Membrane Interactions

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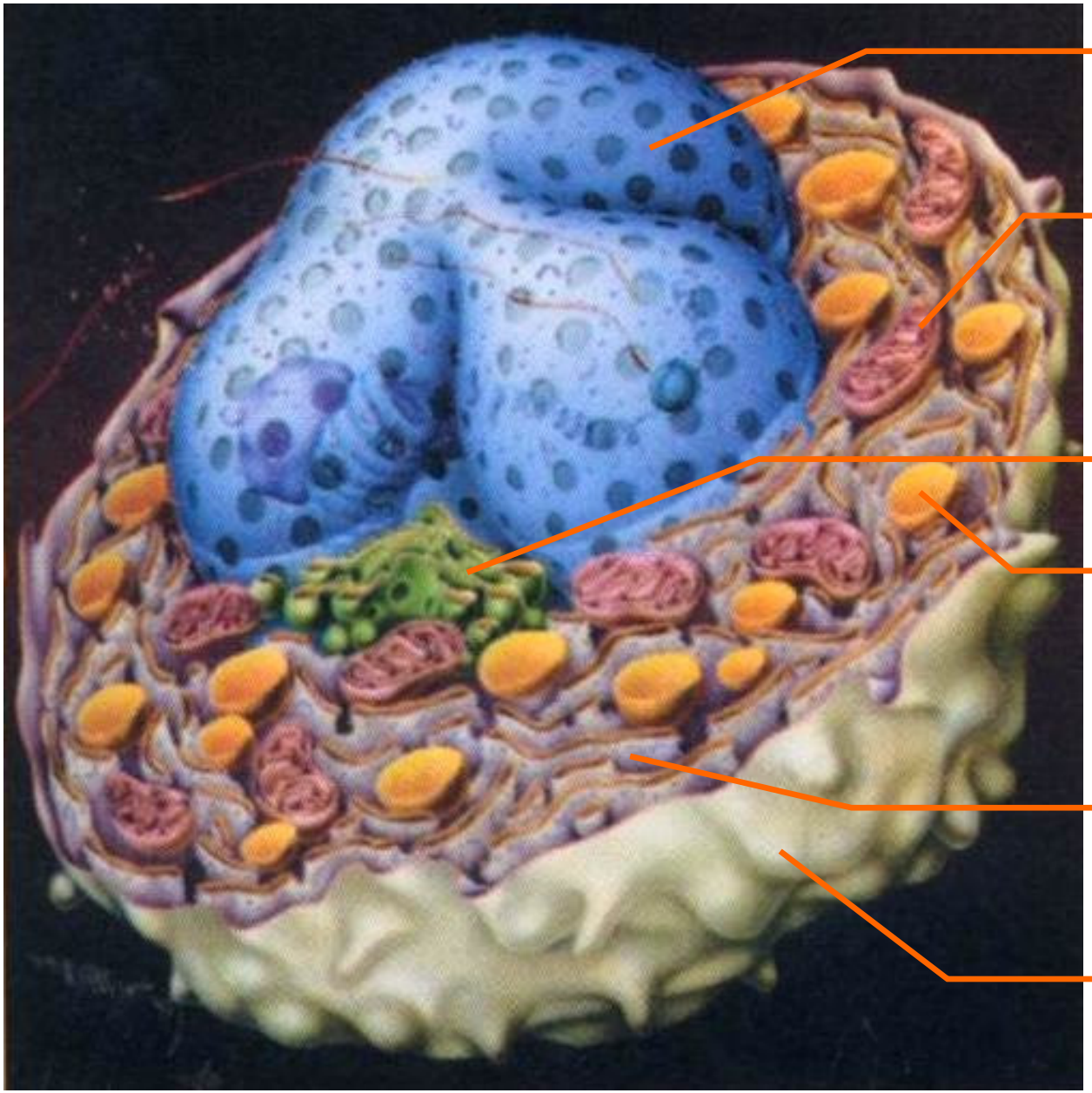
Eukaryotic Cell

