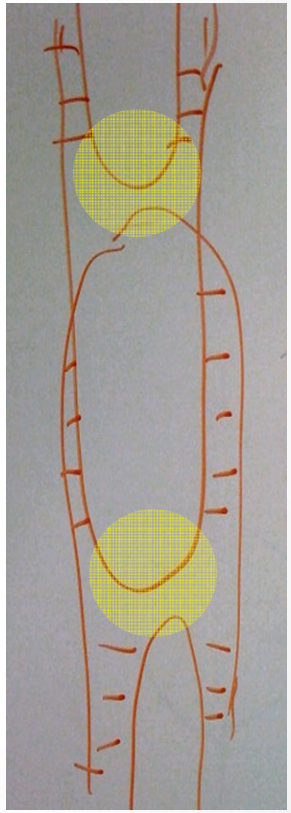


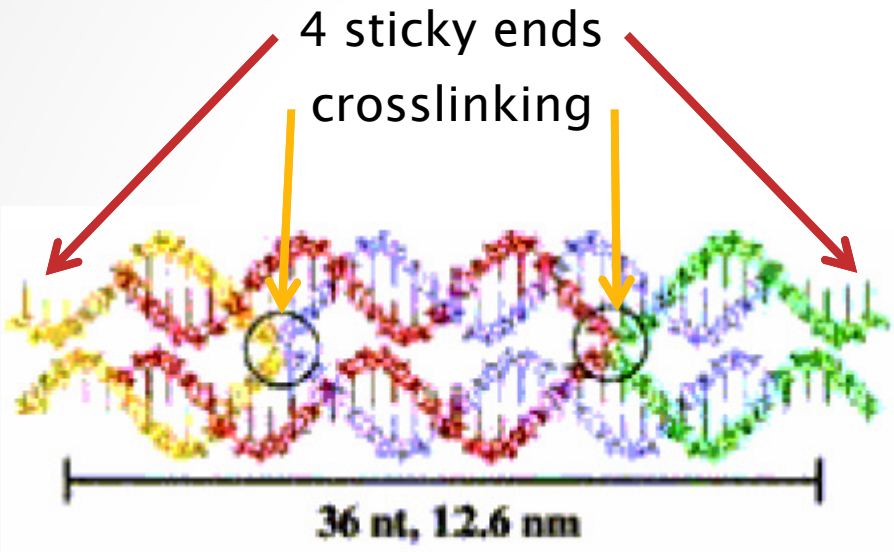


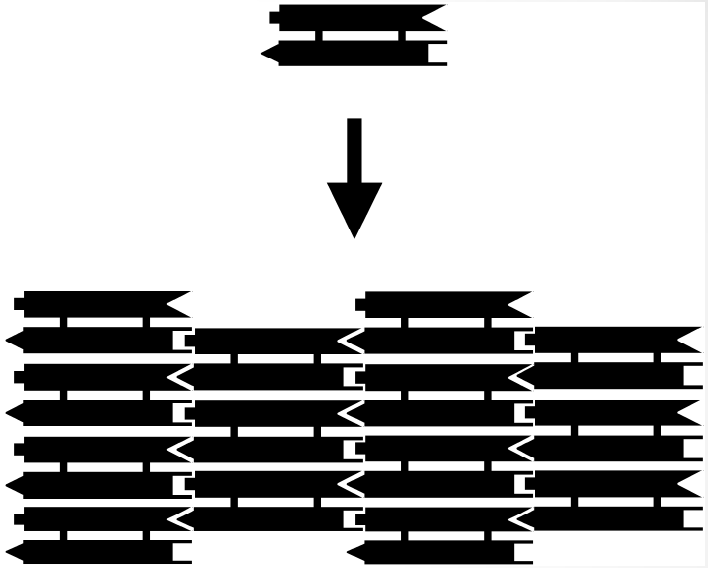
In nature, crosslinking is deadly (blocks DNA replication).

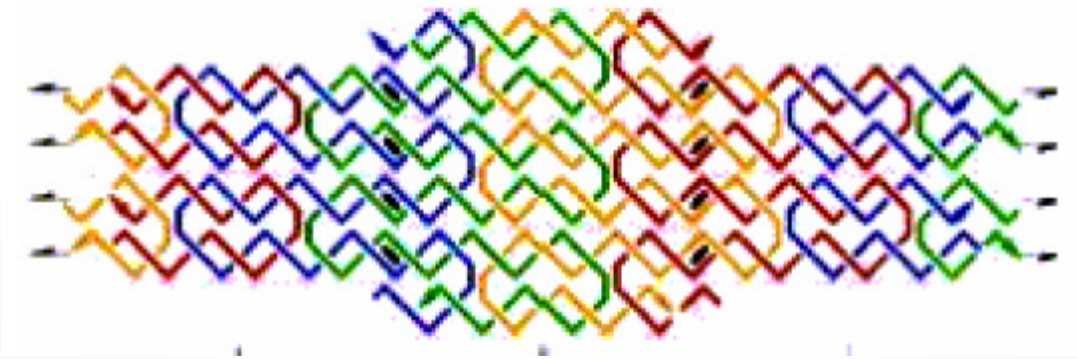


In engineering, crosslinking is the key to using DNA as a construction material.

DNA Tiling



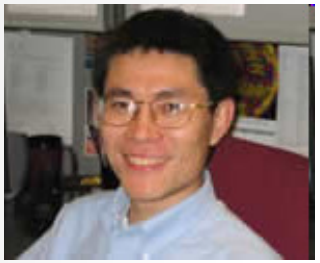




Construction and manipulation of DNA tiles in free space

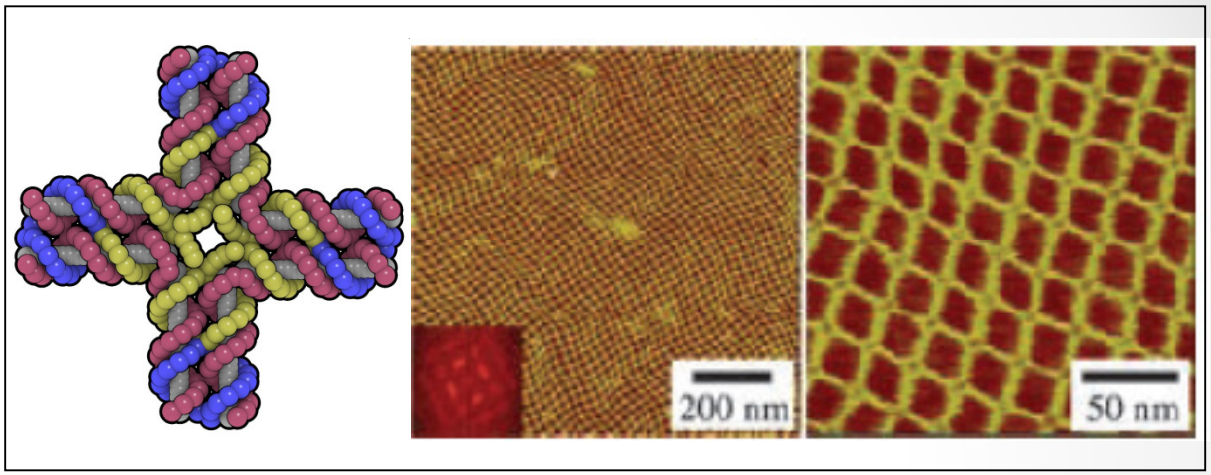
Pankhudi

2D DNA Lattices

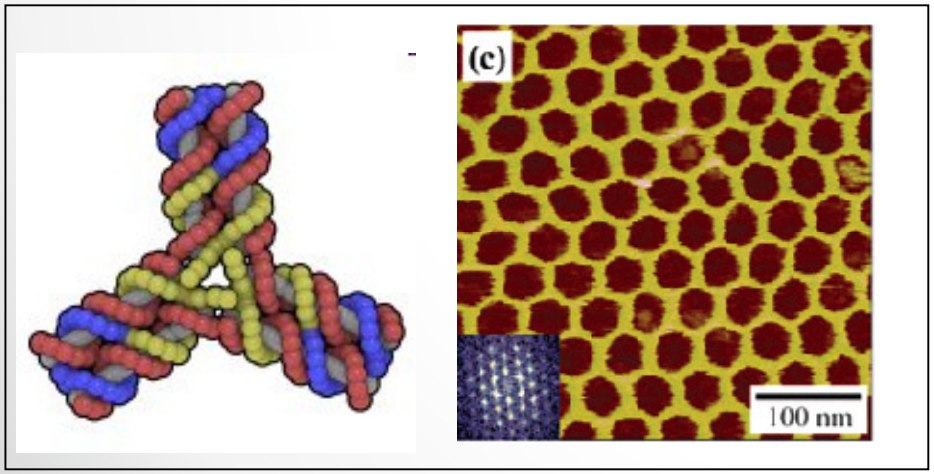


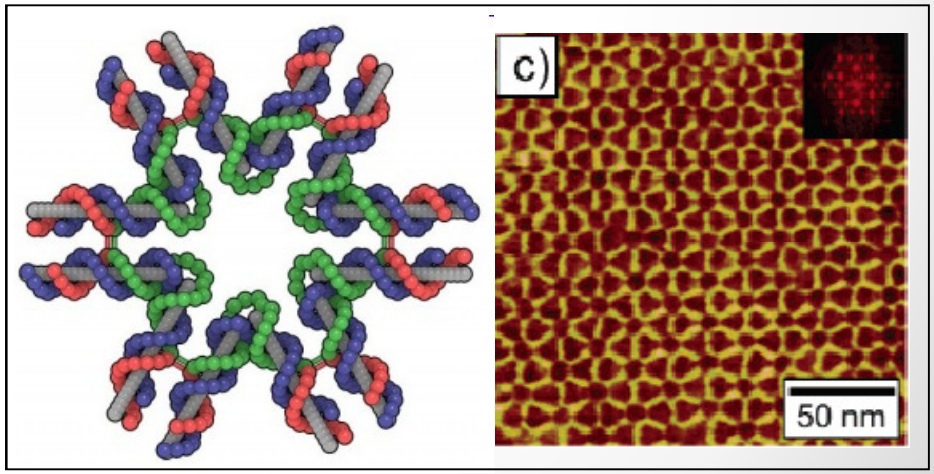
Chengde Mao

Purdue University, USA



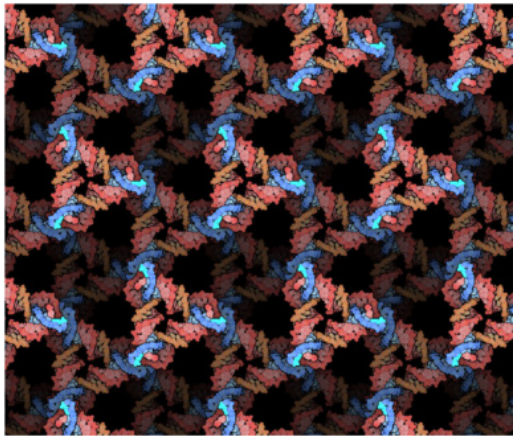
N-point Stars

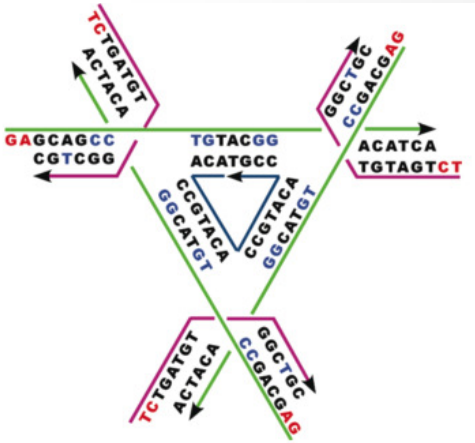




3D DNA Structures

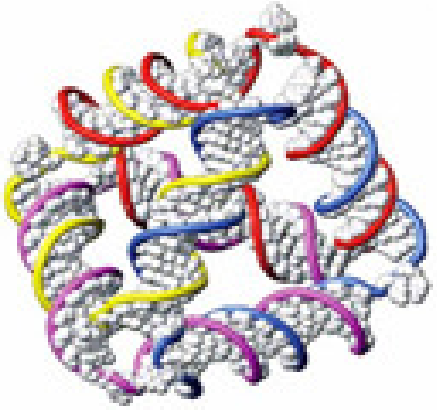
Ned Seeman
NYU

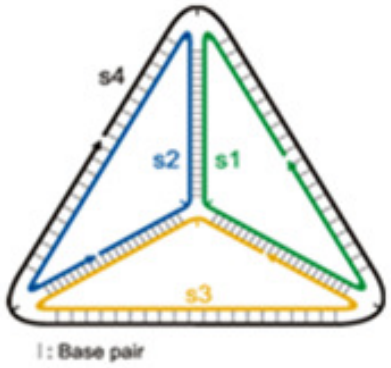




3D Cyrstal

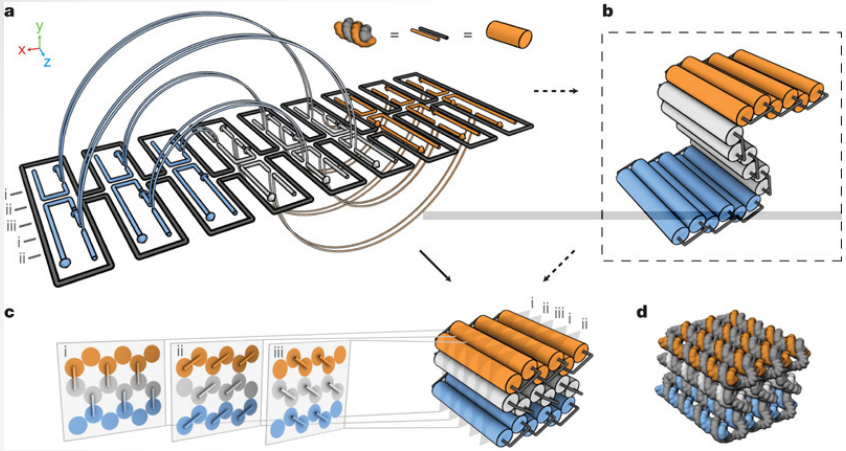
AndrewTuberfield
Oxford

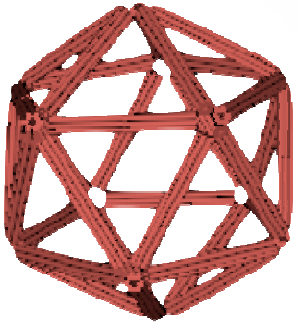




Tetrahedron

CADnano



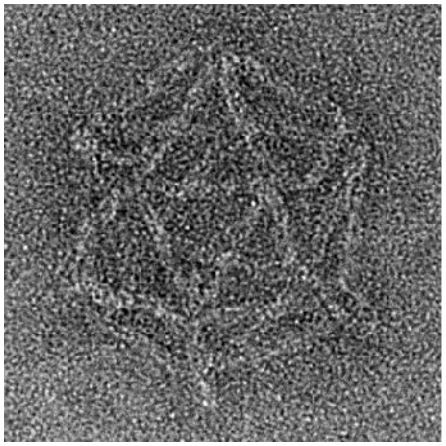


William Shih
Harvard

Folding DNA into Twisted and Curved Nanoscale Shapes

Hendrik Dietz, Shawn M. Douglas, & William M. Shih

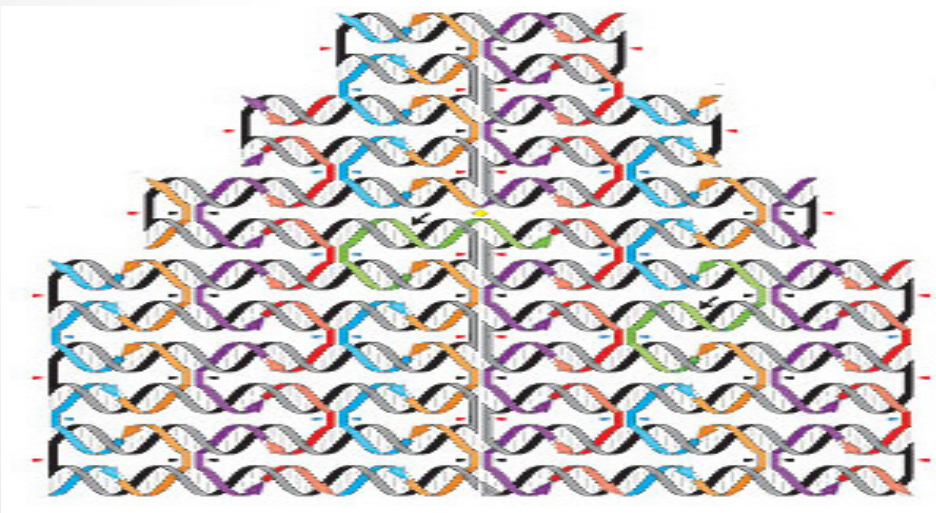
Science, 325:725–730, 7 August 2009.



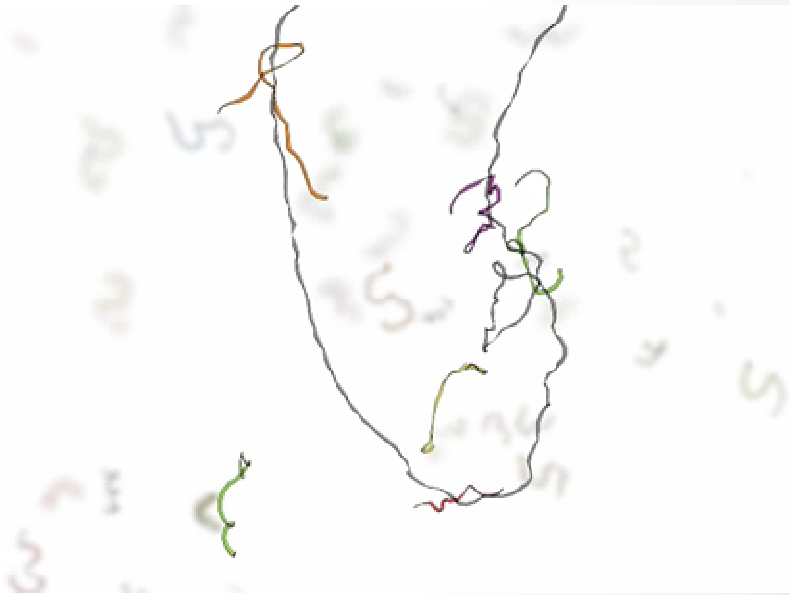
S.M. Douglas, H. Dietz, T. Liedl, B. Högberg, F. Graf and W. M. Shih
Self-assembly of DNA into nanoscale three-dimensional shapes, Nature (2009)

DNA Origami

- *Folding* long (7000bp) naturally occurring (viral) ssDNA
- By lots of short 'staple' strands that constrain it

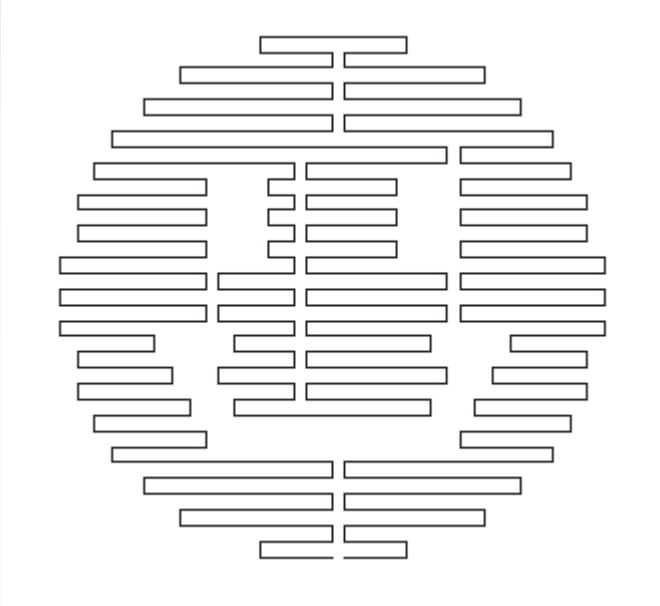


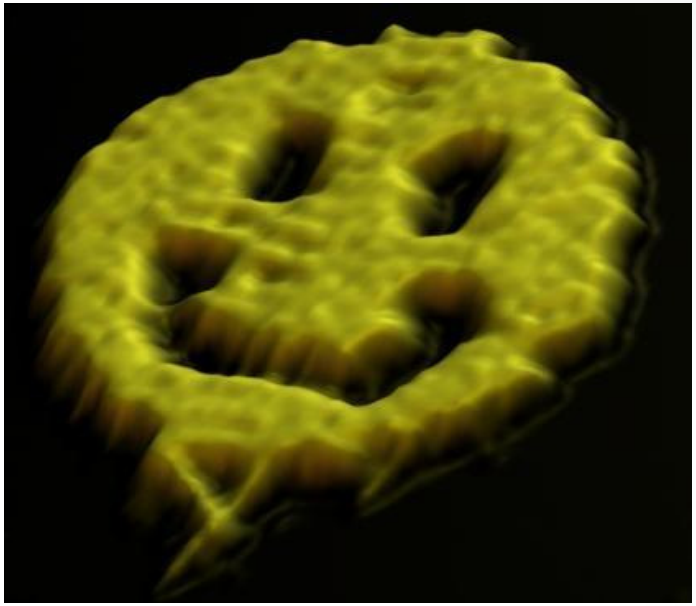
PWK Rothemund, *Nature* 440, 297 (2006)



Black: long viral strand
Color: short staple strands

DNA Origami





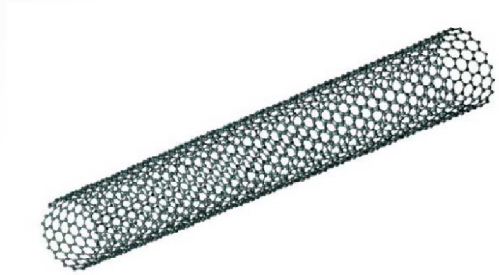
Paul Rothemund's "Disc with three holes" (2006)

Paul W K Rothemund
California Institute of Technology

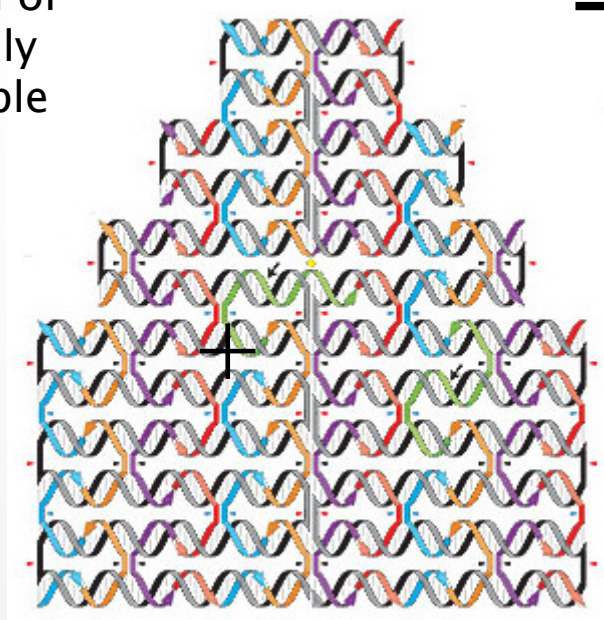
This means we can already self-assemble meso-scale structures.

DNA Circuit Boards

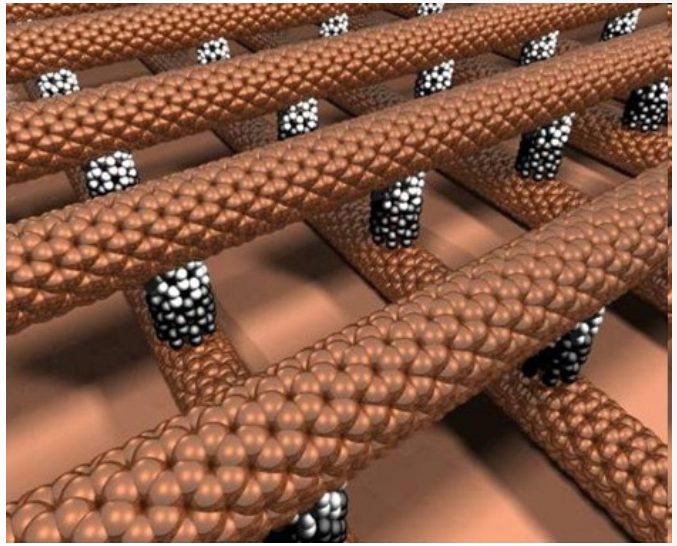
DNA-wrapped nanotubes



6 nm grid of individually addressable pixels

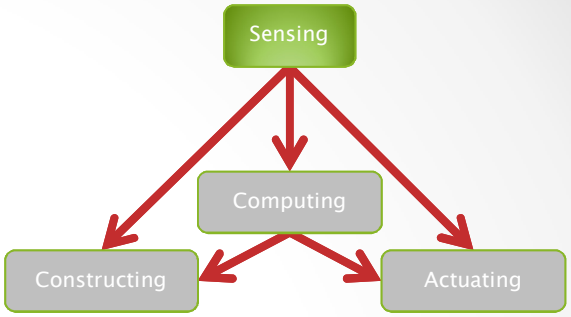


PWK Rothemund, *Nature* 440, 297 (2006)



European Nanoelectronics Initiative Advisory Council

"What we are really making are tiny DNA circuit boards that will be used to assemble other components."
Greg Wallraff, IBM

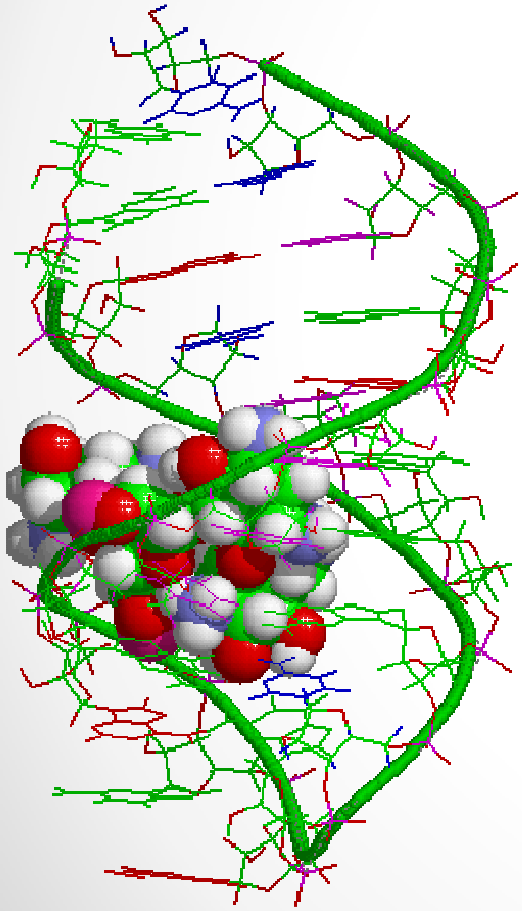


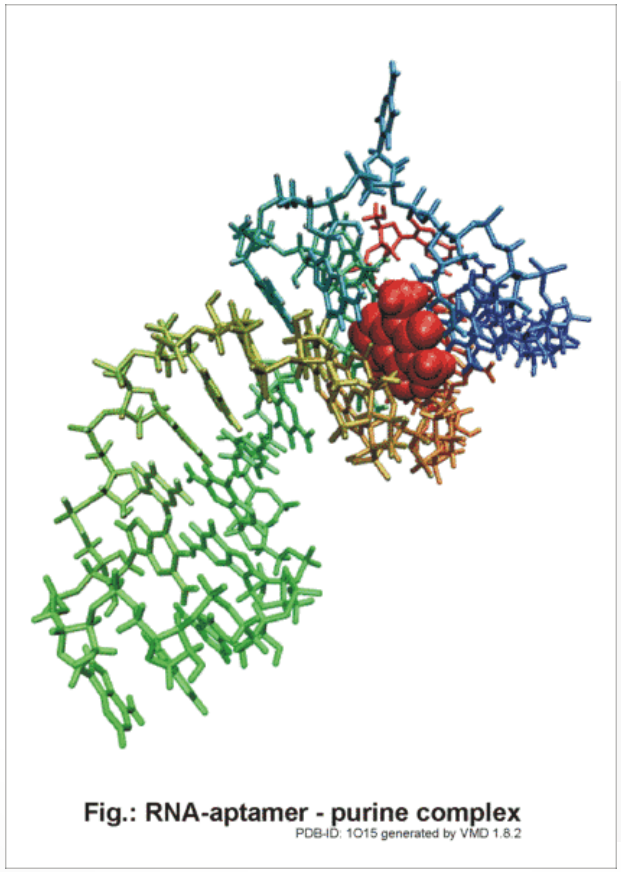
Sensing

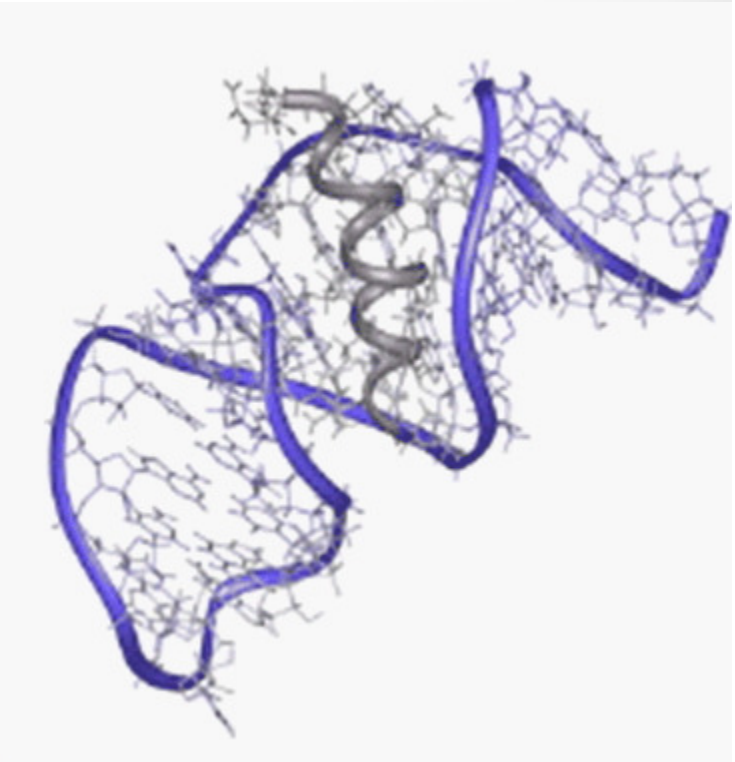
...

Aptamers

- Artificially eveloved DNA molecules that stick to anything you like (highly selectively).

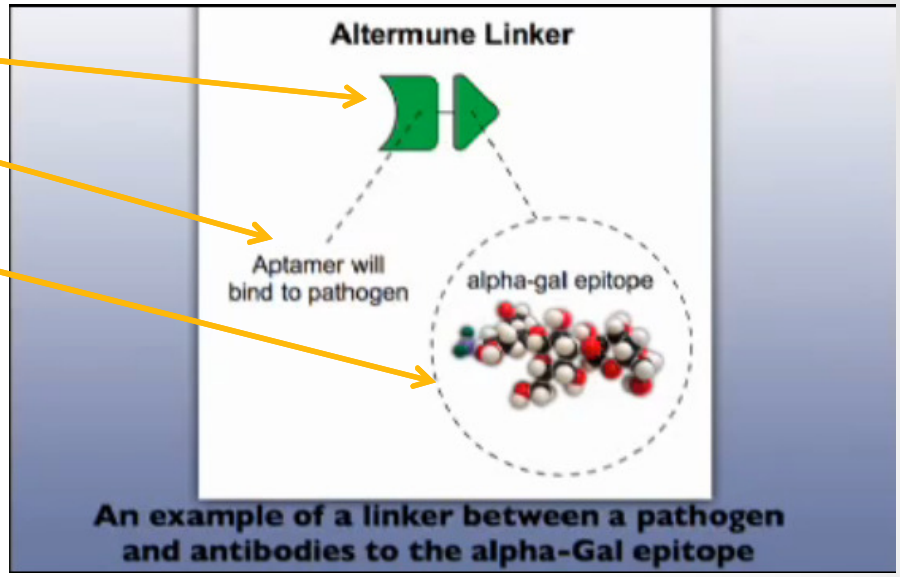




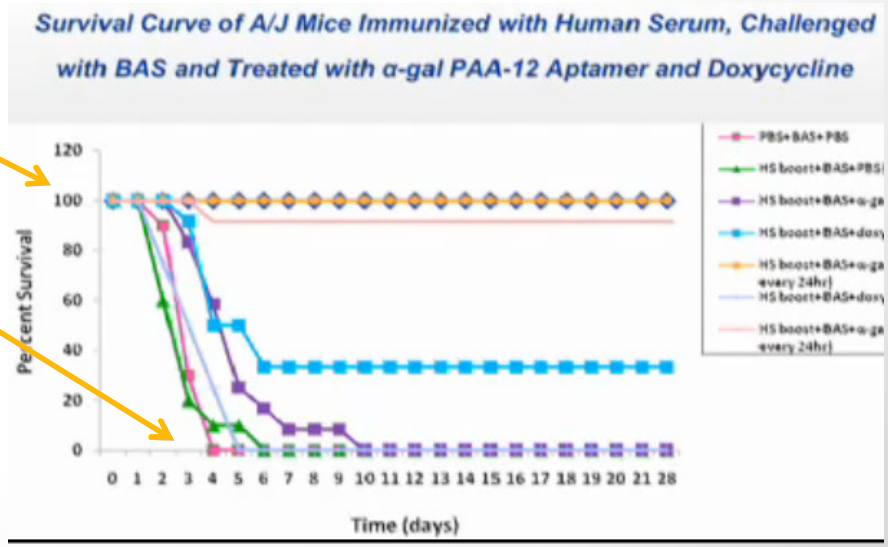


Pathogen Spotlights

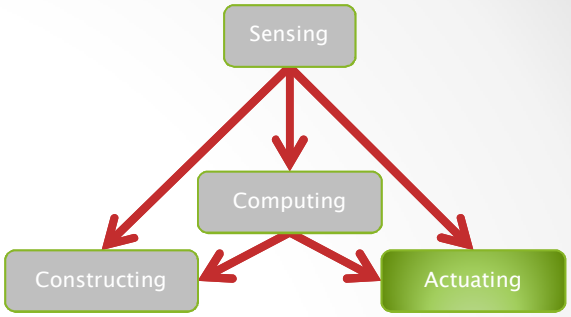
- DNA aptamer binds to:
  - A) a pathogen
  - B) a molecule our immune system already hates and immediately removes (eats) along with anything attached to it



- Result: instant immunity
  - Mice poisoned with Anthrax plus aptamer (100% survival)
  - Mice poinsoned with Anthrax (not so good)



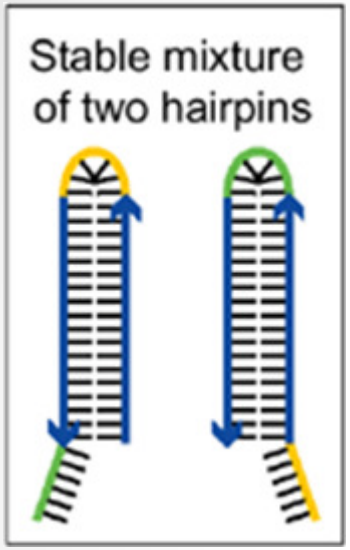
Kary Mullis (incidentally, also Nobel prize for inventing the Polymerase Chain Reaction)

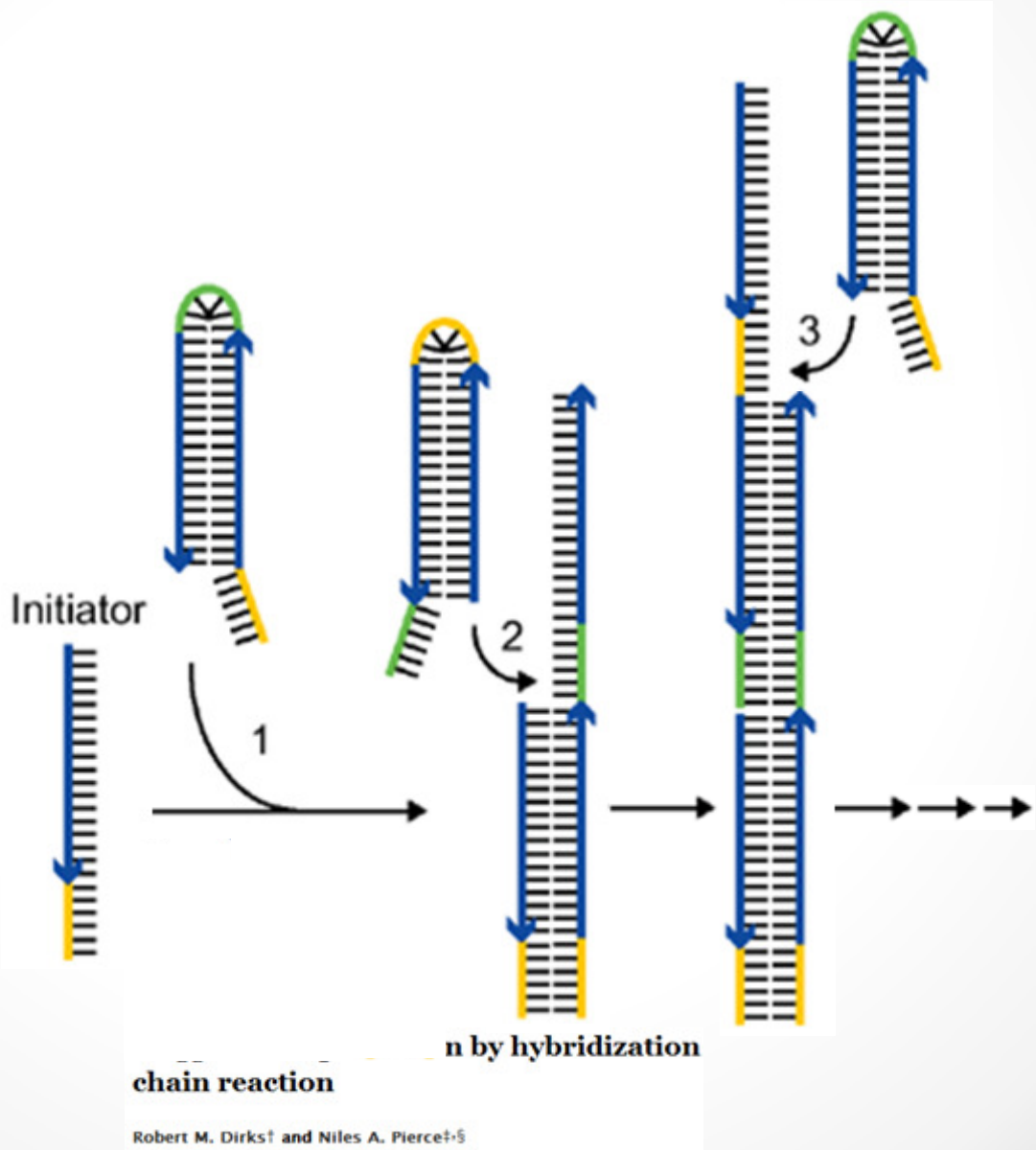


Actuating

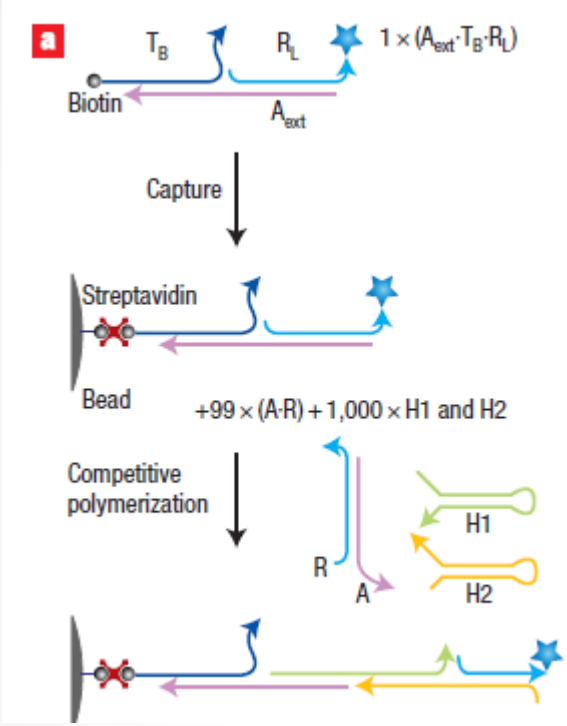
• • •

Hybridization Chain Reaction





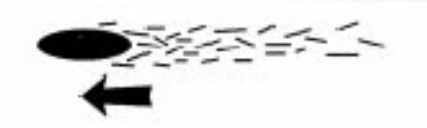
Polymerization Motor

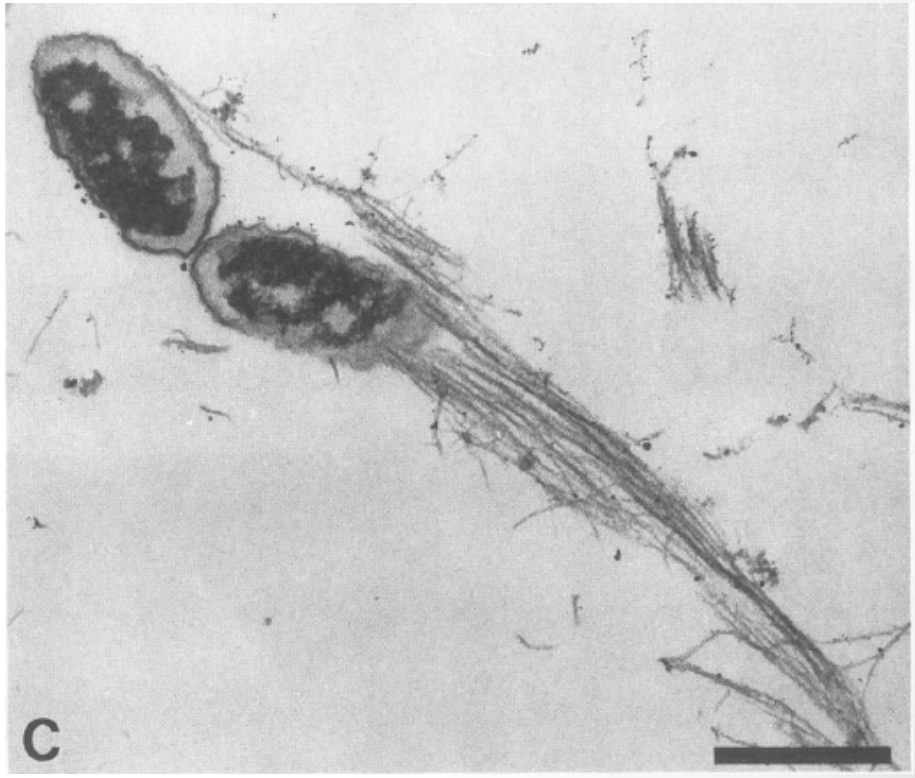


An autonomous polymerization motor powered by DNA hybridization

SUVIR VENKATARAMAN[1], ROBERT M. DIRKS[1], PAUL W. K. ROTHEMUND[2,3], ERIK WINFREE[2,3] AND NILES A. PIERCE[1,4]*

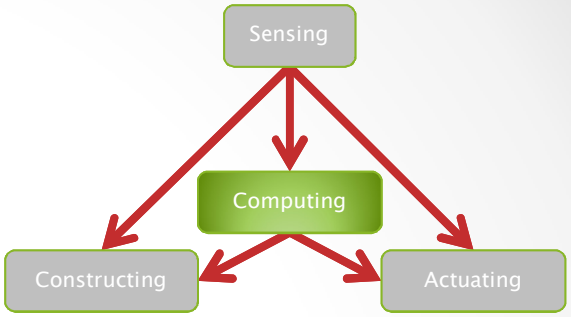
Rickettsia (spotted fever)





Directional Actin Polymerization Associated with Spotted Fever Group Rickettsia Infection of Vero Cells

ROBERT A. HEINZEN, STANLEY F. HAYES, MARIUS G. PEACOCK, AND TED HACKSTADT*



Computing

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Basic Notions

Compositionality

- Sensors and Actuators at the 'edge' of the system
  - They can use disparate kinds of inputs (sensors) and outputs (actuators)
- The 'kernel' of the system computes
  - Must use uniform inputs and outputs
- Compositionality in the kernel
  - Supporting 'arbitrary' computing complexity
  - The output of each computing components must be the same kind of 'signal' as the input
- sdf
  - If the inputs are voltages, the outputs must be voltages
  - If the inputs are DNA, the outputs must be DNA
- Central design question
  - What should our signals (not components!) be?
  - Then design components that manipulate those signals.

What does DNA Compute?

- Electronics has *electrons*
  - All electrons are the same: you can only count them
  - *Few* electrons = False; *lots* of electrons = True
  - But Boolean Logic is only a necessary evil to build symbolic computation
- DNA computing has *symbols* (DNA words)
  - DNA words are not all the same
  - Symbolic computation on abstract signals can be done *directly*
  - Signals are presented concurrently (in a soup)
  - No requirement to do Boolean Logic
- Then, what are our 'gates' (if not Boolean?)
  - Theory of Concurrency
  - Process Algebra as the "Boolean Algebra" of DNA Computing

Why Compute with DNA?

- Not to solve NP-complete problems.
- Not to put Intel out of business.
- Not to orchestrate protein production.
- To precisely control the organization and dynamics of matter and information at the molecular level.
  - The use of DNA is “accidental”.
  - No genes involved.
  - In fact, no material of biological origin.