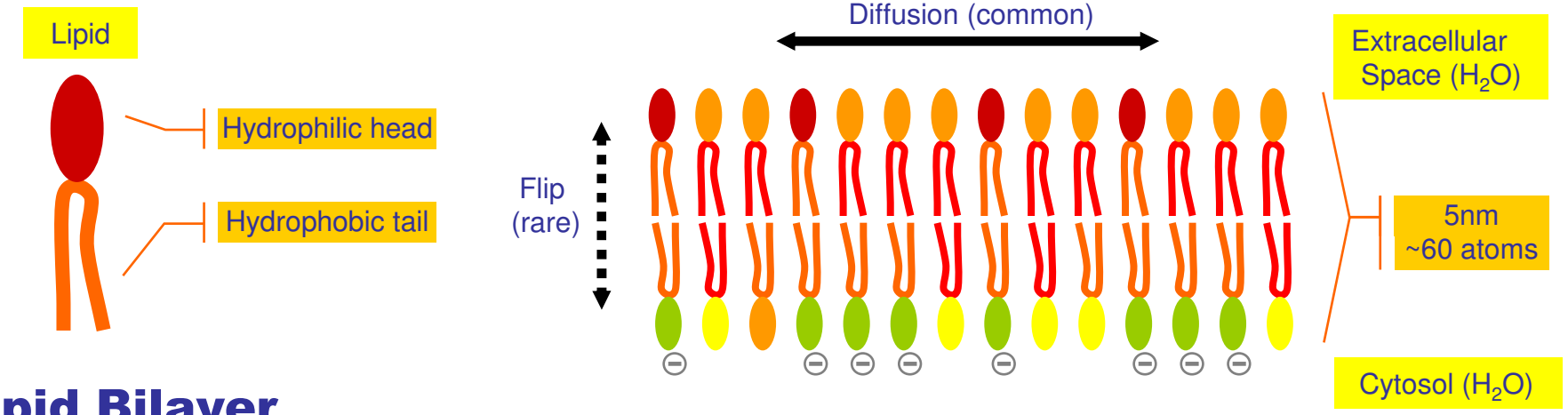
Membranes everywhere



Membrane-based Systems

- Many cellular processes operate on membranes, through membranes, via membrane transformations, and via active membrane transport. It's *very far* from a "chemical soup":
  - **For a cell to function properly, each of its numerous proteins must be localized to the correct cellular membrane or aqueous compartment. [MCB p.675]**
- What is the dynamics of these complex configurations of membranes? (Still poorly understood in biology.)
- In modeling it, we *must* use abstractions, to avoid combinatorial explosion: 1 membrane $\approx \infty$ molecules.
- Emerging area of *Systems Biology* (~ above molecules, ~ study of biological processes).

Membranes are Oriented 2D Surfaces



Lipid Bilayer

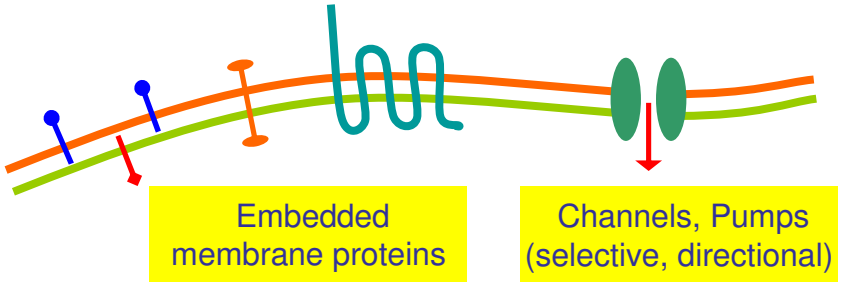
Self-assembling

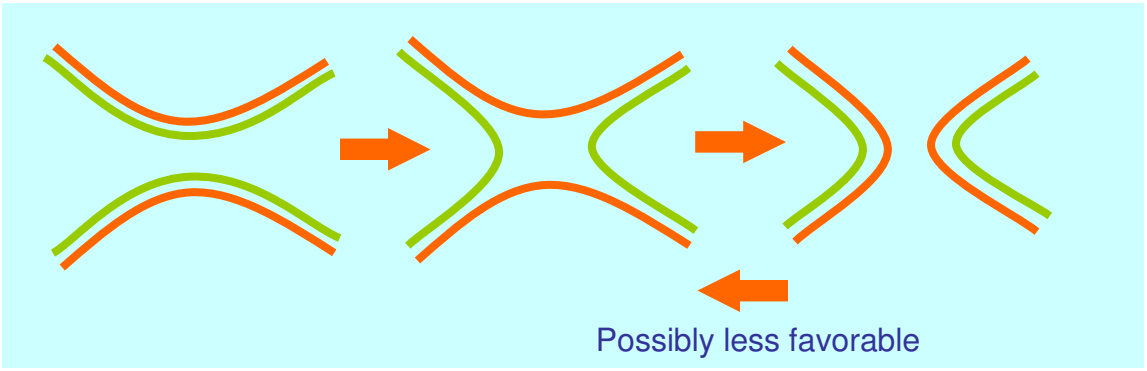
Largely impermeable

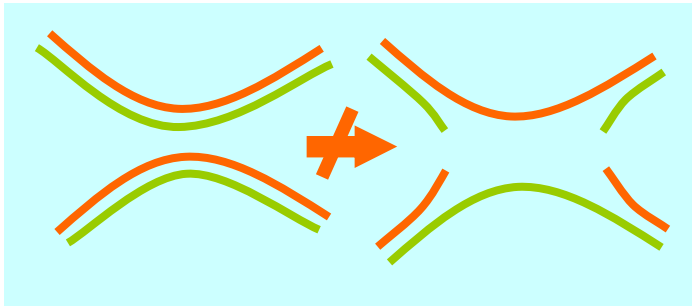
Asymmetrical (in real cells)

Embedded proteins

A 2D fluid inside a 3D fluid!