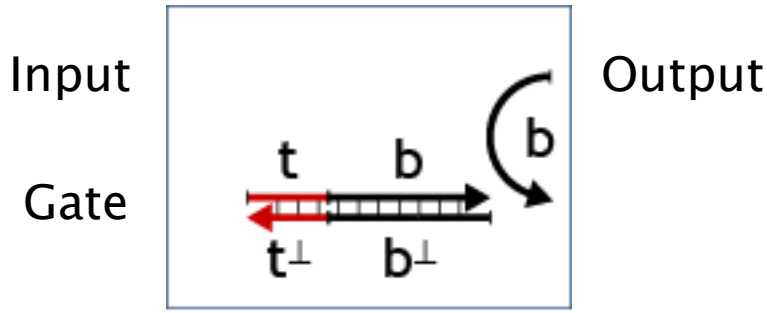
Rules of the Game

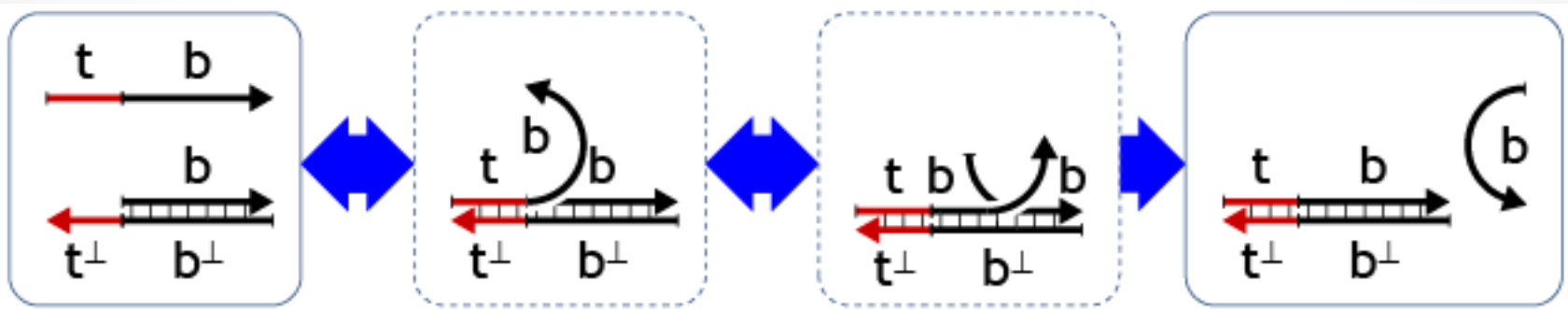
- Short complementary segments hybridize reversibly

- Long complementary segments hybridize irreversibly

DNA Strand Displacement

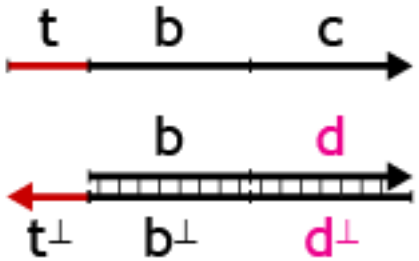
- Short strand (toehold): reversible binding
- Long strand (body): irreversible binding



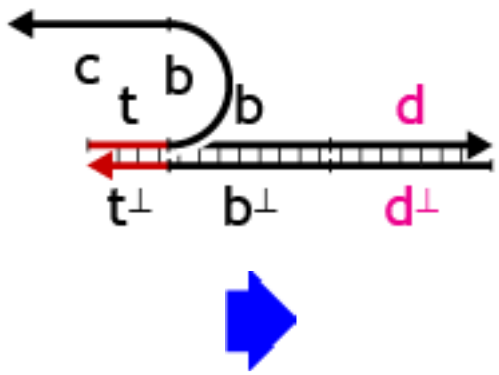


Failed Strand Displacement

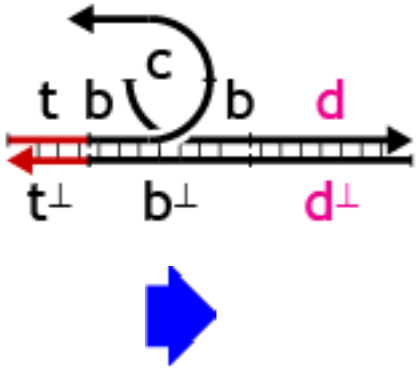
- What if the input does not match the gate?



Failed Strand Displacement



Failed Strand Displacement



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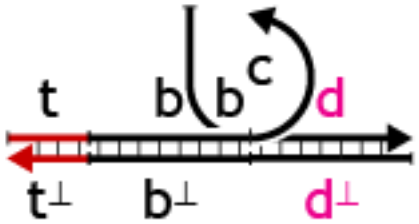

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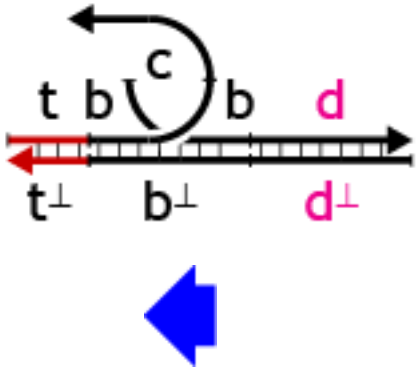
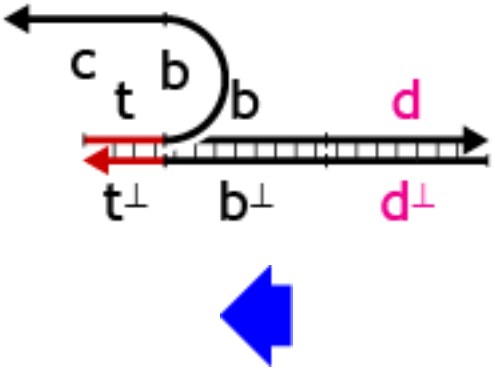
Failed Strand Displacement

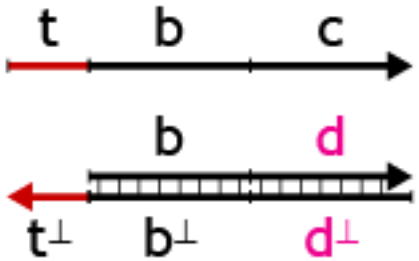


Failed Strand Displacement

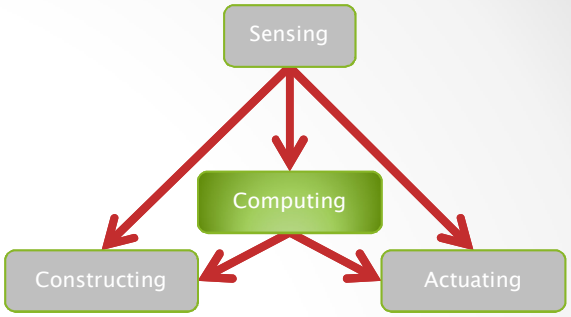


Failed Strand Displacement

- Hence an incorrect binding will undo
  - That's why toeholds must bind reversibly



- Matching depends on the long segment only
  - Strand displacement succeeds iff the whole long segment matches
  - The address space is determined by the size of the long segment, which is unbounded (not by the size of the toehold)
  - The toehold is just a 'cache' of the address



Computing

...

Implementing “Arbitrary” Computing Functions

Signals

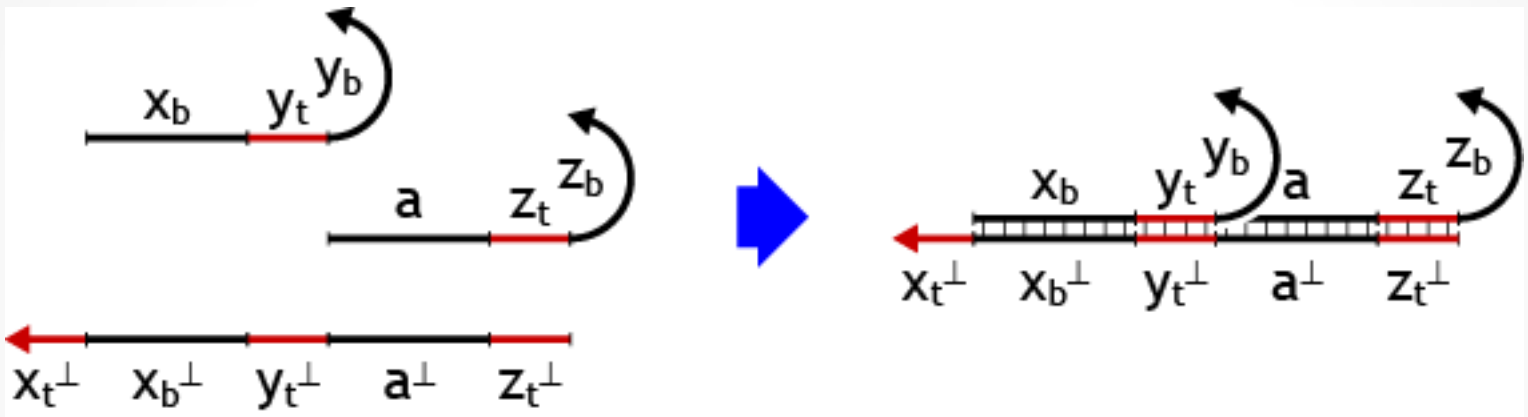
- A signal is the representation of an abstract event
  - E.g. generated by a sensor
  - E.g. accepted by an effector
  - We are not limited to true/false

- 3-domain signals
  - x_h: hystory (ignore)
  - x_t: toehold (binding)
  - x_b: body (recognition)

- Signals (single stranded DNA) are prepared by (artificial) DNA synthesis

Gates

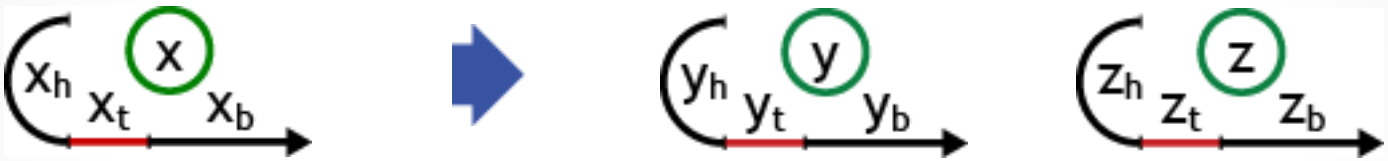
- Double-stranded structures with free toeholds



- Gates are prepared by self-assembly from single-stranded DNA that is synthesized

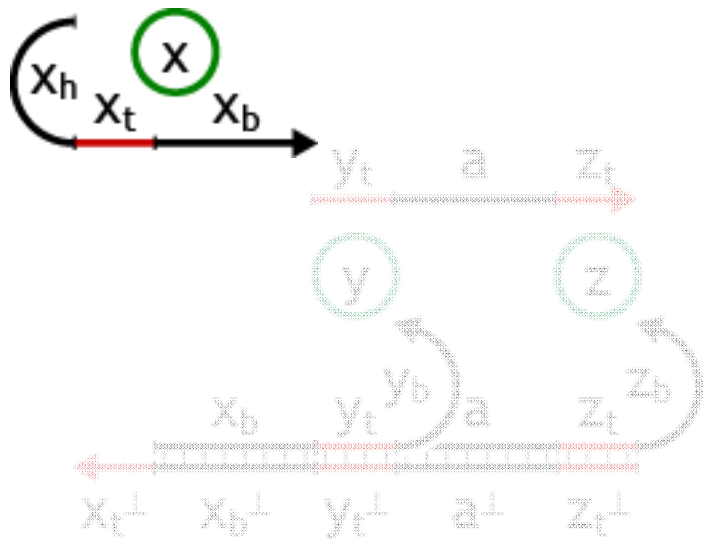
Fork Gate

- $x \rightarrow y + z$



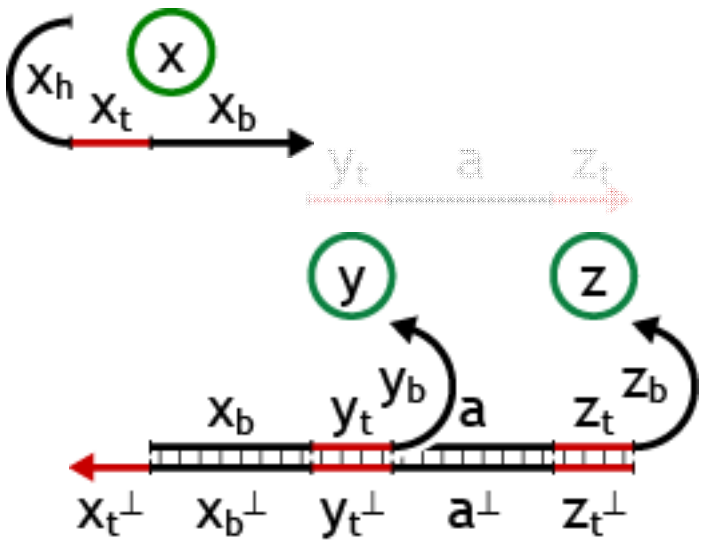
- $x \rightarrow y + 0$ transform x to y (transducer)
- $x \rightarrow x + y$ linear production of y (catalyst)
- $x \rightarrow x + x$ exponential production of x (amplifier)

Fork Gate

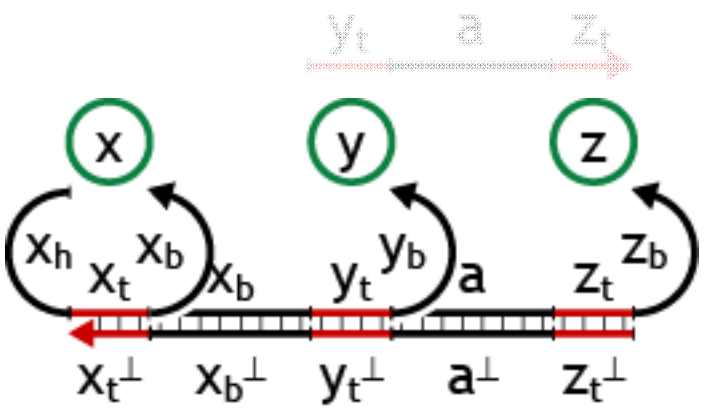


This is the Fork Gate structure

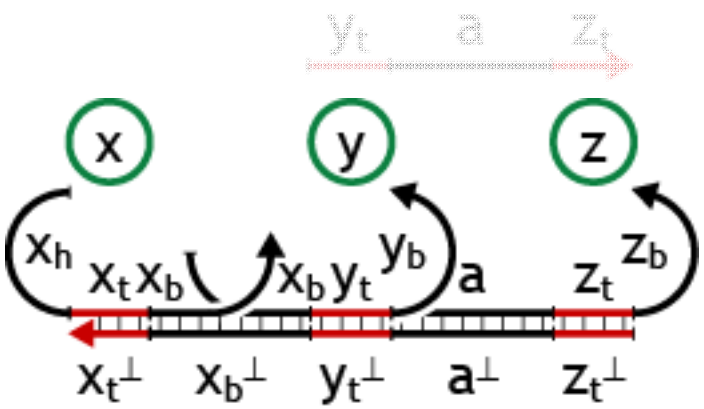
Fork Gate


x_h
x
x_t
x_b
y_t
a
z_t
y
z
y_b
z_b
x_b
y_t
a
z_t
$x_t^\perp$
$x_b^\perp$
$y_t^\perp$
$a^\perp$
$z_t^\perp$

Fork Gate


y_t a z_t
x
y
z
x_h x_t x_b
x_b y_t y_b
a z_t z_b
$x_t^\perp$ $x_b^\perp$ $y_t^\perp$ $a^\perp$ $z_t^\perp$

Fork Gate


y_t a z_t
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x_h
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x_b y_t
y_b
a
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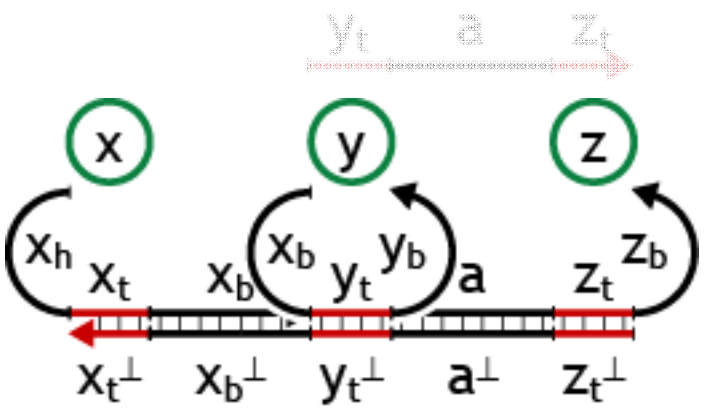
$x_t^\perp$
$x_b^\perp$
$y_t^\perp$
$a^\perp$
$z_t^\perp$

Fork Gate


y_t a z_t
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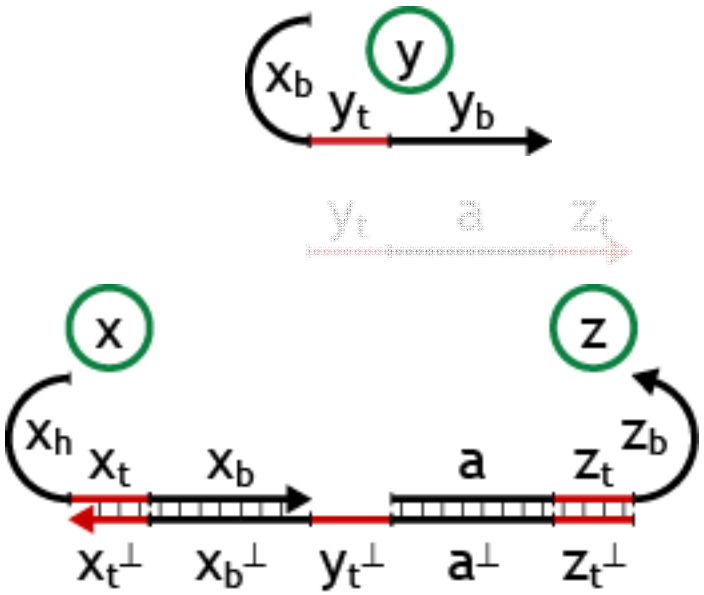
x_h x_t x_b x_b y_t y_b a z_t z_b
$x_t^\perp$ $x_b^\perp$ $y_t^\perp$ $a^\perp$ $z_t^\perp$

Fork Gate


y
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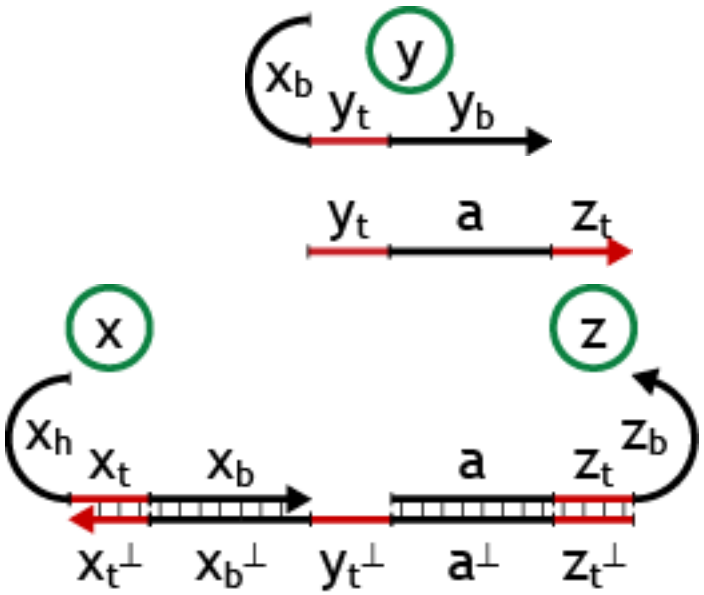
Fork Gate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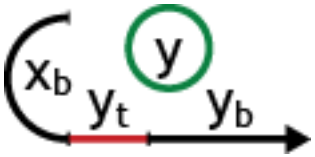
Fork Gate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x_b
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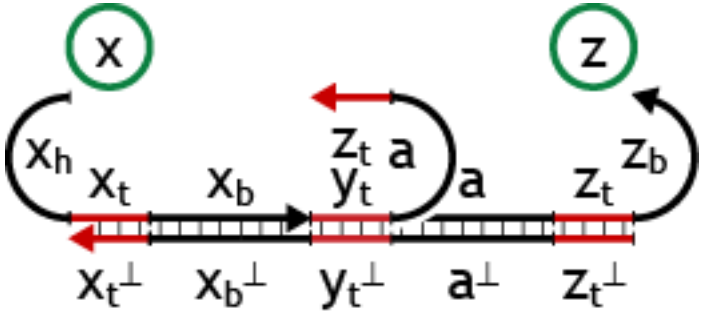
y_b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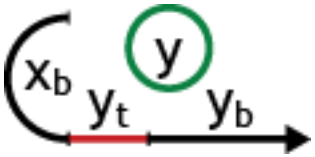
Fork Gate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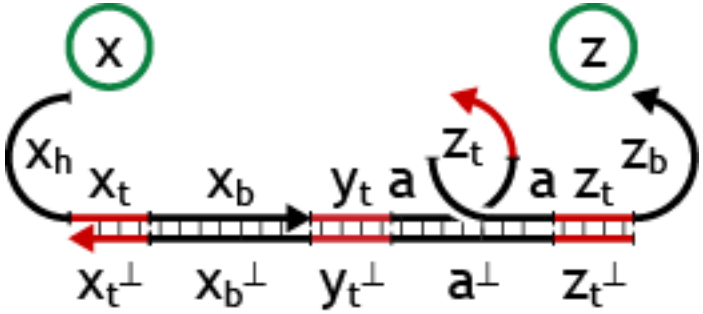
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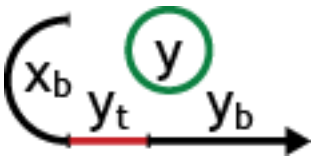
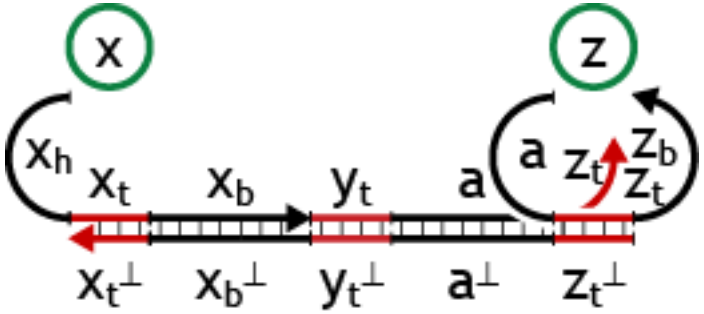
$a^\perp$
$z_t^\perp$

Fork Gate


x_b
y
y_t
y_b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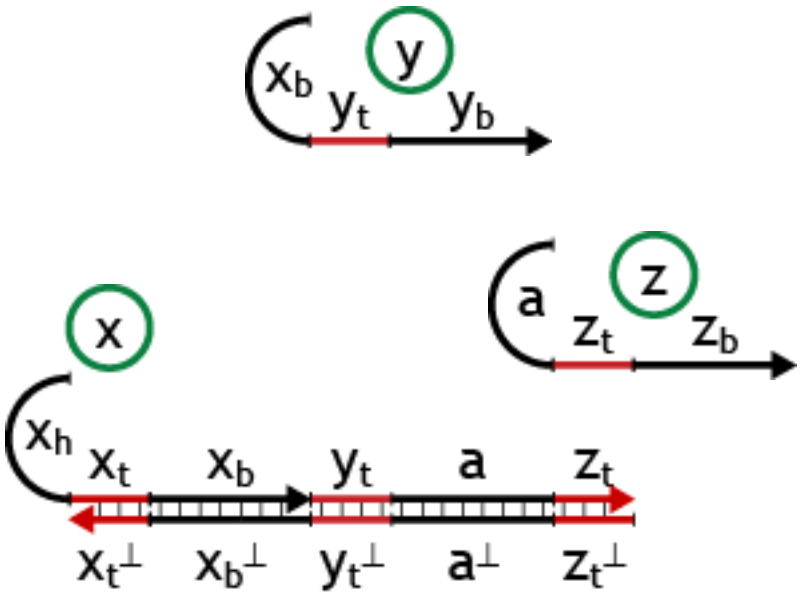
Fork Gate


x_b
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$x_t^\perp$
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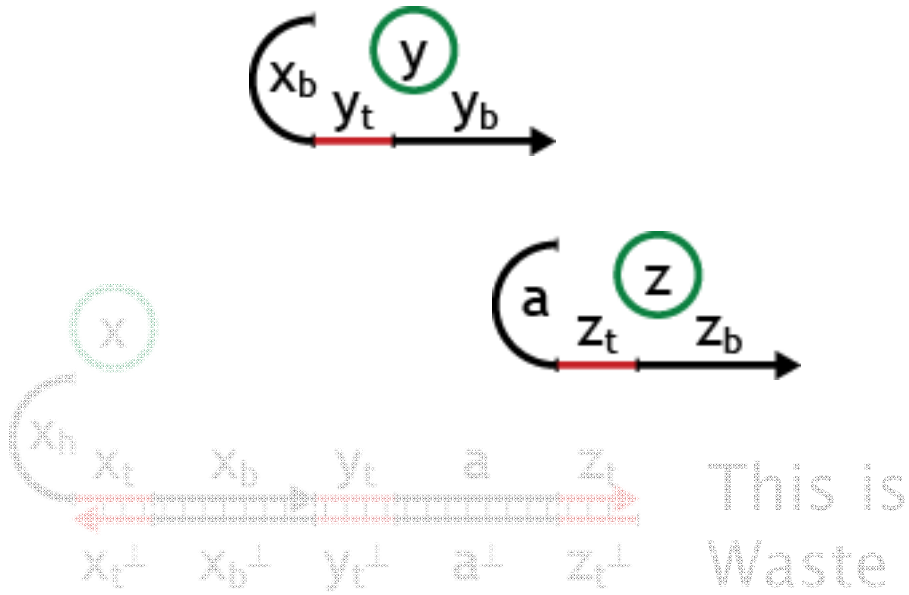
$y_t^\perp$
$a^\perp$
$z_t^\perp$

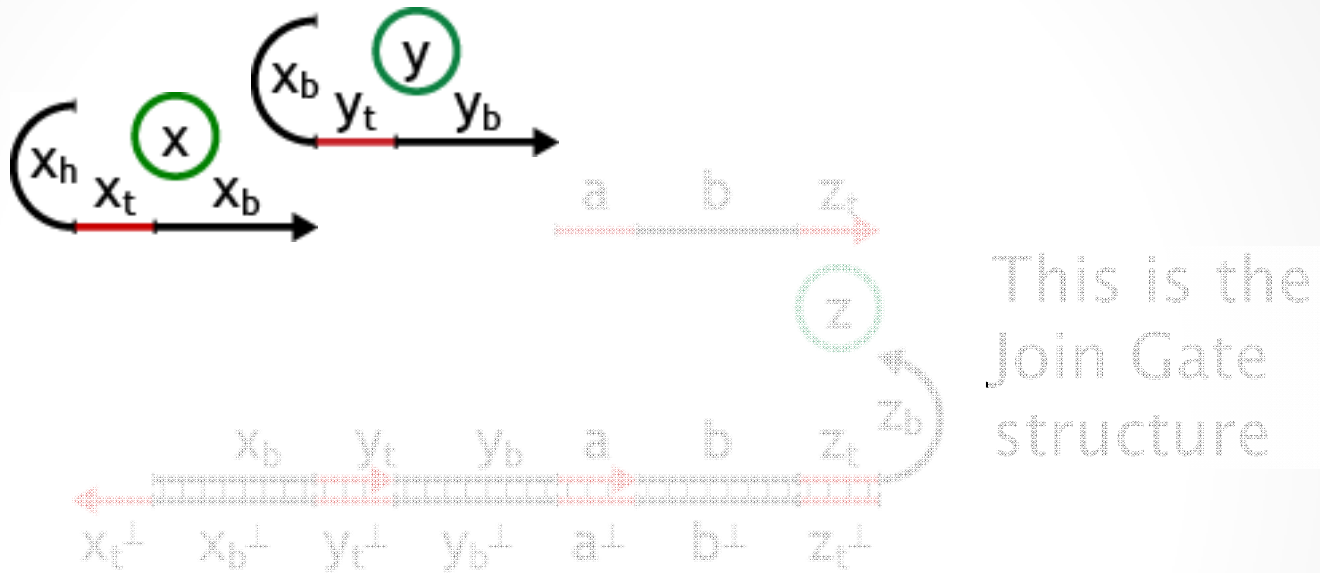
Fork Gate


x_b y_t y y_b
a z_t z z_b
x
x_b x_t x_b y_t a z_t
$x_t^\perp$ $x_b^\perp$ $y_t^\perp$ $a^\perp$ $z_t^\perp$
This is
Waste

Join Gate

- $x + y \rightarrow z$

Join Gate


x_h
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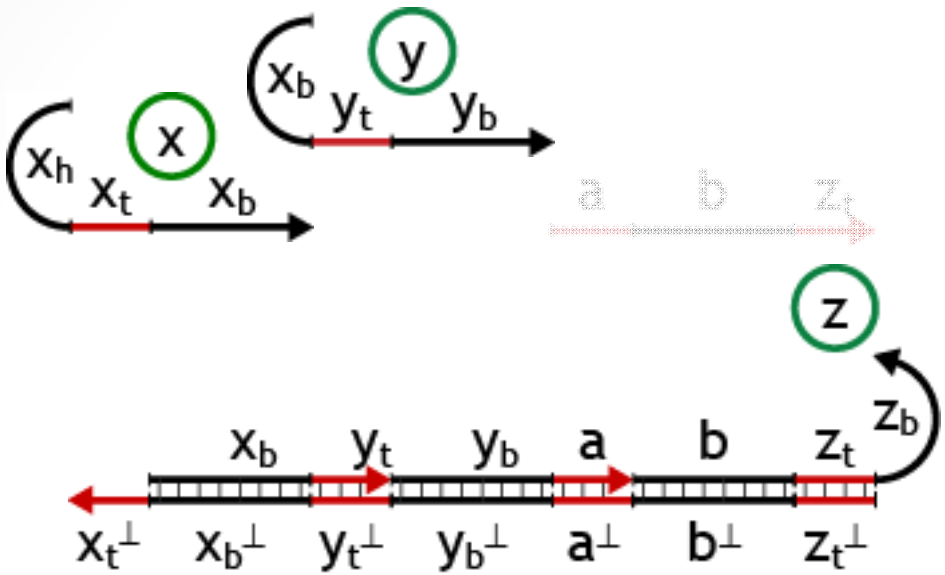
z_t
$x_t^\perp$
$x_b^\perp$
$y_t^\perp$
$y_b^\perp$
$a^\perp$
$b^\perp$
$z_t^\perp$
This is the Join Gate structure

Join Gate


x
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y_b
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a
b
z_t
$x_t^\perp$
$x_b^\perp$
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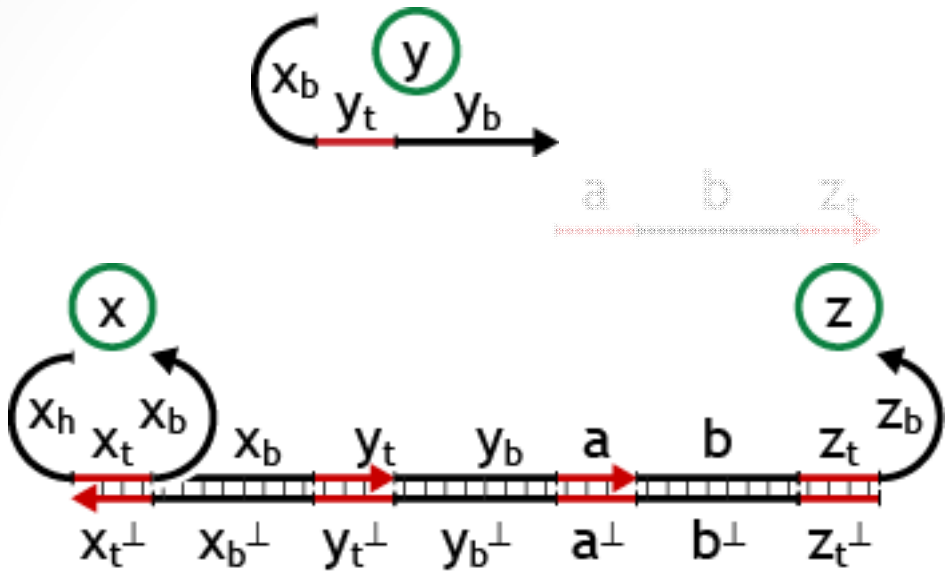
$y_b^\perp$
$a^\perp$
$b^\perp$
$z_t^\perp$

Join Gate


x_b
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z
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x_b
x_b
y_t
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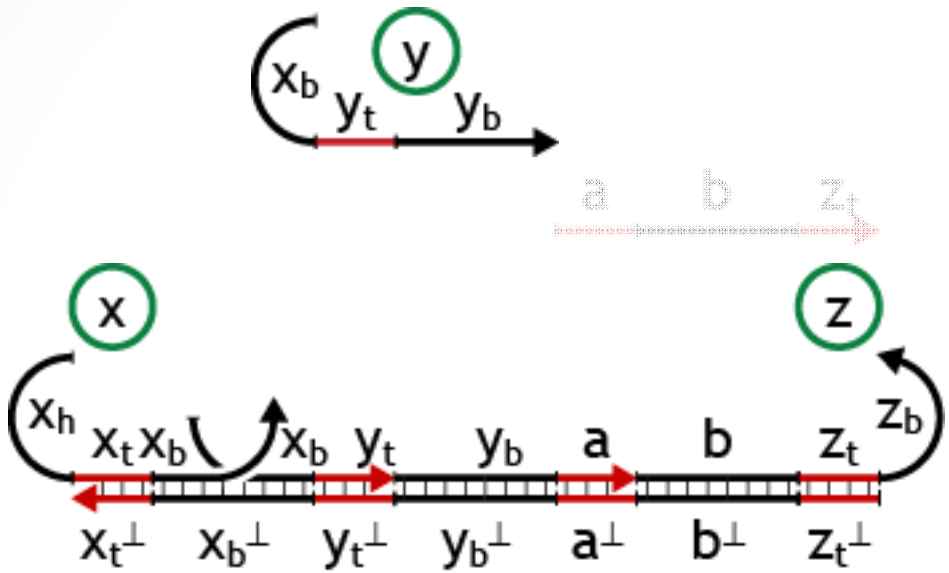
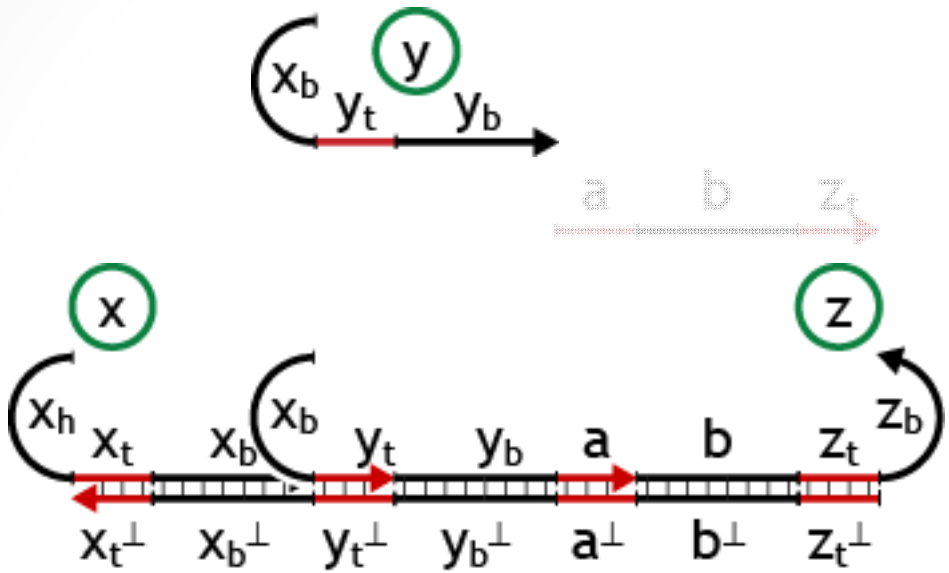
a
b
z_t
z_b
$x_t^\perp$
$x_b^\perp$
$y_t^\perp$
$y_b^\perp$
$a^\perp$
$b^\perp$
$z_t^\perp$

Join Gate


x_b
y
y_t
y_b
a
b
z_t
x
z
x_h
x_t
x_b
x_b
y_t
y_b
a
b
z_t
z_b
$x_t^\perp$
$x_b^\perp$
$y_t^\perp$
$y_b^\perp$
$a^\perp$
$b^\perp$
$z_t^\perp$

Join Gate


x_b
y
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y_b
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$y_b^\perp$
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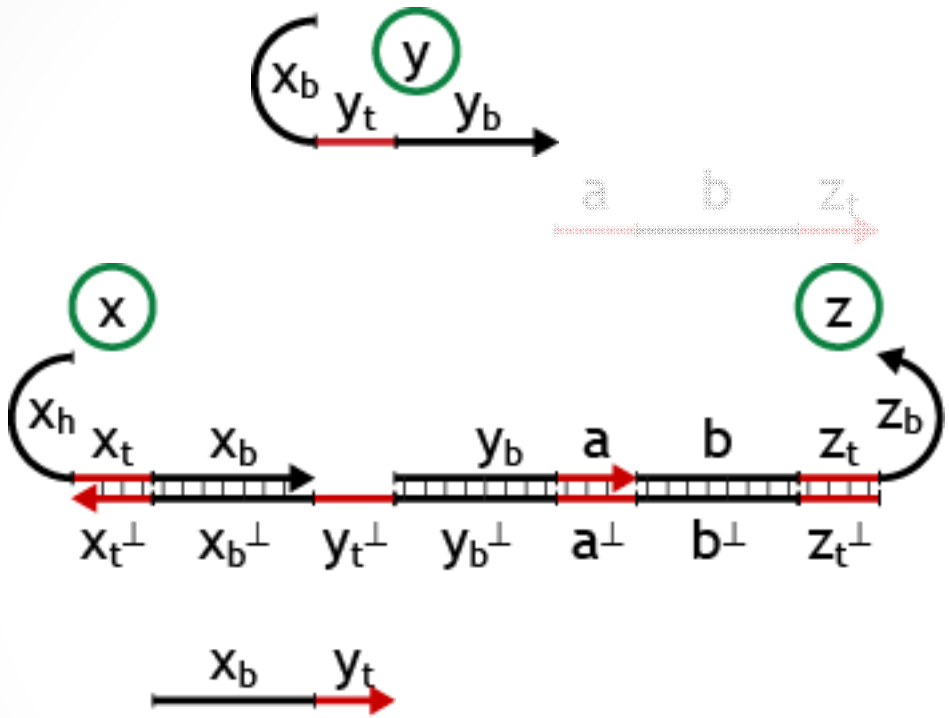
$b^\perp$
$z_t^\perp$

Join Gate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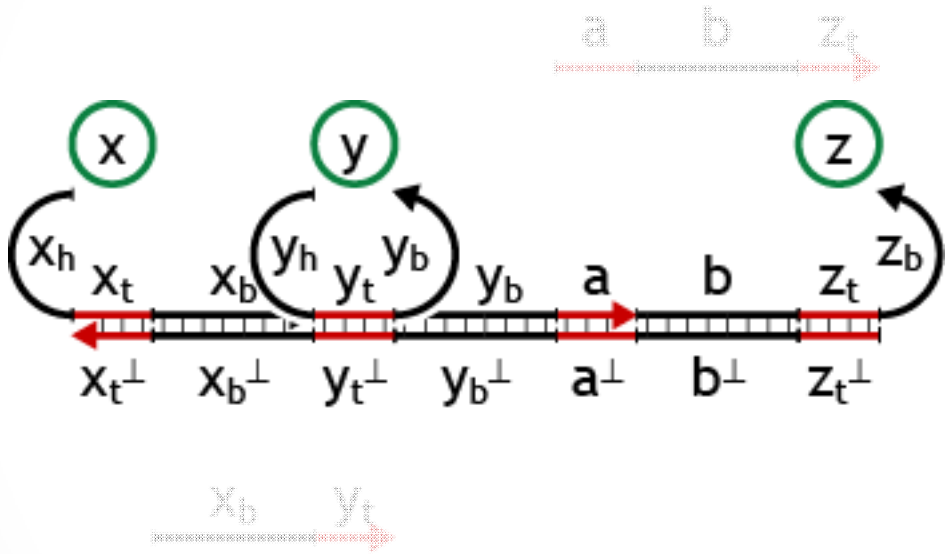
z_b
$x_t^\perp$
$x_b^\perp$
$y_t^\perp$
$y_b^\perp$
$a^\perp$
$b^\perp$
$z_t^\perp$
x_b
y_t

Join Gate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y_b
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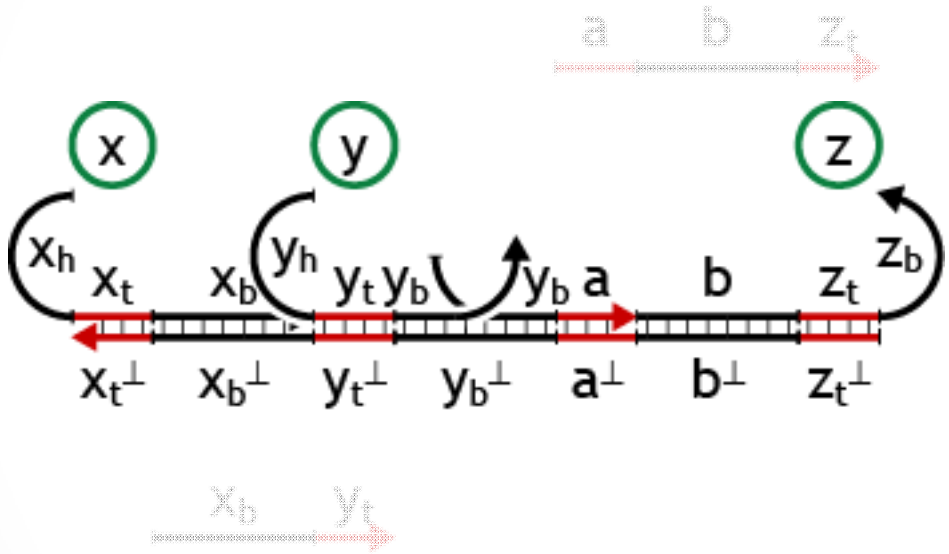
z_t
z_b
$x_t^\perp$
$x_b^\perp$
$y_t^\perp$
$y_b^\perp$
$a^\perp$
$b^\perp$
$z_t^\perp$
x_b
y_t

Join Gate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z_b
$x_t^\perp$
$x_b^\perp$
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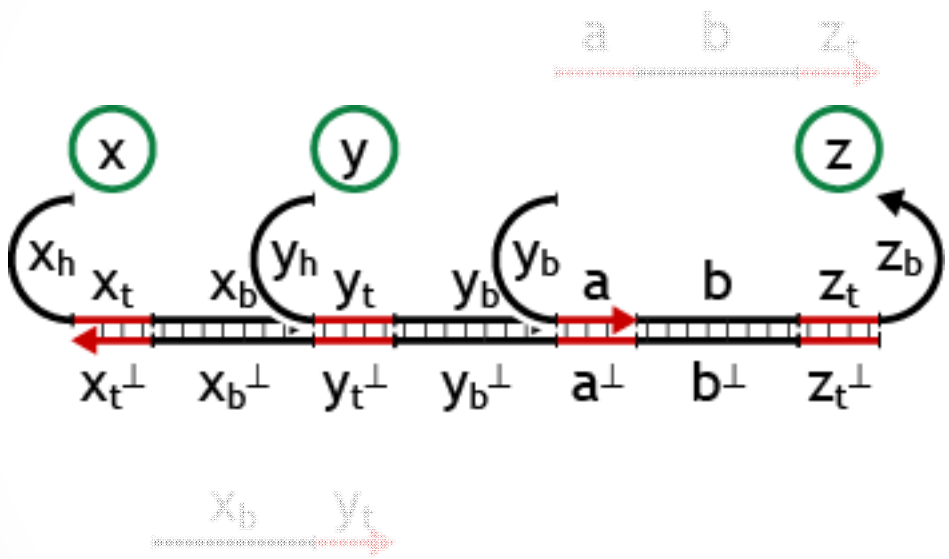
$a^\perp$
$b^\perp$
$z_t^\perp$
x_b
y_t