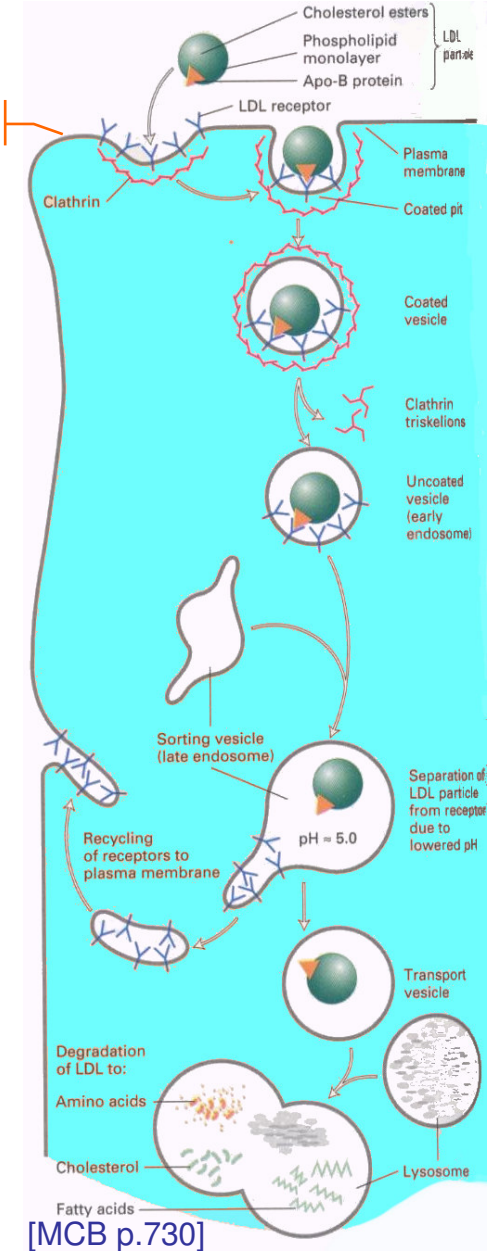






A Biological Algorithm

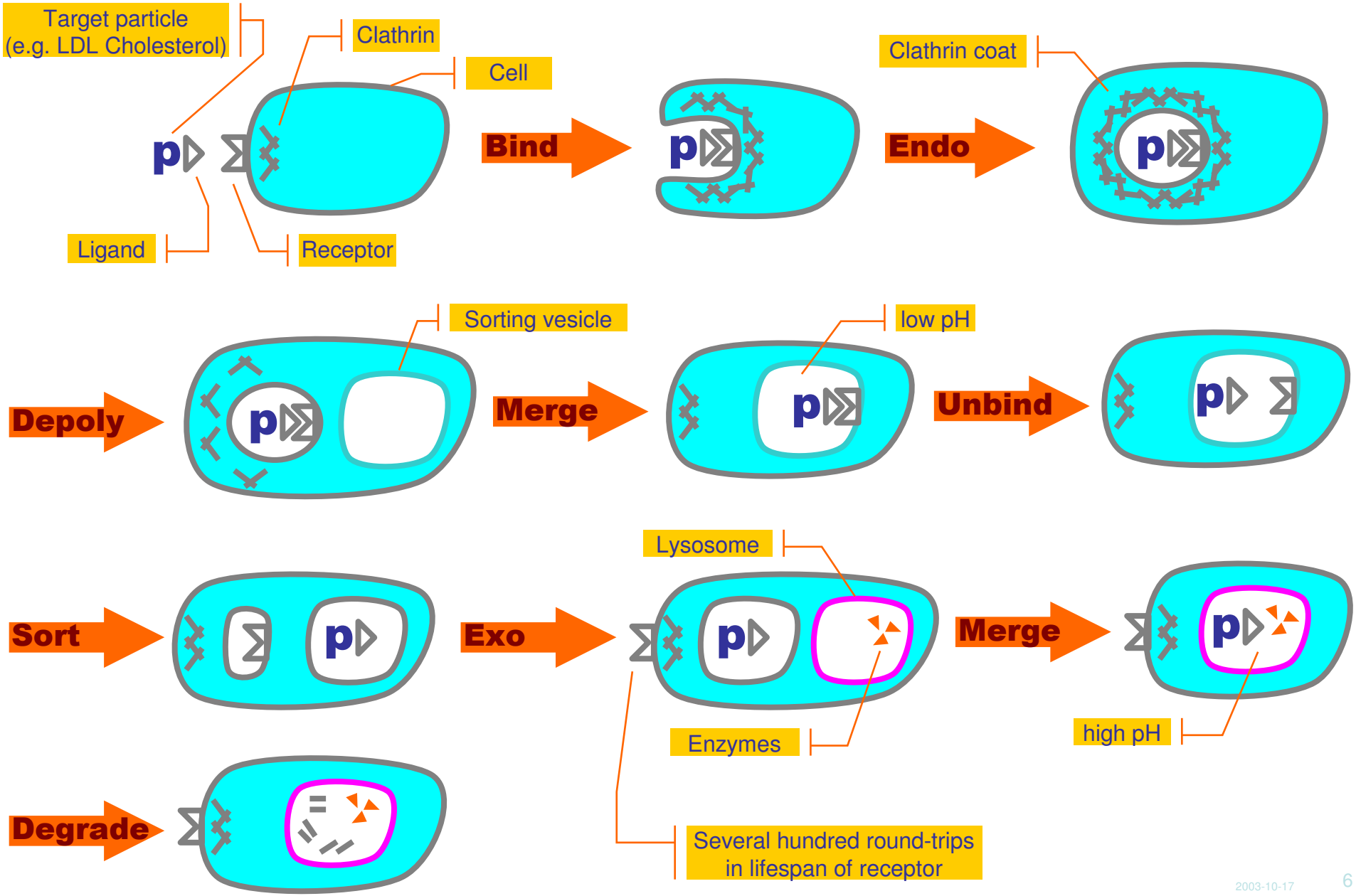
- LDL-Cholesterol Degradation
  - A cast of many thousands (molecules) just to get one molecule from A to B.
  - Membranes are key to the algorithm, we want to model *them*, not their individual molecules.
- How do people know all that?
  - They take pictures, see all stages of the algorithm in the same snapshot.
  - Stop genes, see what stages survive; build temporal sequence of stages.
  - Identify key molecules. Model them and play with them to see what they do.
  - Many steps still murky. Not possible to model them in detail even if wanted to.



[MCB p.730]

Receptor-Mediated Degradation Pathway

(Abstract view)



Aims

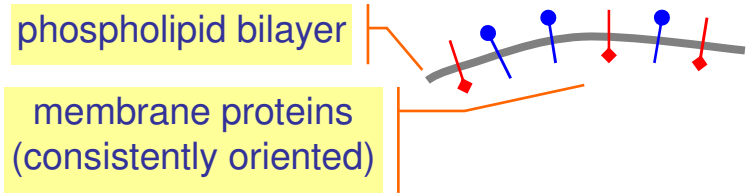
- Describing biological processes
  - Avoid informal diagrams.
  - Write bioalgorithms in something close to a language.
- Abstraction options
  - Start too low ⇒ get lost in a mess of details.
  - Start too high ⇒ ignore too many details.
- Strategy (for now)
  - 1) Start too high (but learn basic gameplay).
  - 2) Move one or two levels down.
- Approaches considered here
  - Algebras (*Bitonal Algebra*)
  - Rewriting systems (BiGraphs, Gamma, P-Systems, etc.)
  - Calculi (BioSPi, BioAmbients, and now *Brane Calculi*)