Join Gate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$x_t^\perp$
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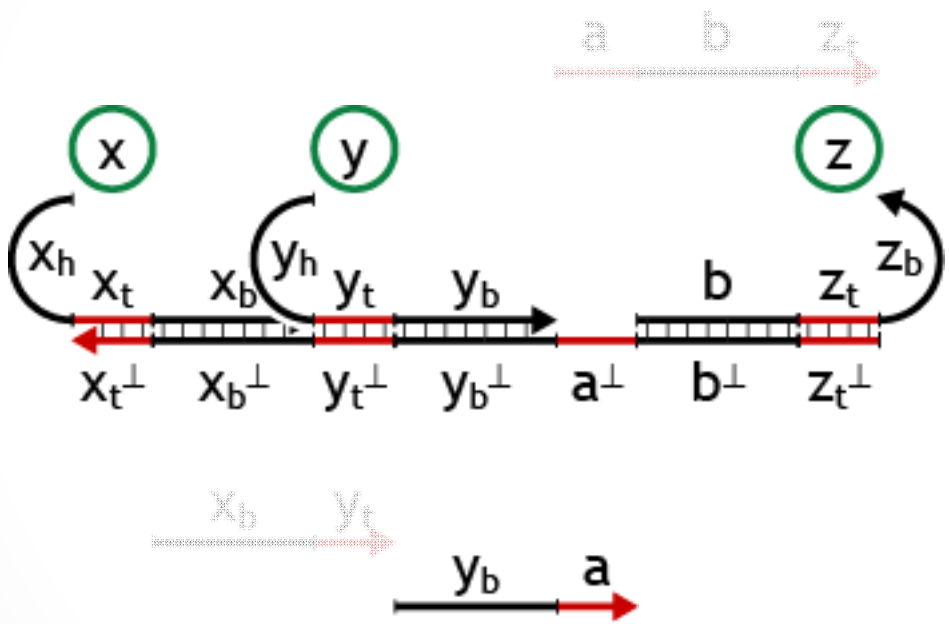
$a^\perp$
$b^\perp$
$z_t^\perp$
x_b
y_t

Join Gate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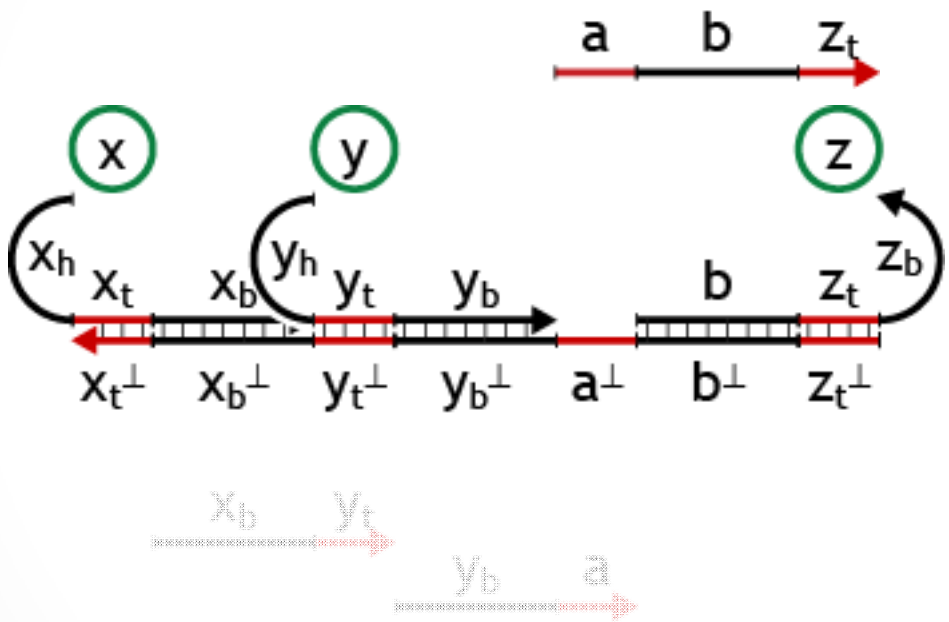
x_b
y_t
y_b
a

Join Gate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x_b
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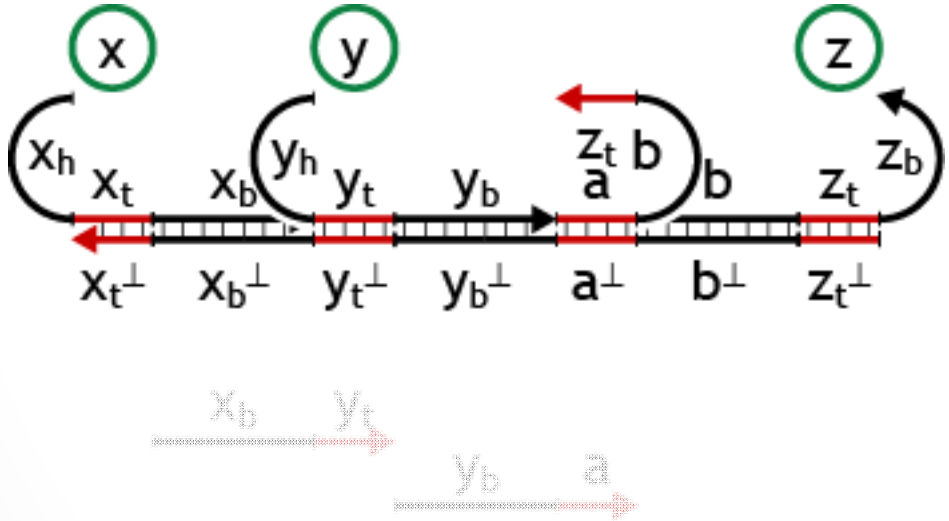
y_b
a

Join Gate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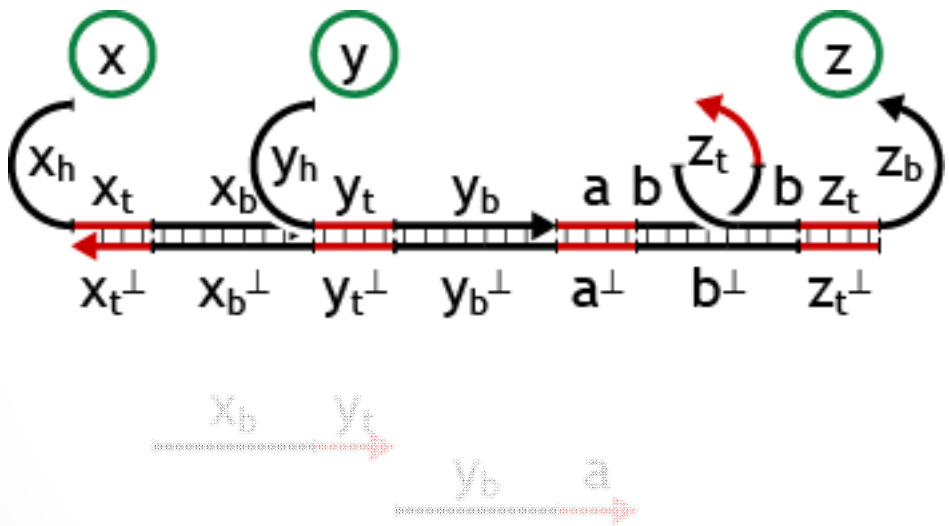
x_b
y_t
y_b
a

Join Gate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$y_t^\perp$
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$z_t^\perp$
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y_b
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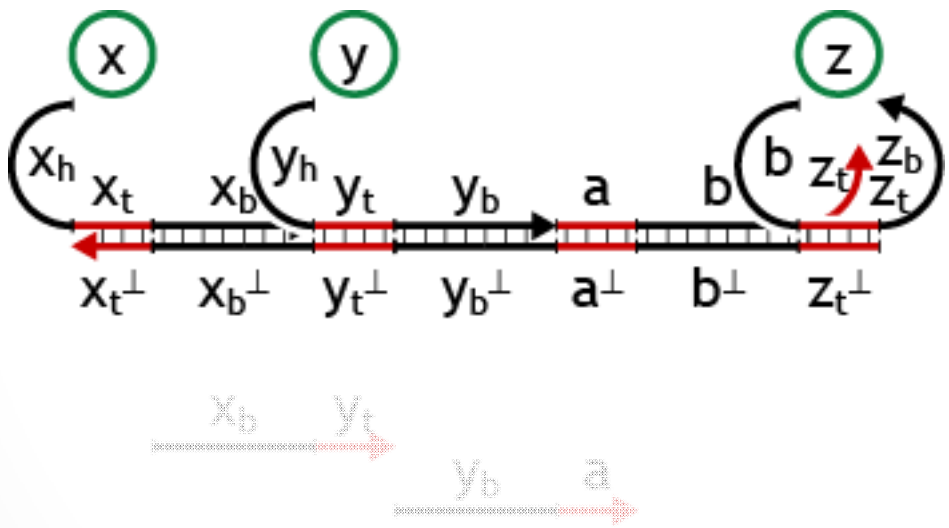
Join Gate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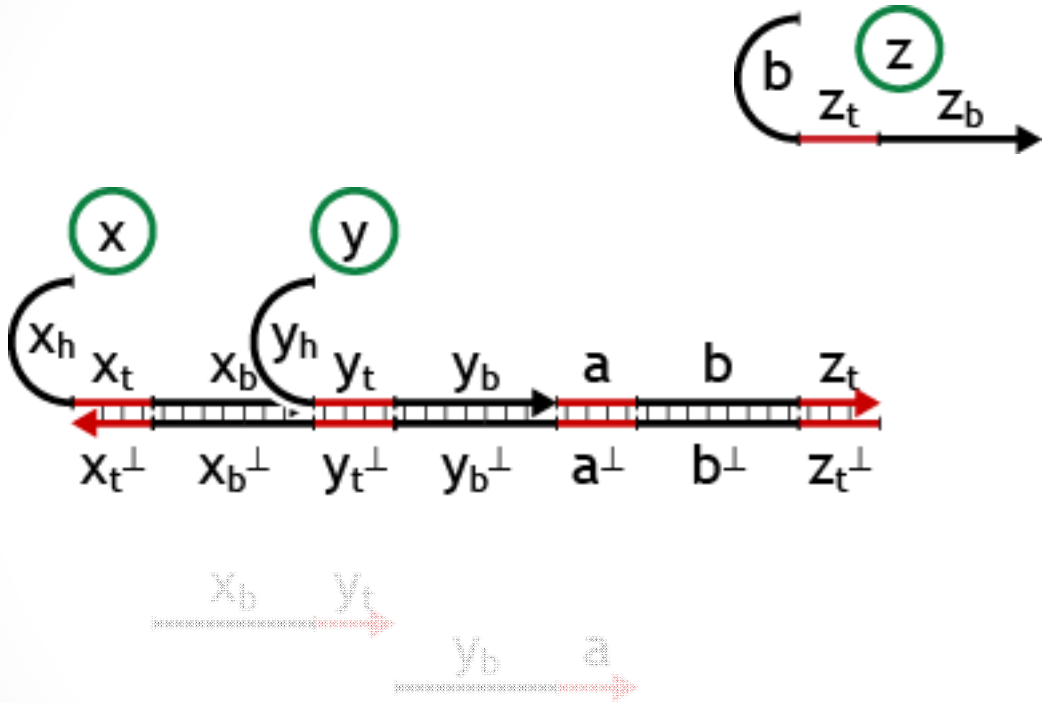
x_b
y_t
y_b
a

Join Gate


b
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z_b
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x_t
x_b
y_h
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a
b
z_t
$x_t^\perp$
$x_b^\perp$
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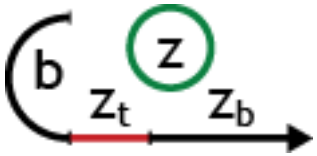
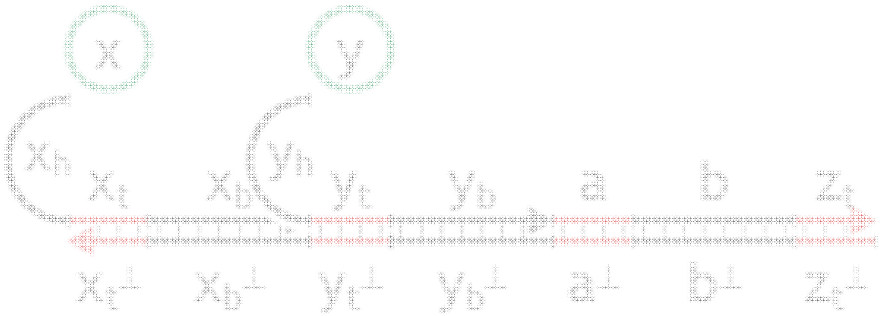
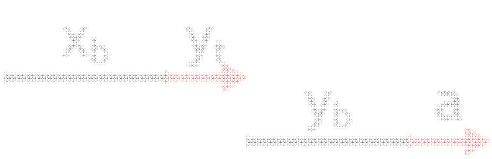
$b^\perp$
$z_t^\perp$
x_b
y_t
y_b
a

Join Gate

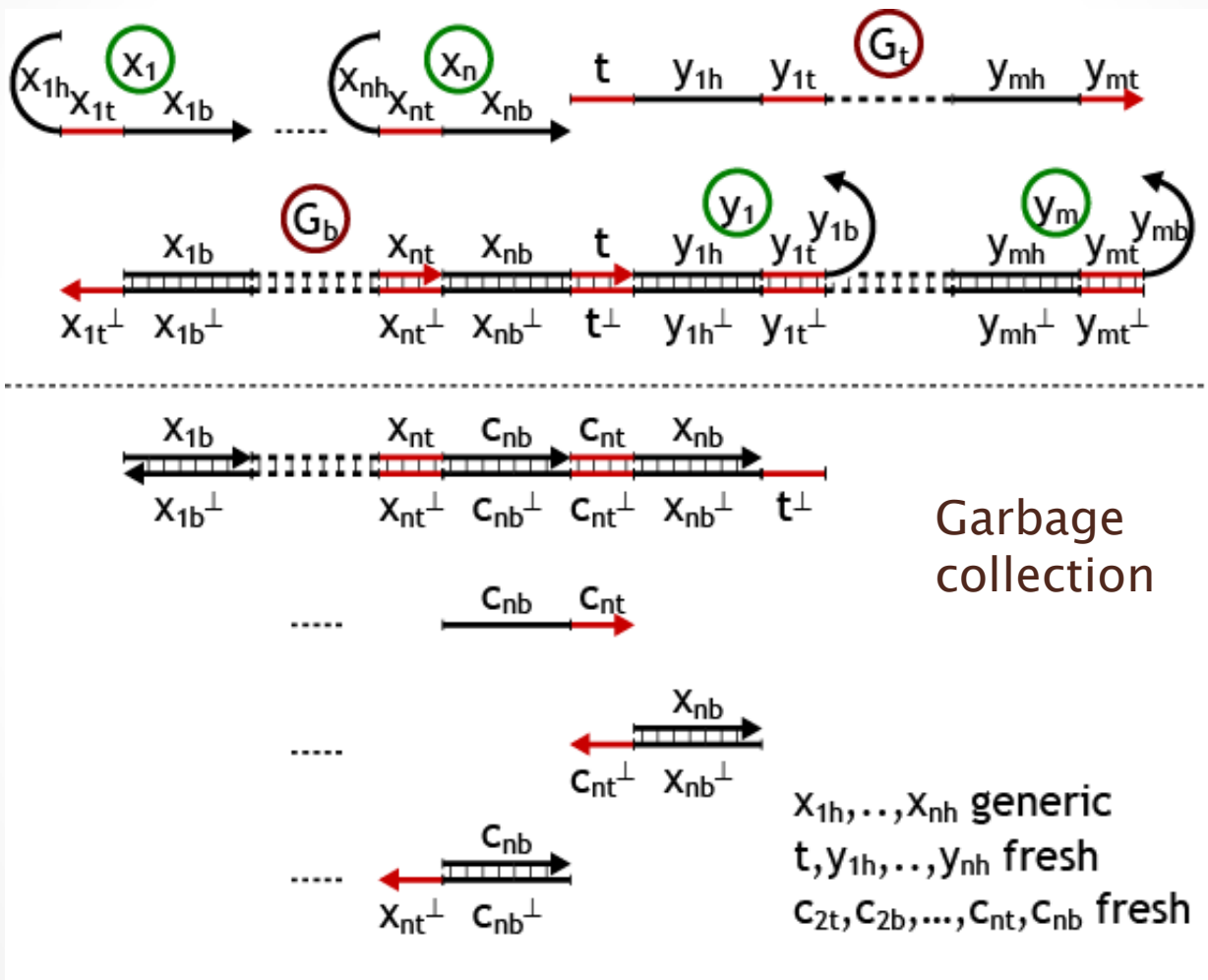






This is Waste

General n-Join/m-Fork Gate



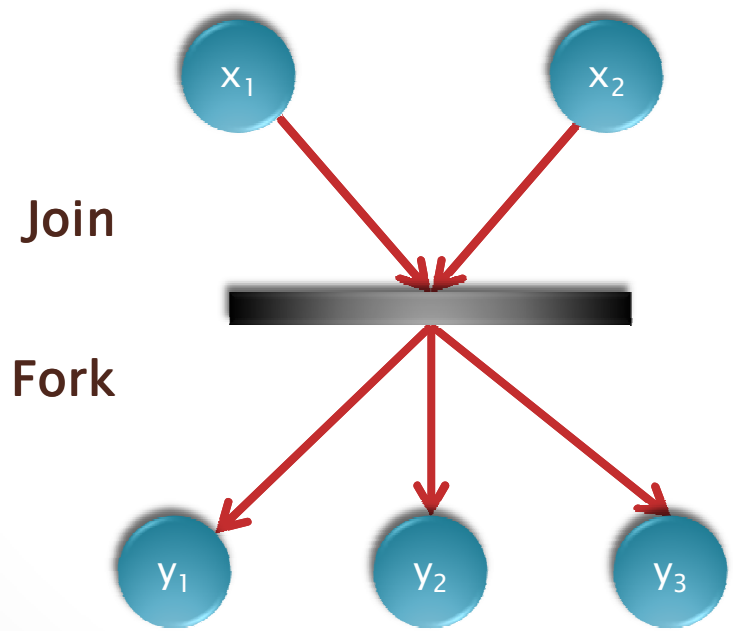
Gate Design Verification

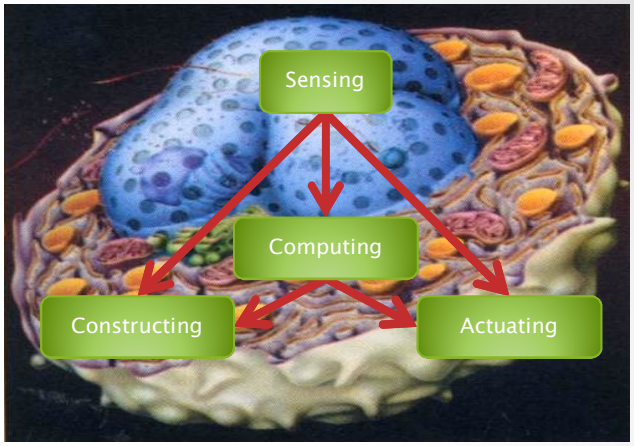
- Active garbage
  - The active join residuals slow down the performance of following joins.
  - → Add a garbage collector to remove the active residuals.
- Interference between gates
  - The join garbage collector interferes with the fork gate.
  - → Modify the fork gate to remove the interference.
- What else could go wrong?
  - Endless possibilities.
  - → Prove that the fork/join gate structures correctly implement fork/join in all larger circuits.

Strand Algebra

$$x_1 \mid .. \mid x_n \mid [x_1,..,x_n].[y_1,..,y_m] \rightarrow y_1 \mid .. \mid y_m$$

- Join + Fork + Populations = (Stochastic) Petri Nets





Curing

...

A Doctor in Each Cell

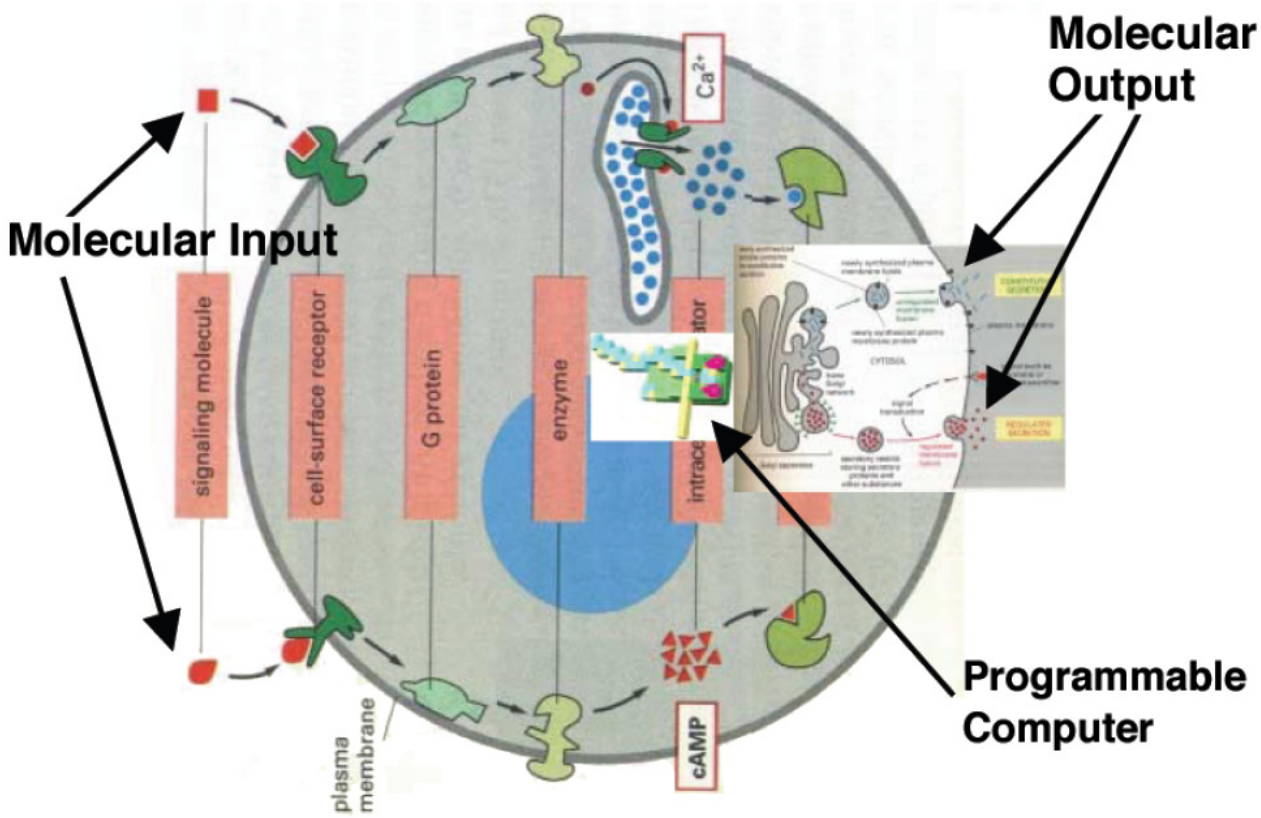


Fig. 1 Medicine in 2050: "Doctor in a Cell"

Ehud Shapiro

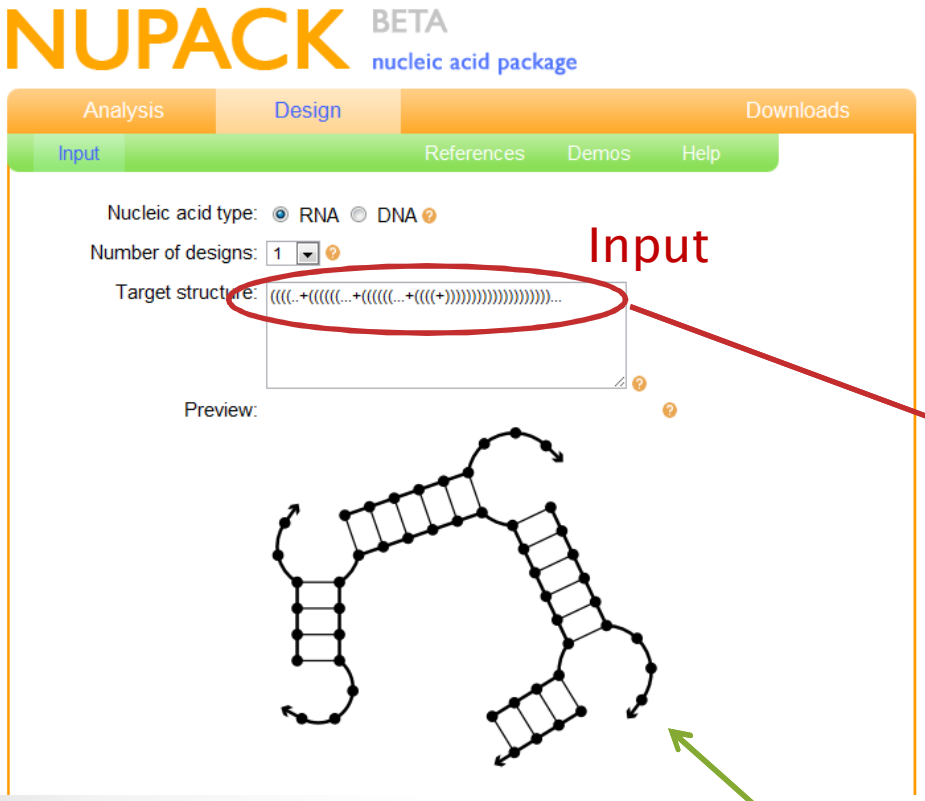
Rivka Adar
Kobi Benenson
Gregory Linshitz
Aviv Regev
William Silverman

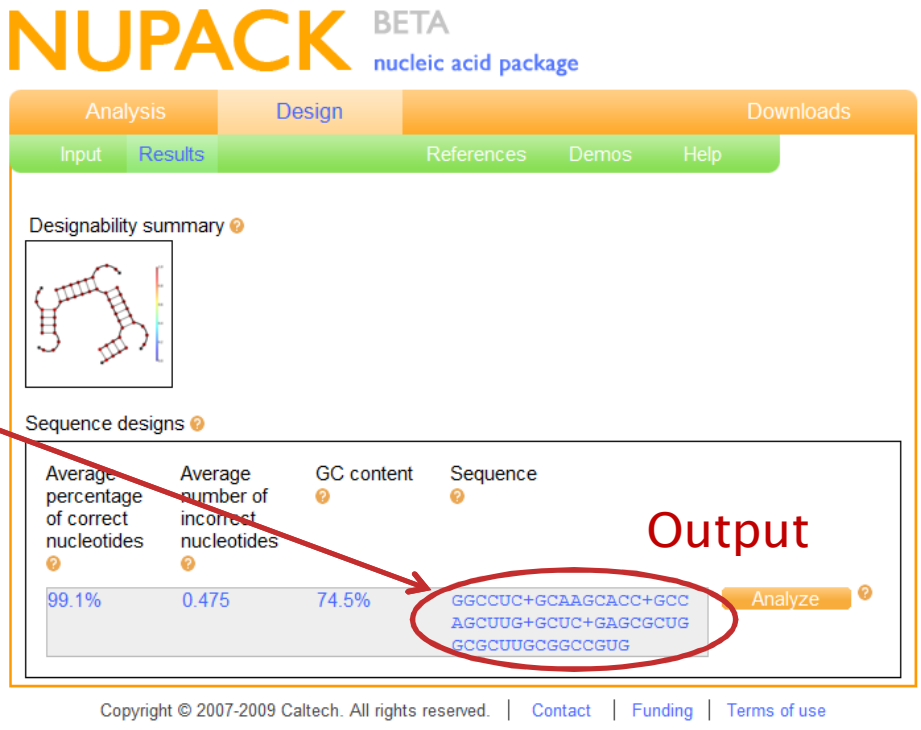
Molecules and computation

Tools

…

Sequence Design





So we can in principle work at this level.

Visual DSD
A Strand Displacement Simulator

Matthew Lakin, Simon Youssef, Andrew Phillips

http://lepton.research.microsoft.com/webdna/

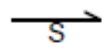
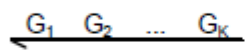
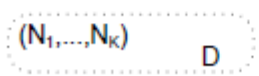
Syntax

Interface focus
IET
FirstCite e-publishing
J. R. Soc. Interface
doi:10.1098/rsif.2009.0072.focus
Published online

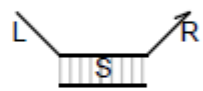
A programming language for composable DNA circuits

Andrew Phillips* and Luca Cardelli

A. Syntax of DNA molecules *D*

Upper strand with sequence complementary to S	 `<S>`
Molecule with segments $G_1,\ldots,G_K$	 `G1:G2:...:GK`
Parallel molecules $D_1,\ldots,D_K$	D_1 D_2 ... D_K `D1 | D2 | ... | DK`
Molecules *D* with private domains $N_1,\ldots,N_K$	 `new (N1,...,NK) D`

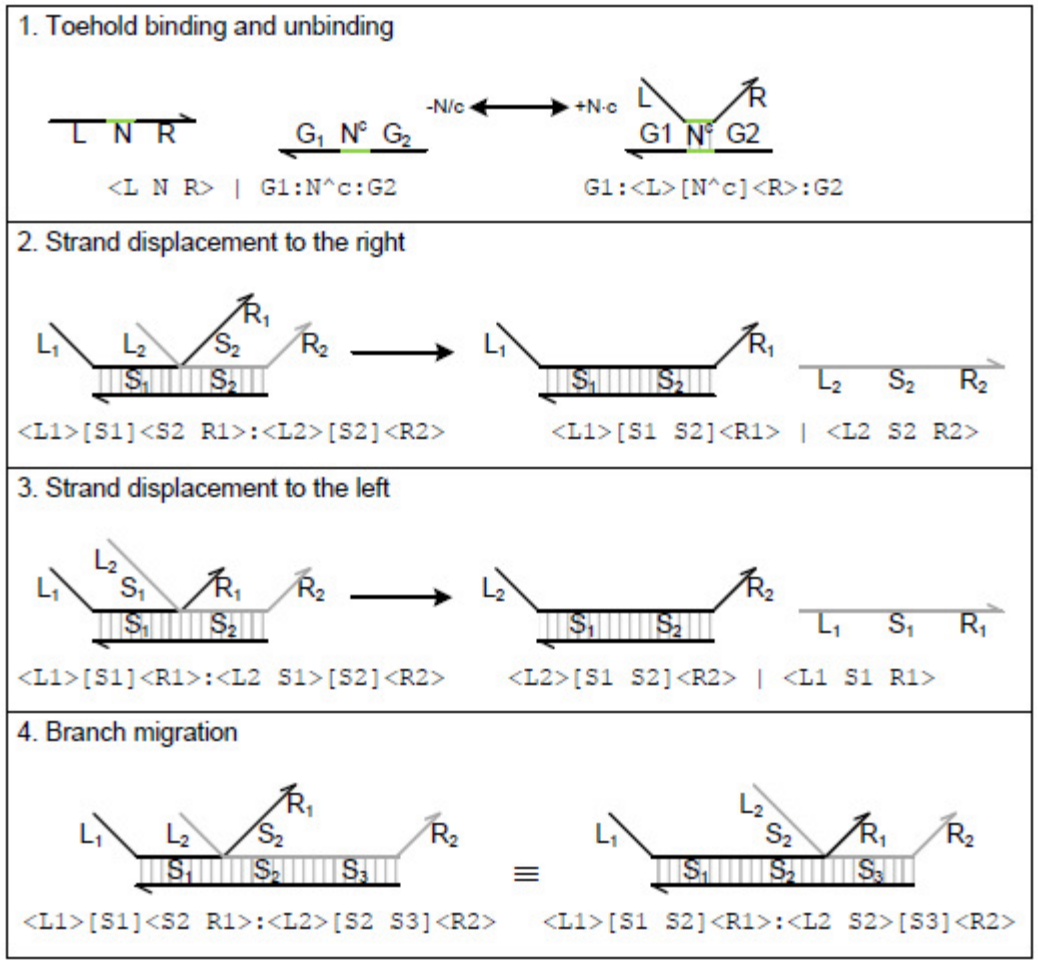
B. Syntax of DNA segments *G*

Lower strand with toehold N^c	 `N^c`
Double strand with sequence S and overhangs L, R	 `<L>[S]<R>`

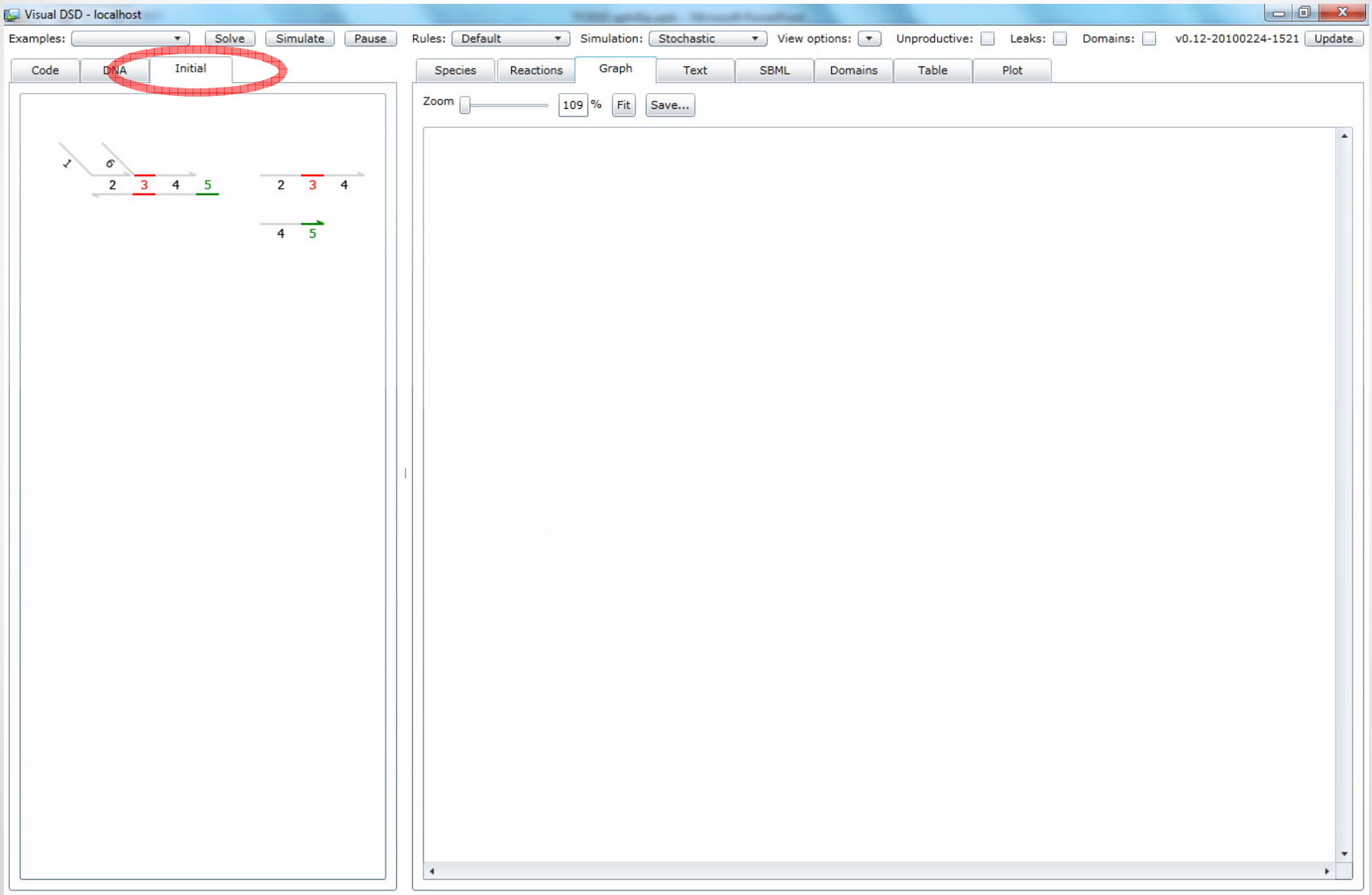
C. Syntax of DNA sequences *S,L,R*

Sequence of domains $O_1,\ldots,O_K$	O_1 O_2 ... O_K `O1 O2 ... OK`

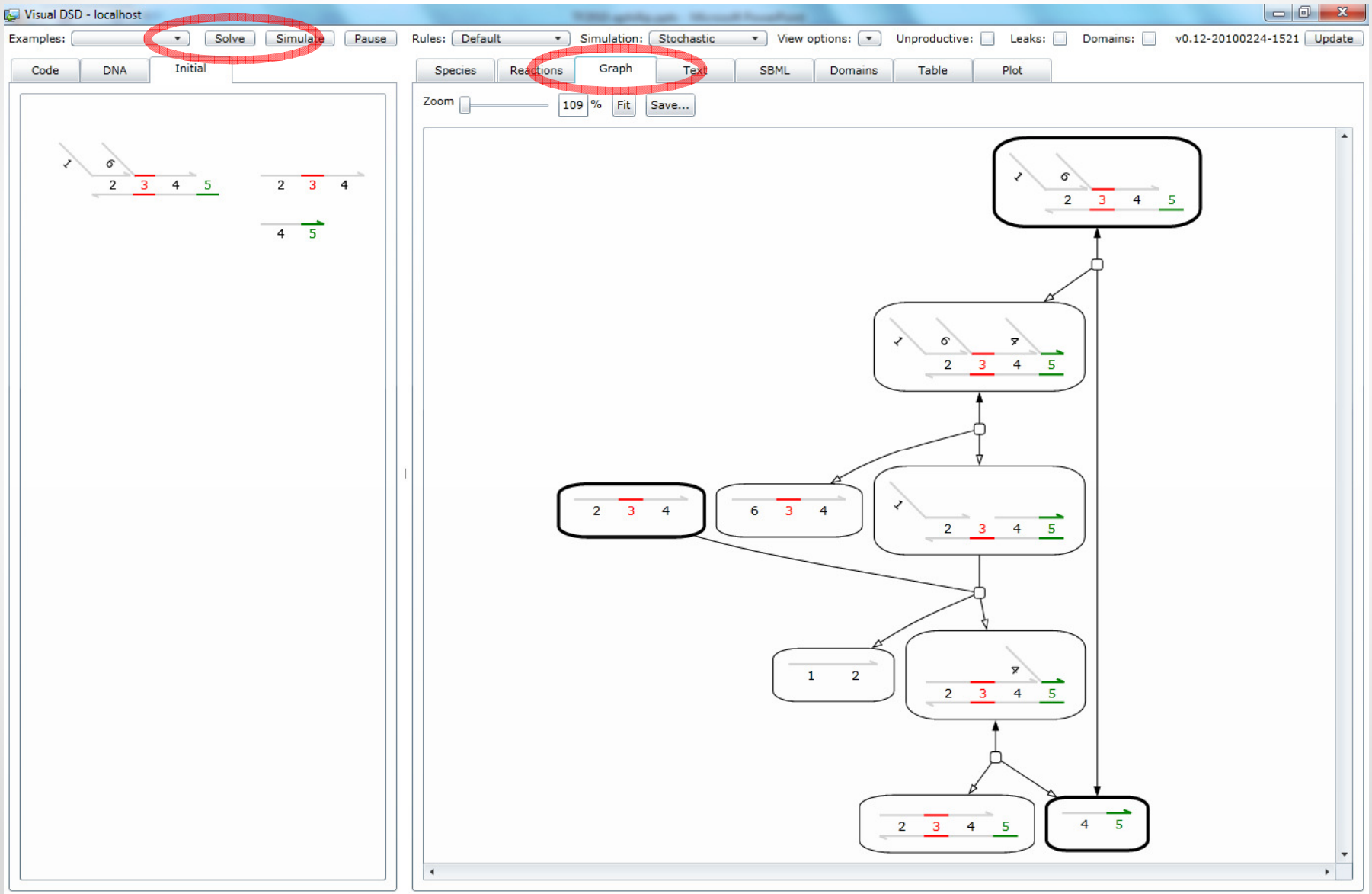
Dynamics


1. Toehold binding and unbinding
L N R
G1 N^c G2
-N/c
+N·c
L R
G1 N^c G2
<L N R> | G1:N^c:G2
G1:<L>[N^c]<R>:G2
2. Strand displacement to the right
L1 L2 S1 S2 R1 R2
<L1>[S1]<S2 R1>:<L2>[S2]<R2>
<L1>[S1 S2]<R1> | <L2 S2 R2>
3. Strand displacement to the left
<L1>[S1]<R1>:<L2 S1>[S2]<R2>
<L2>[S1 S2]<R2> | <L1 S1 R1>
4. Branch migration
L1 L2 S1 S2 S3 R1 R2
≡
<L1>[S1]<S2 R1>:<L2>[S2 S3]<R2>
<L1>[S1 S2]<R1>:<L2 S2>[S3]<R2>

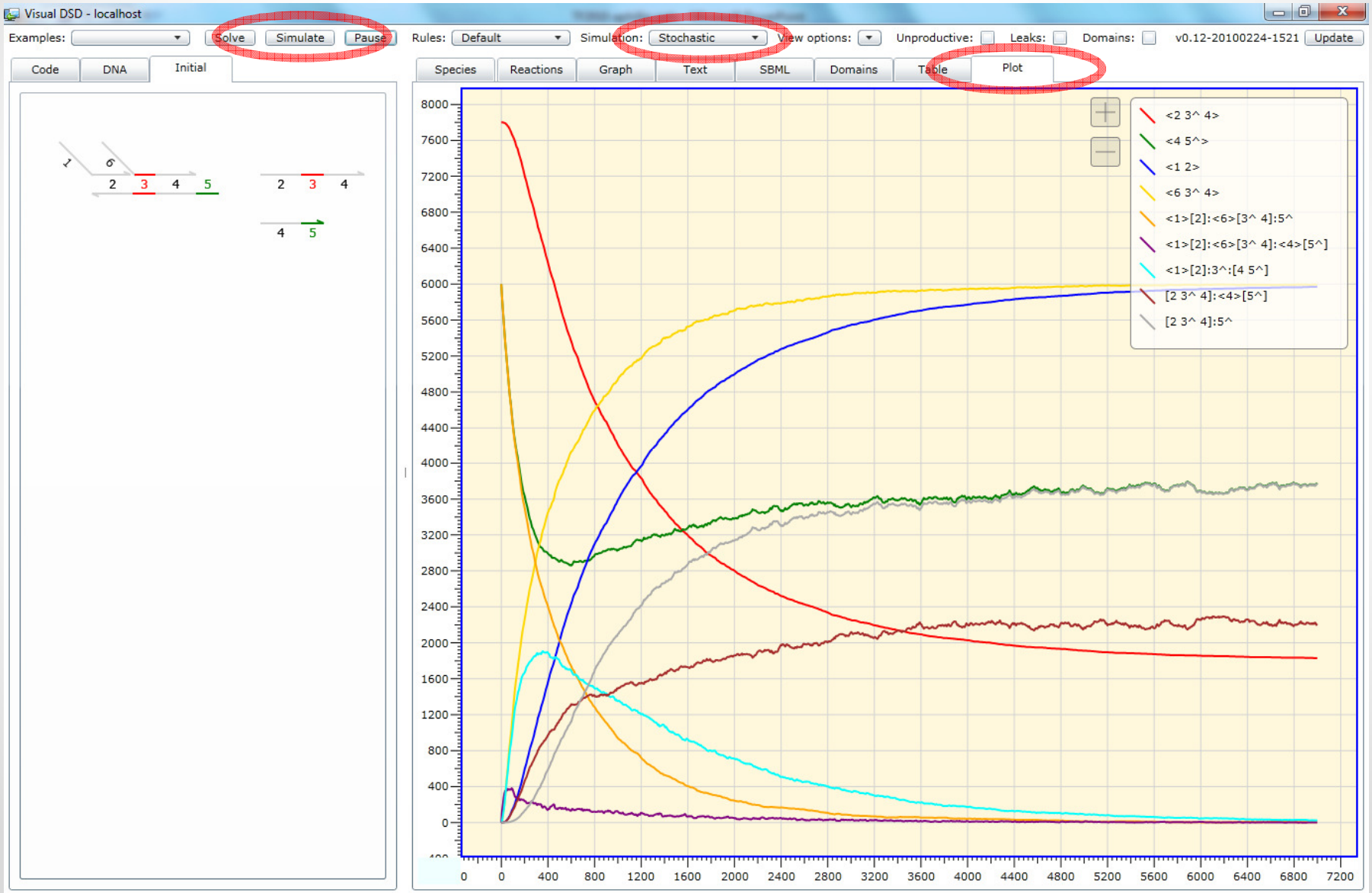
Initial Species



Reaction Graph



Simulation


Visual DSD - localhost
Examples:
Solve
Simulate
Pause
Rules: Default
Simulation: Stochastic
View options:
Unproductive:
Leaks:
Domains:
v0.12-20100224-1521
Update
Code
DNA
Initial
Species
Reactions
Graph
Text
SBML
Domains
Table
Plot
<2 3^ 4>
<4 5^>
<1 2>
<6 3^ 4>
<1>[2]:<6>[3^ 4]:5^
<1>[2]:<6>[3^ 4]:<4>[5^]
<1>[2]:3^:[4 5^]
[2 3^ 4]:<4>[5^]
[2 3^ 4]:5^

DNA Sequences

Visual DSD - localhost

Examples: | Solve | Simulate | Pause | Rules: Default | Simulation: Stochastic | View options: | Unproductive: | Leaks: | Domains: | v0.12-20100302-1033 | Update

Code | DNA | Initial | Species | Reactions | Graph | Text | SBML | Domains | Table | Plot