Part I

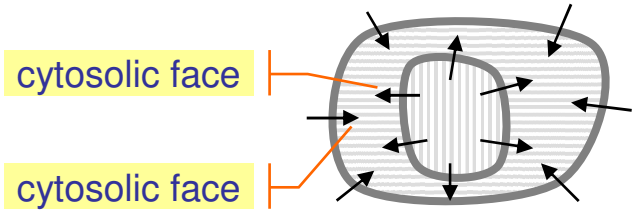
Bitonal Systems

Systems of Oriented Membranes

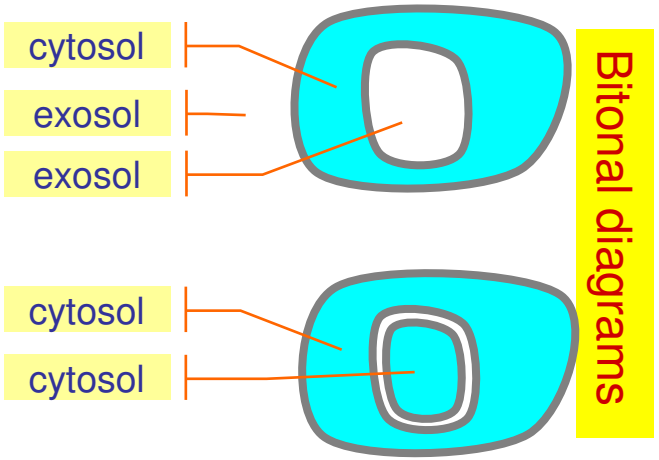
Membranes are closed non-intersecting curves, with an orientation[1].



Each membrane has two faces. A cytosolic (~*inner*) face and an exoplasmic (~*outer*) face. Nested membranes alternate orientation. (E.g. cytosolic faces always face each other.)



This alternation is illustrated by using two tones: blue (cytosol[2]) and white (exosol[3]). Bitonal diagrams.



Double membranes (e.g. the nuclear membrane) can be used for blue-in-blue components.

(1) A membrane is built from a phospholipid bilayer that is asymmetrical. Moreover, all real membranes are heavily sprinkled with proteins: “each type of integral membrane protein has a single specific orientation with respect to the cytosolic and exoplasmic faces of a cellular membrane, and all molecules of any particular integral membrane protein share this orientation. This absolute asymmetry in protein orientation confers different properties on the two membrane faces.” MCB p162.
(2) Short for Cytoplasmic Solution. (3) Short for Exoplasmic Region (I am making this one up).

Bitonal Diagrams

Bitonal Postulate

Blue and white areas alternate.

Bitonal Invariant

Bitonality is preserved by reactions.

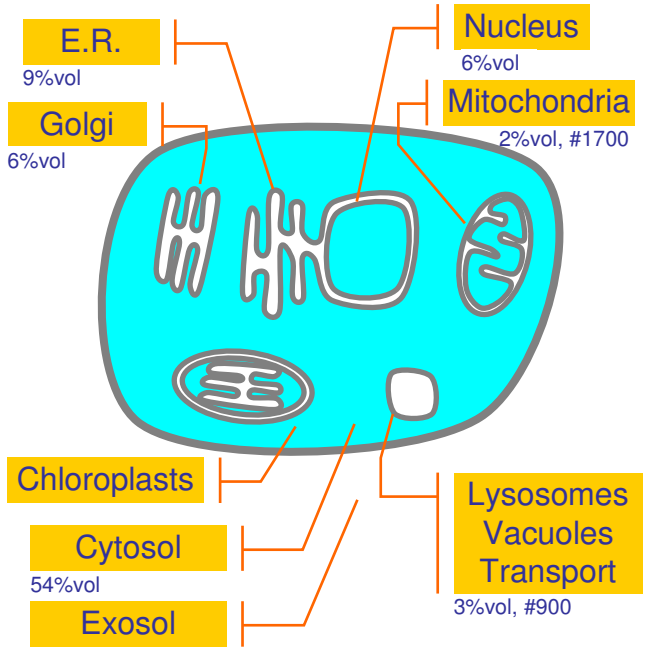
Tonal Duality Postulate

The tone-dual of a reaction is a reaction.