Check sequences | Reset

TOEHOLD SEQUENCES

TATTCC
GCTA
GTCA
TACCAA
CATCG
ACTACAC
CTCAG
CTCAATC
CCTACG
TCTCCA
CCCT
GACA
ACCT
TAGCCA
CACACA
AGAC

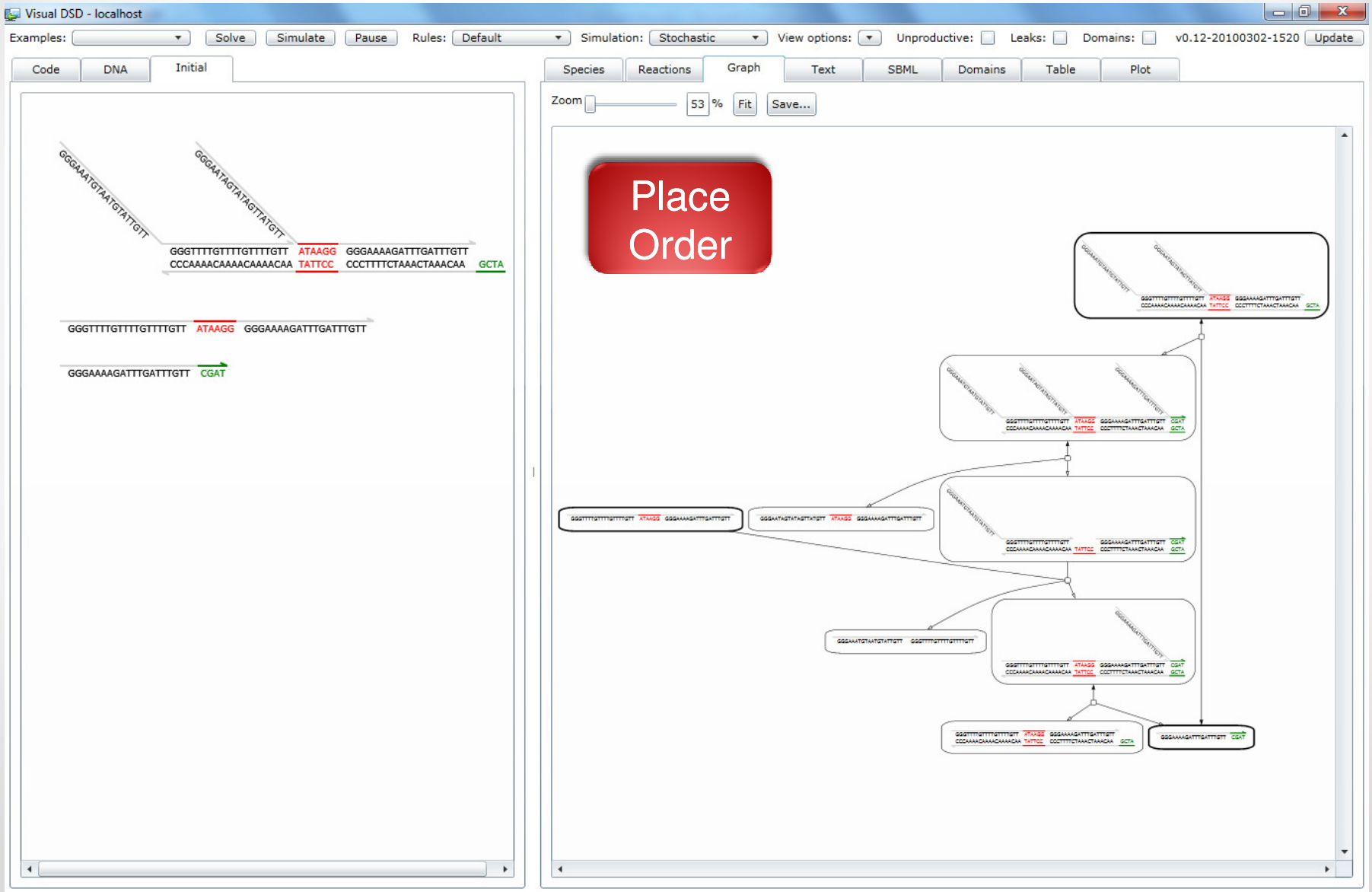
SPECIFICITY SEQUENCES

CCCAAAACAAAACAAAACAA
CCCTTTTCTAAACTAAACAA
CCCTTTACATTACATAACAA
CCCTTATCATATCAATACAA
CCCTTAACTTAACAAATCTA
CCCTATTCAATTCAAATCAA
CCCTATACTATACAATACTA
CCCTAATCTAATCATAACTA
CCCTAAACTTATCTAAACAT
CCCATTTCAAATCAAAACTT
CCCATTACTAATCAATTCAA
CCCATATCTATACATTACAA
CCCATAACTATTCTAAACTA
CCCAATTCTTAACATATCAA
CCCAATACTATTCATAACAT
CCCAAATCTTAACTATACTA
CCTATACCTTAACTTAACAA
CCATATCCATAACTTTACAA
CCATAACCTATACTTATCAA
CCATTTCCTTTTCTTAACTA
CCATTACCATATCTTATCAT
CCAAAACCATAACATAACTT

Zoom

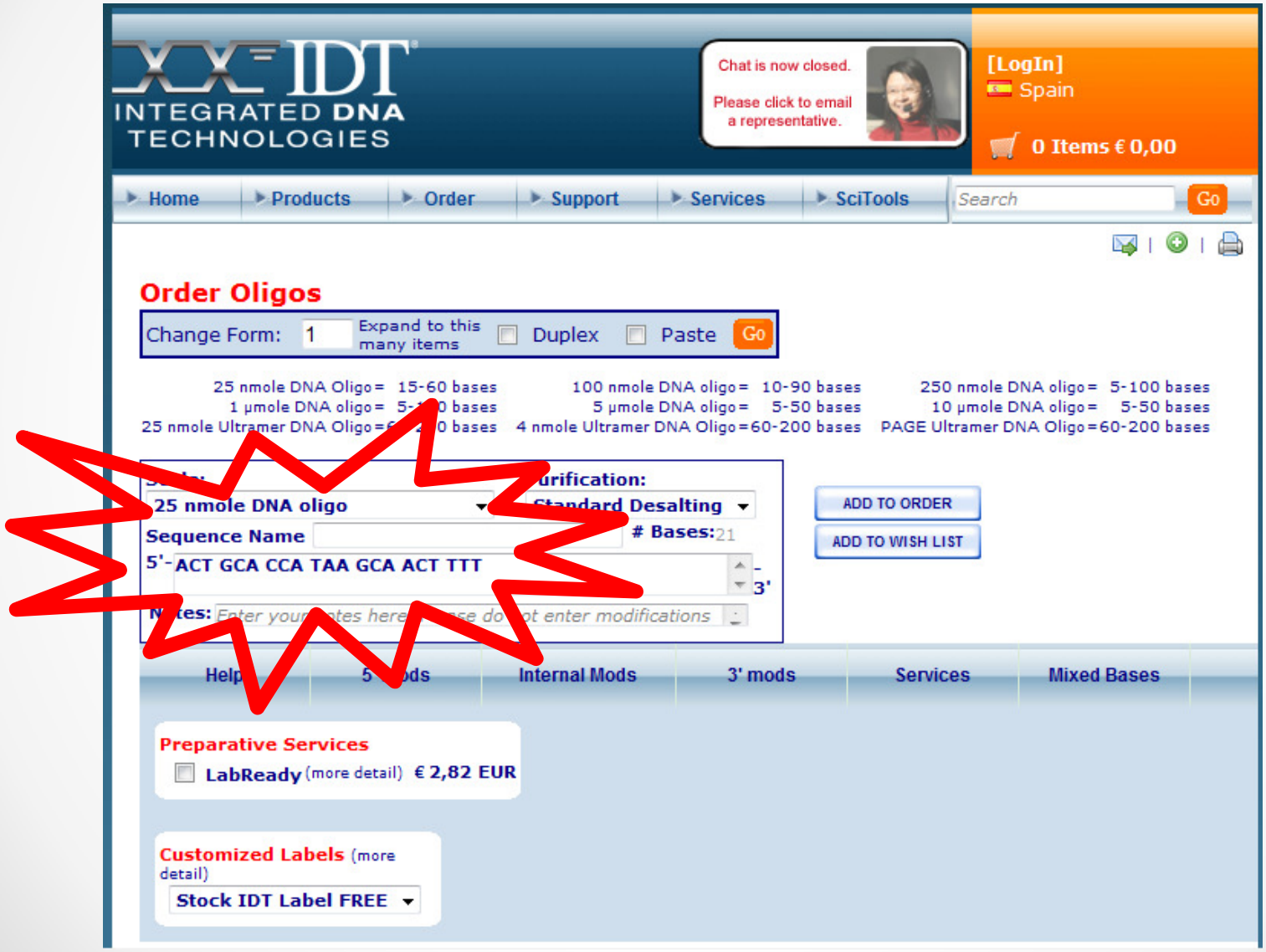
3^ --> TATTCC
5^ --> GCTA
1 --> CCCTTTACATTACATAACAA
2 --> CCCAAAACAAAACAAAACAA
4 --> CCCTTTTCTAAACTAAACAA
6 --> CCCTTATCATATCAATACAA

Final DNA Circuit

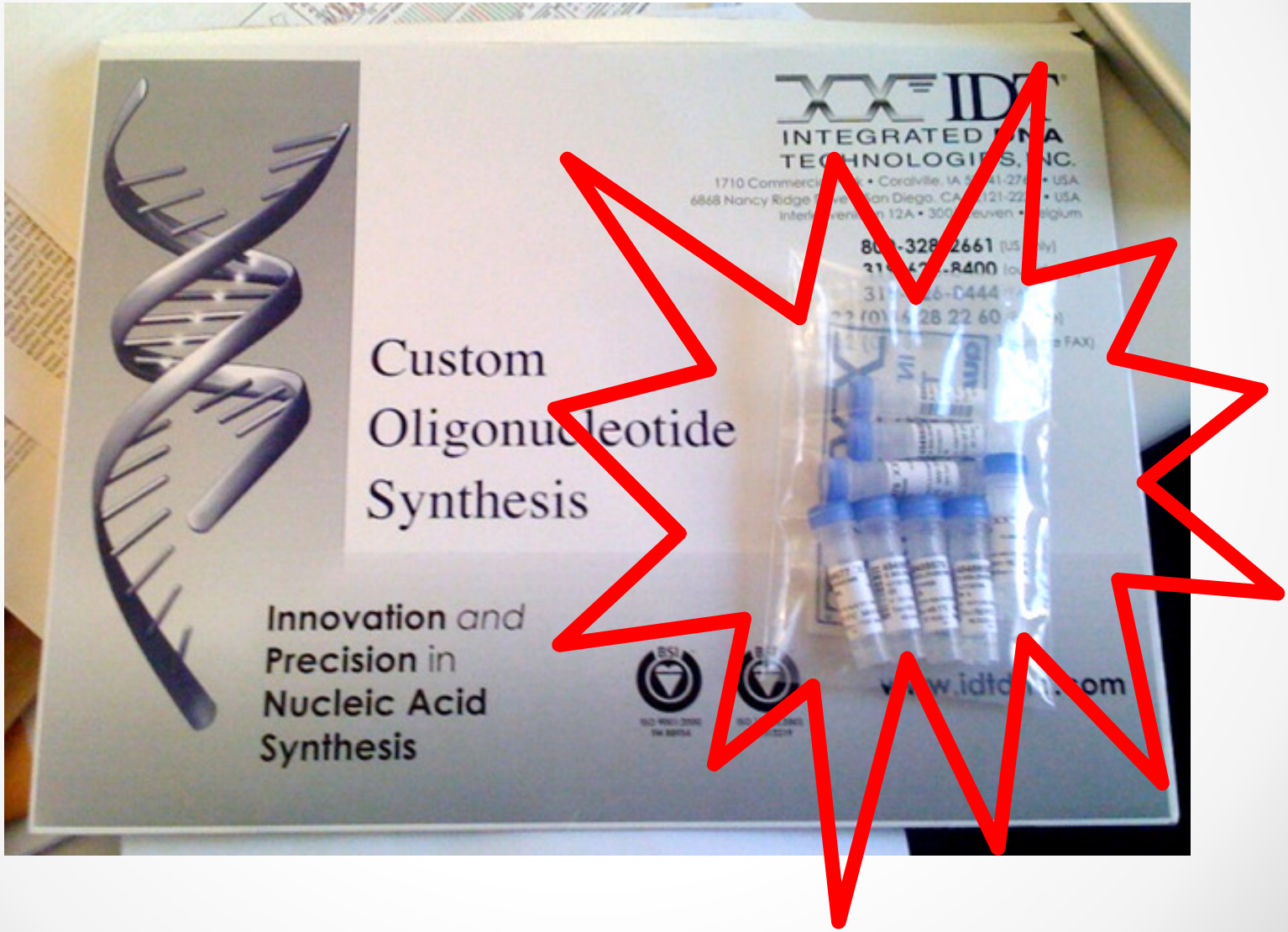


Next-Day Oligos!

Compilation



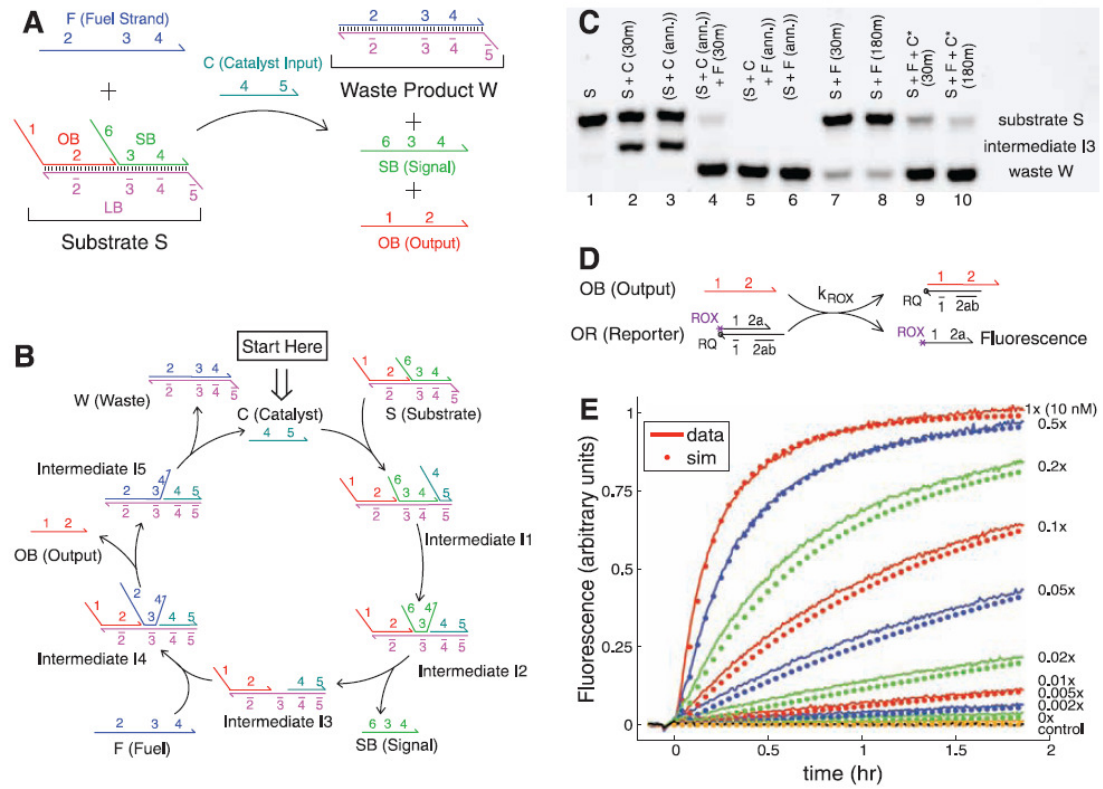
Code Generation



Add Water

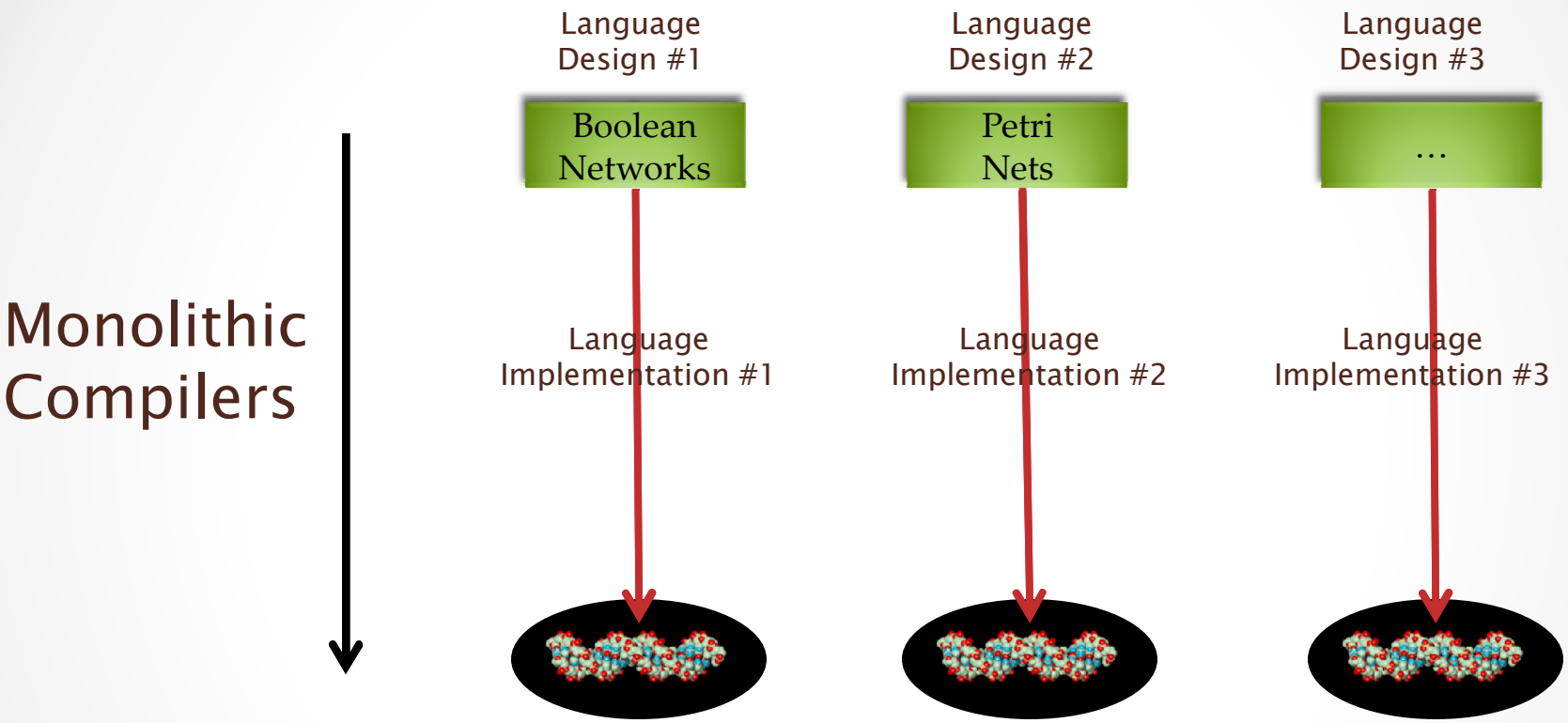
Execution!

Engineering Entropy-Driven Reactions and Networks Catalyzed by DNA
David Yu Zhang, *et al.*
Science **318**, 1121 (2007);
DOI: 10.1126/science.1148532

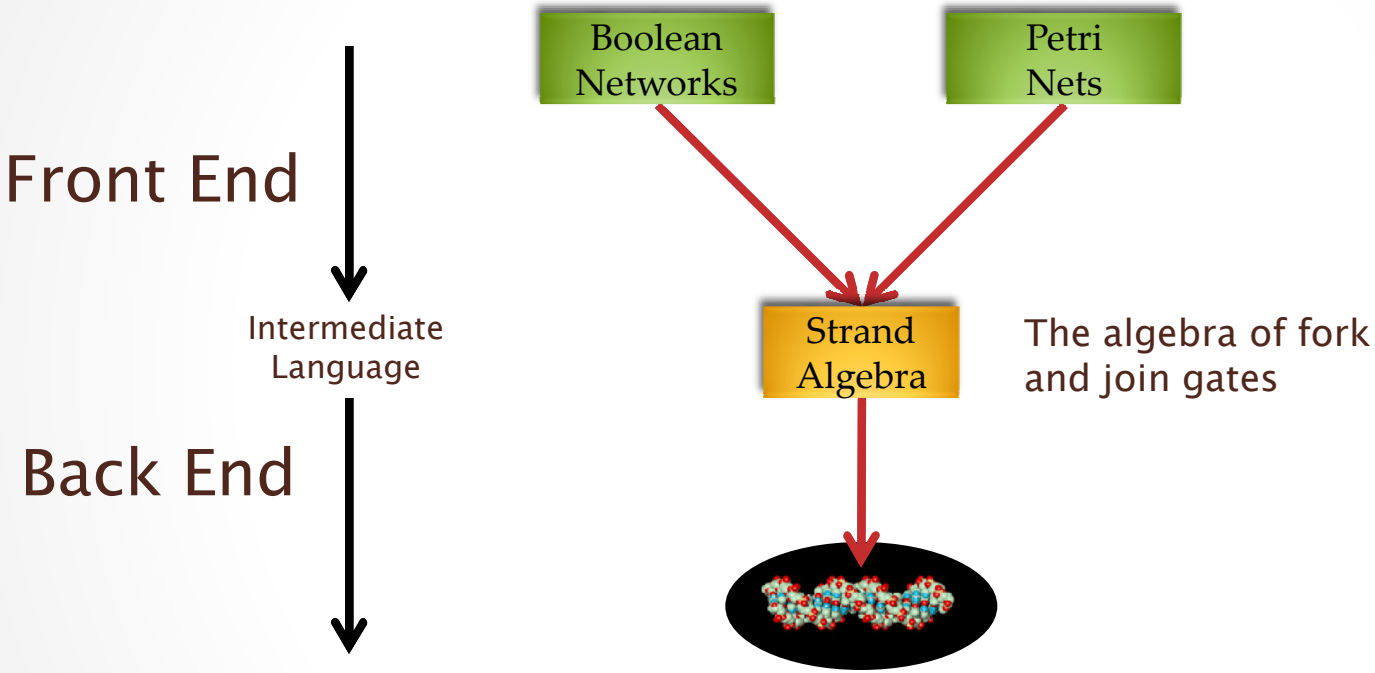


DNA Compilation

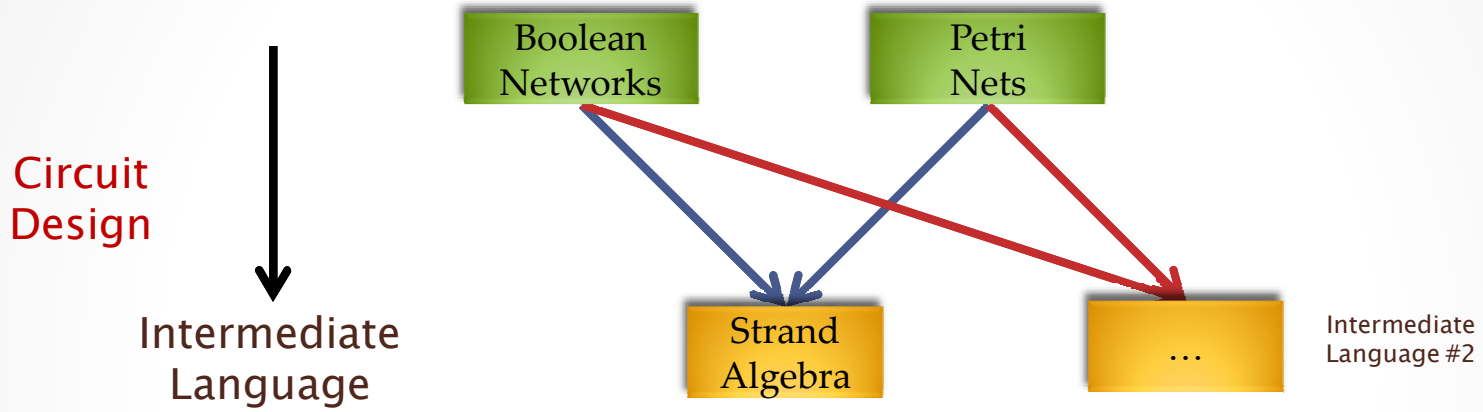
Compilers


Monolithic Compilers
Language Design #1
Boolean Networks
Language Implementation #1
Language Design #2
Petri Nets
Language Implementation #2
Language Design #3
...
Language Implementation #3

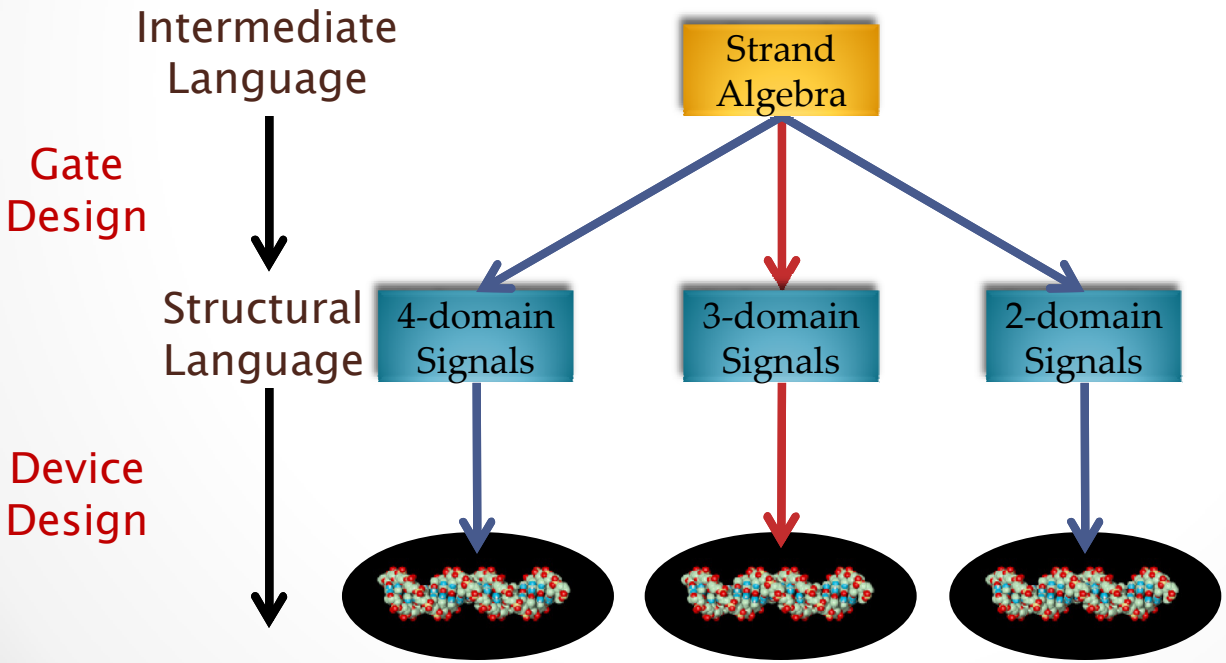
Intermediate Languages



Front Ends


Circuit Design
Intermediate Language
Boolean Networks
Petri Nets
Strand Algebra
...
Intermediate Language #2

Back Ends


Intermediate Language
Gate Design
Structural Language
Device Design
Strand Algebra
4-domain Signals
3-domain Signals
2-domain Signals

Strand Algebra

Strand Algebra

$$P ::= x \;\vdots\; [x_1,..,x_n].[y_1,..,y_m] \;\vdots\; 0 \;\vdots\; P|P \;\vdots\; P^* \qquad n\geq 1,\ m\geq 0$$

n x m gates

x	is a *signal*
$[x_1,..,x_n].[y_1,..,y_m]$	is a *gate*
0	is an *inert solution*
$P\|P$	is *parallel composition* of signals and gates
P^*	is a *population* (multiset) of signals and gates

Reaction Rule

$$x_1 \mid .. \mid x_n \mid [x_1,..,x_n].[y_1,..,y_m] \rightarrow y_1 \mid .. \mid y_m$$

Auxiliary rules (axioms of diluted well-mixed solutions)

$$P \rightarrow P' \;\Rightarrow\; P \mid P'' \rightarrow P' \mid P'' \qquad \text{Dilution}$$

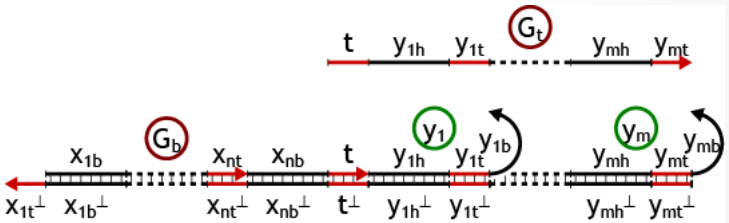
$$P \equiv P_1,\ P_1 \rightarrow P_2,\ P_2 \equiv P' \;\Rightarrow\; P \rightarrow P' \qquad \text{Well Mixing}$$

Where $\equiv$ is a congruence relation (syntactical 'chemical mixing') with $P^* \equiv P \mid P^*$ for unbounded populations.

Compiling Strand Algebra to DNA

$P ::= x \;\vdots\; [x_1,..,x_n].[y_1,..,y_m] \;\vdots\; 0 \;\vdots\; P|P \;\vdots\; P^* \qquad n \geq 1,\ m \geq 0$

- compile(x) = 

- compile($[x_1,..,x_n].[y_1,..,y_m]$) = 

- compile(0) = empty solution

- compile(P | P') = mix(compile(P), compile(P'))

- compile(P*) = population(compile(P))

More in the DNA15 Paper

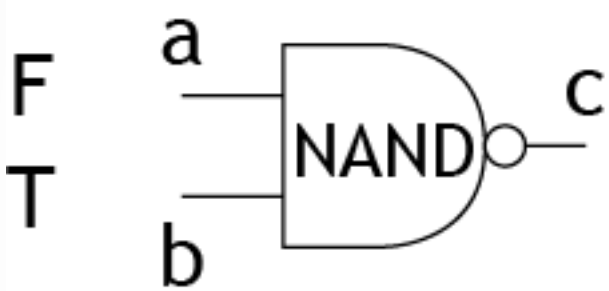
- Stochastic strand algebra
  - Matches the stochastic semantics of chemistry
  - Uses a technique for implementing constant buffered populations, to replace P* with finite populations
- Nested strand algebra
  - An higher-level language (with nested expressions)
  - A compilation algorithm into the basic strand algebra

Compiling Abstract Machines

• • •

Boolean Networks

Boolean Networks to Strand Algebra



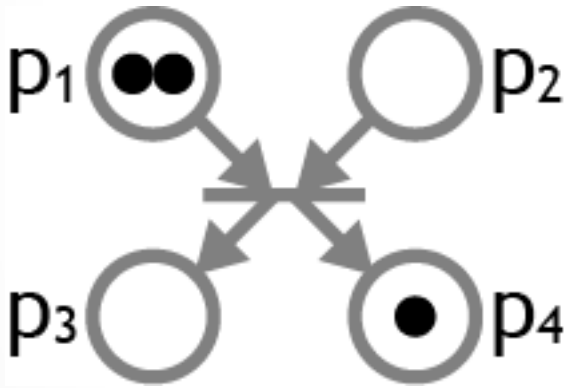
$$([a_F,b_F].c_T)^* \mid$$
$$([a_F,b_T].c_T)^* \mid$$
$$([a_T,b_F].c_T)^* \mid$$
$$([a_T,b_T].c_F)^* \mid$$
$$a_F \mid b_T$$

This encoding is *compositional*, and can encode *any* Boolean network:
- multi-stage networks can be assembled (combinatorial logic)
- network loops are allowed (sequential logic)

Petri Nets

Petri Nets to Strand Algebra

Transitions as Gates
Place markings as Signals



$$([p_1,p_2].[p_3,p_4])^* \mid p_1 \mid p_1 \mid p_4$$

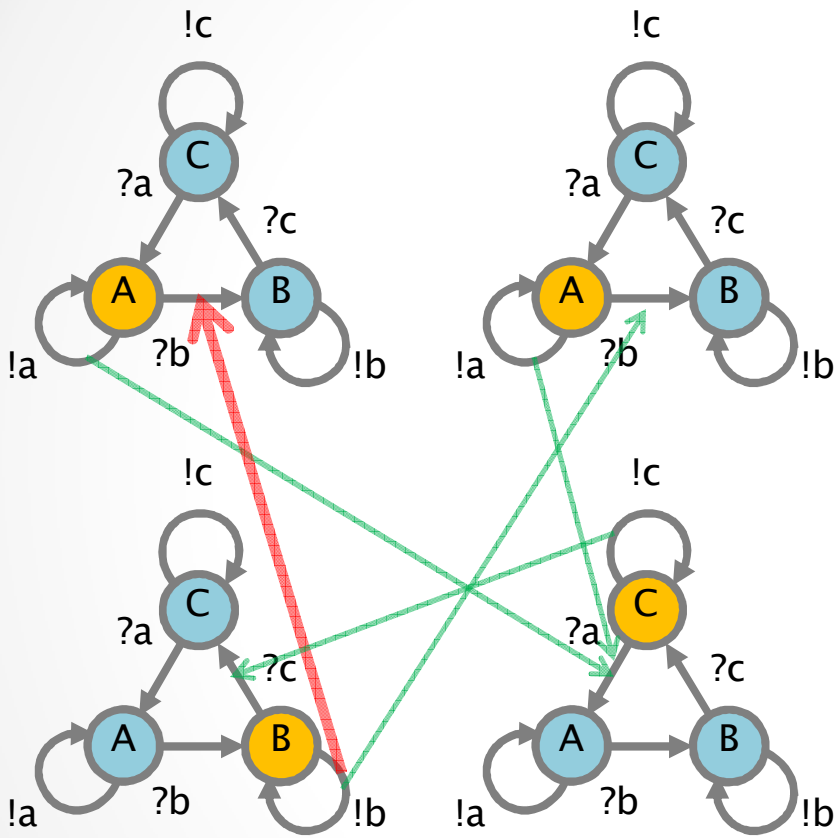
Chemical Reaction Networks

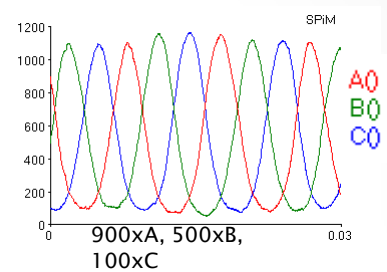
Implementing an arbitrary finite chemical system in DNA with asymptotically correct kinetics
Soloveichick & al. DNA 15

Species become signals
Reactions become gates

$A + B \rightarrow C + D \quad \Rightarrow \quad [A,B].[C,D]$

Interacting Automata

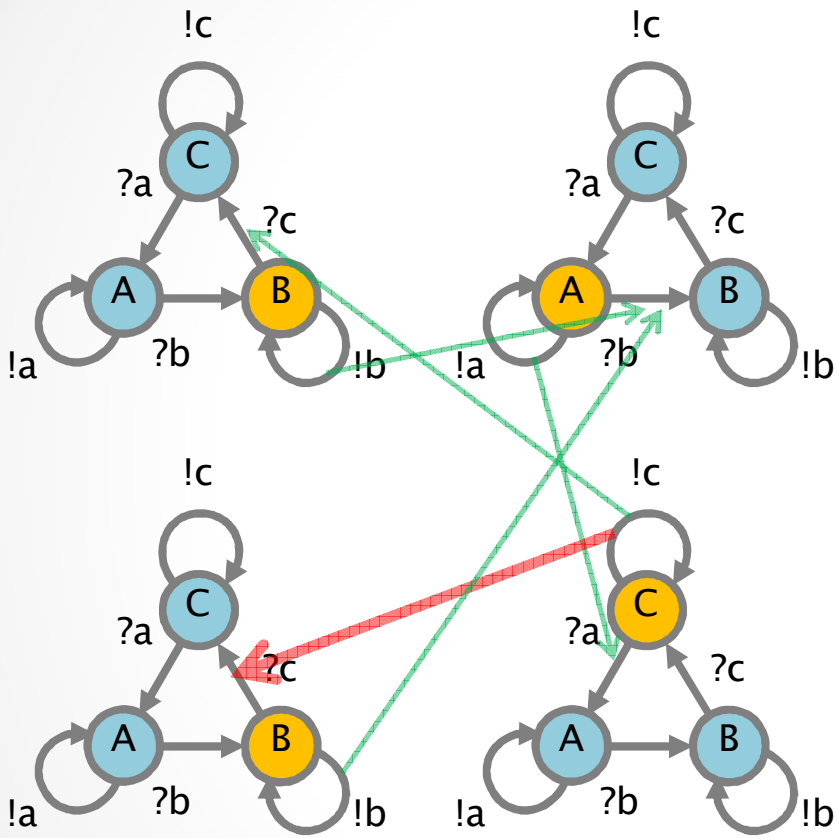


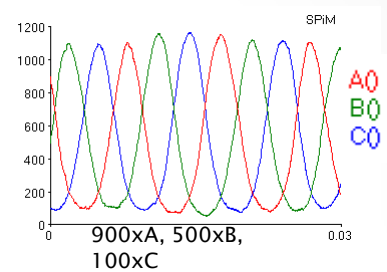


([A,B].[B,B])* |
([B,C].[C,C])* |
([C,A].[A,A])* |
A | A | B | C

This is a uniform population of identical automata,
but heterogeneous populations of interacting automata can be similarly handled.

Interacting Automata





([A,B].[B,B])* |
([B,C].[C,C])* |
([C,A].[A,A])* |
A | B | B | C

This is a uniform population of identical automata,
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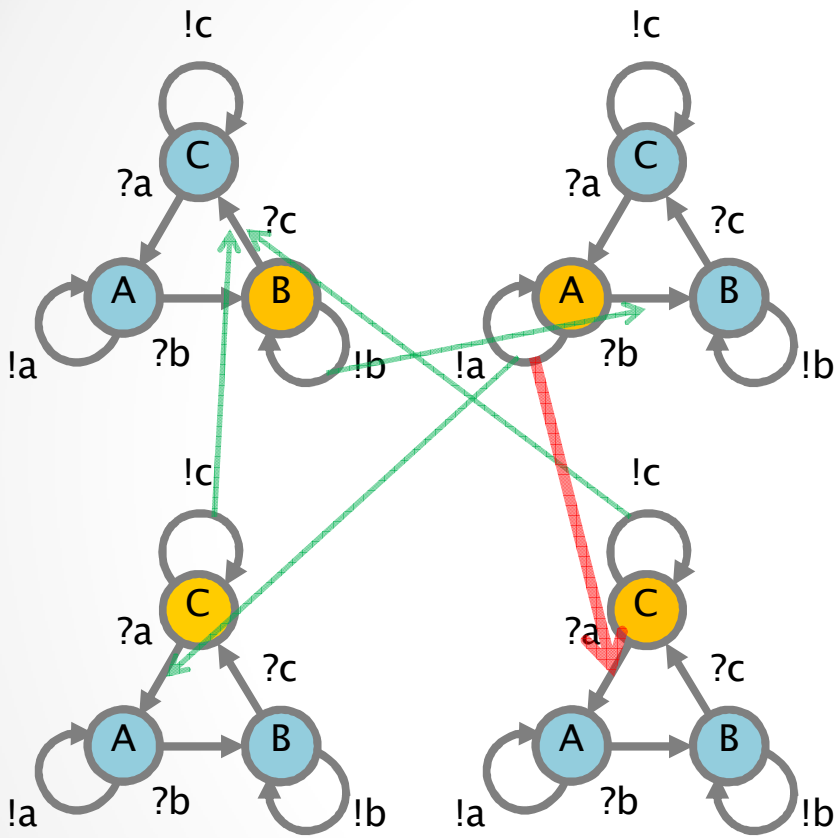
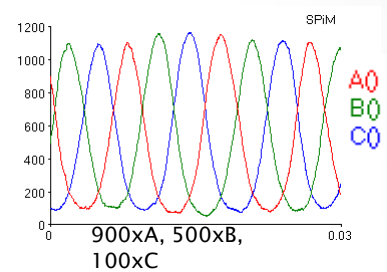
Interacting Automata





([A,B].[B,B])* |
([B,C].[C,C])* |
([C,A].[A,A])* |
A | B | C | C

This is a uniform population of identical automata,
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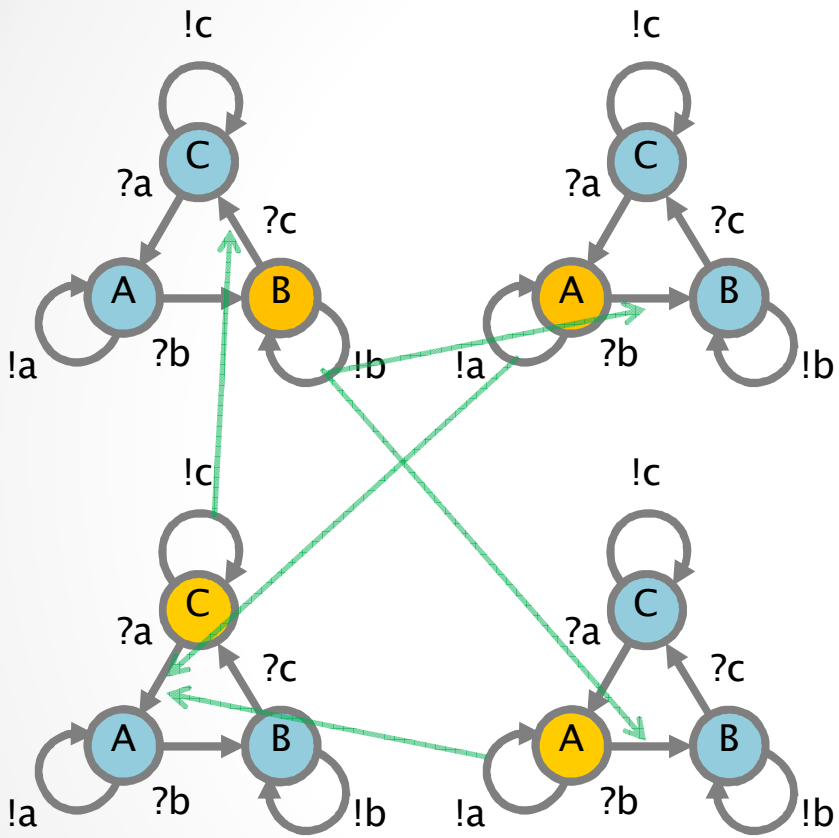
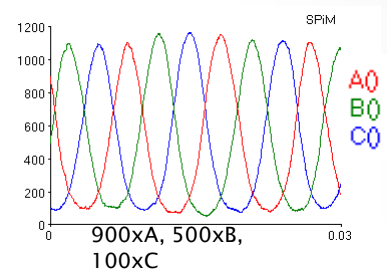
Interacting Automata





([A,B].[B,B])* |
([B,C].[C,C])* |
([C,A].[A,A])* |
A | A | B | C

This is a uniform population of identical automata,
but heterogeneous populations of interacting automata can be similarly handled.

And finally...

...

Summary

- Abstract Machines to Strand Algebra
  - Or other intermediate language
- Strand Algebra to DSD
  - Or other structural language
- Simulation, analysis, etc.
  - Design iteration
- DSD to Sequences
  - E.g. NuPack, or pre-build strand libraries
- Sequences to DNA
  - Web order
- DNA experiments
  - Fairly basic wet lab
- Deployable Nanotech

Conclusions

- Programmable Matter
  - Nucleic acids
- Molecular Computation
  - DNA strand displacement
- Molecular Compilation
  - From programming abstractions (Petri Nets, Process Algebra, etc.), through intermediate language (Strand Algebra) to molecule synthesis (DNA).
- Correctness
  - Ensuring molecular programs work as intended
  - Through thermodynamic analysis, simulation, formal verification.

Acknowledgments

- Illustrations
  - John Reif, Duke
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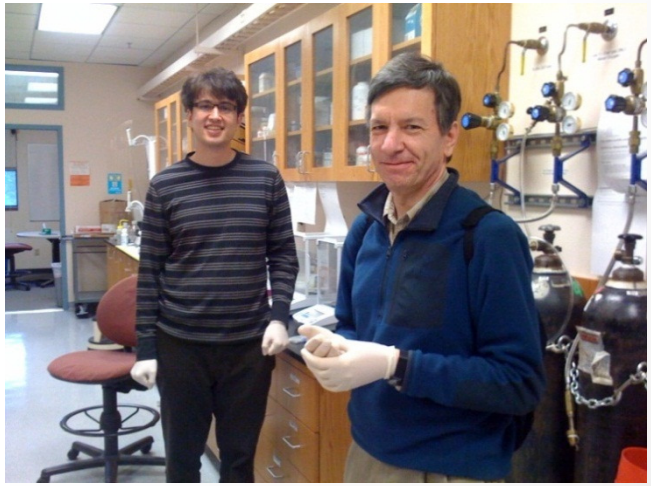
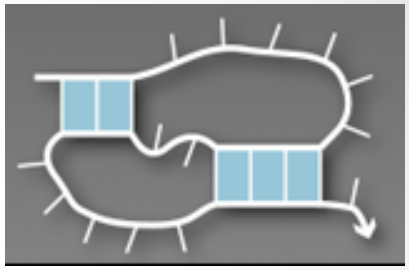
  - Erik Winfree, Caltech
  - Bernard Yurke, Boise State
  - Molecular movies by Drew Berry
  - Wikipedia, YouTube
- David Soloveichik



The Molecular Programming Project



- Caltech & U.Washington
  - National Science Foundation's Expeditions in Computing
  - Shuki Brooks, Erik Klavins, Richard Murray, Niles Pierce, Paul Rothemund, Erik Winfree.
- Goals
  - Create a functional abstraction hierarchy and use this hierarchy to construct programming languages and compilers.
  - Create a theoretical framework for the analysis and design of molecular programs, one that serves as the underpinning for an actual practice of molecular programming.
  - Validate our compilers and theoretical framework with experimental systems utilizing molecular programs with 10 to 100 times the number of components currently used.
  - Test our molecular programming technologies on real-world applications.
  - Recruit and train a generation of molecular programmers with the insight and skills necessary to conceive, design, and implement complex molecular systems.

Q?

http://lucacardelli.name