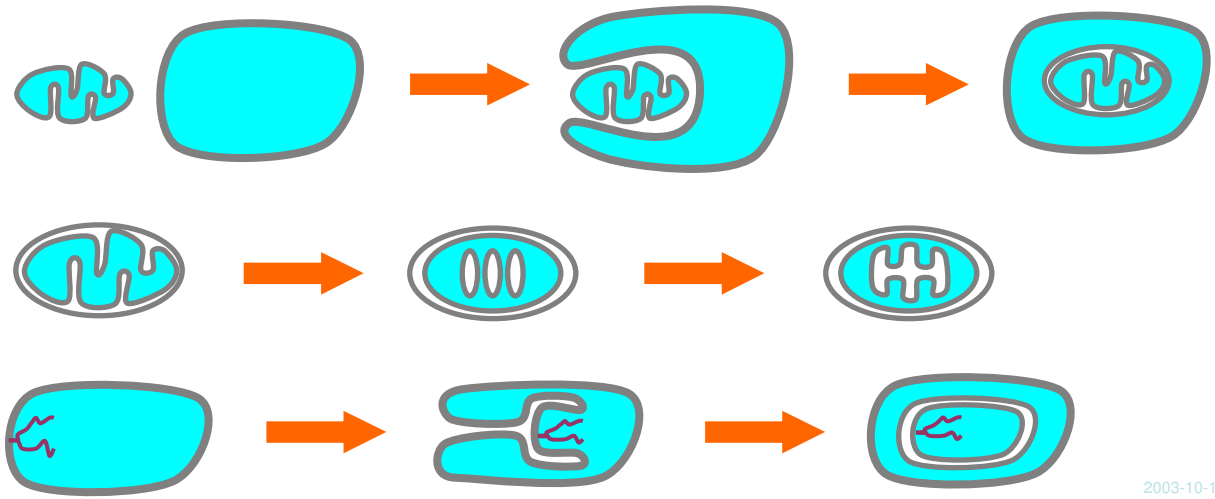
Tonal Stability Invariant

Reactions do not re-tone the background.

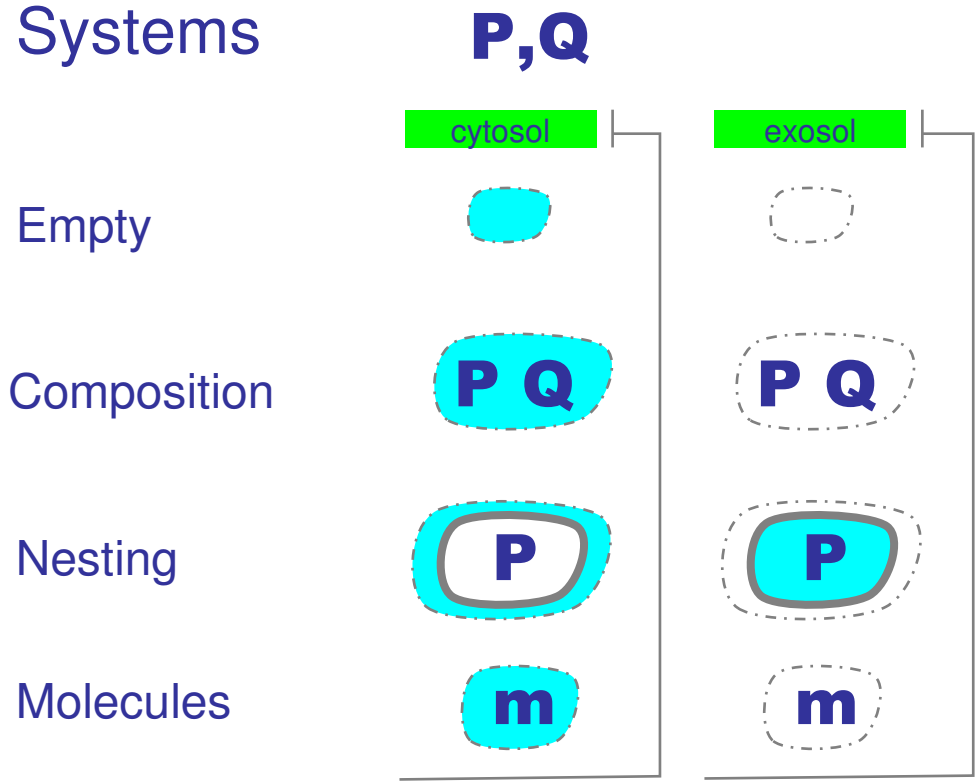
Reactions do not re-tone whole subsystems.

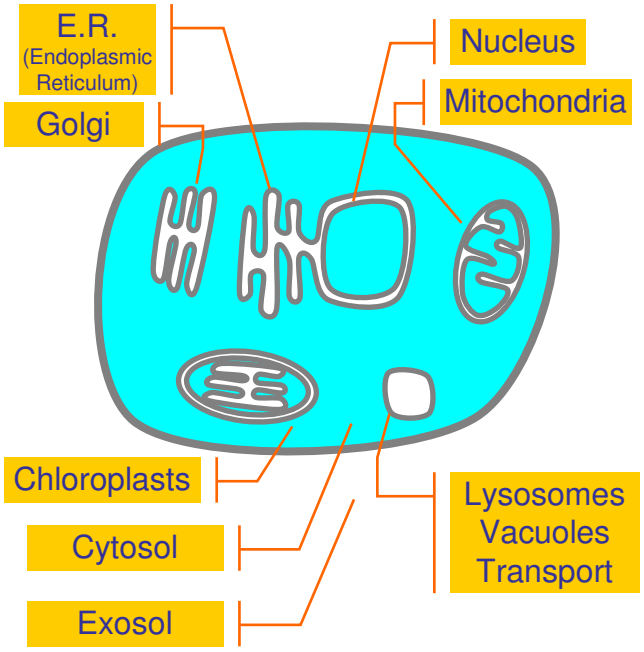


Evolutionary explanations of bitonality



Bitonal Systems





A *bitonal* system **P** has proper tone alternation.
The *tonality* of **P** is the tone of its background, also drawn as:

P a system **P** of blue tonality ("**P** swims in cytosol")

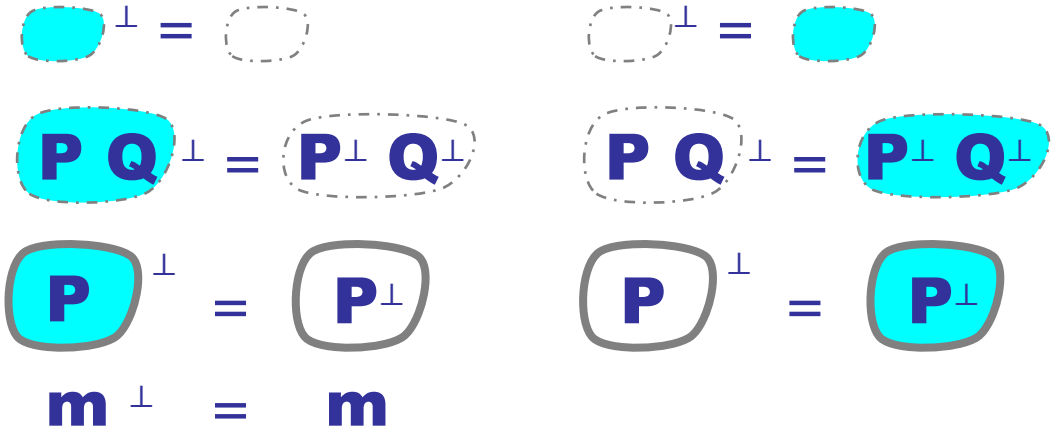
P a system **P** of white tonality ("**P** swims in exosol")

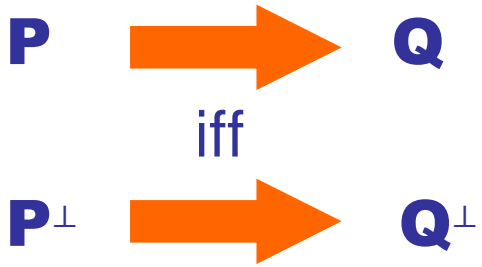
Bitonal Reactions

Directed reaction
P,**Q** same tonality

Reversible reaction
P,**Q** same tonality

Dual Reactions



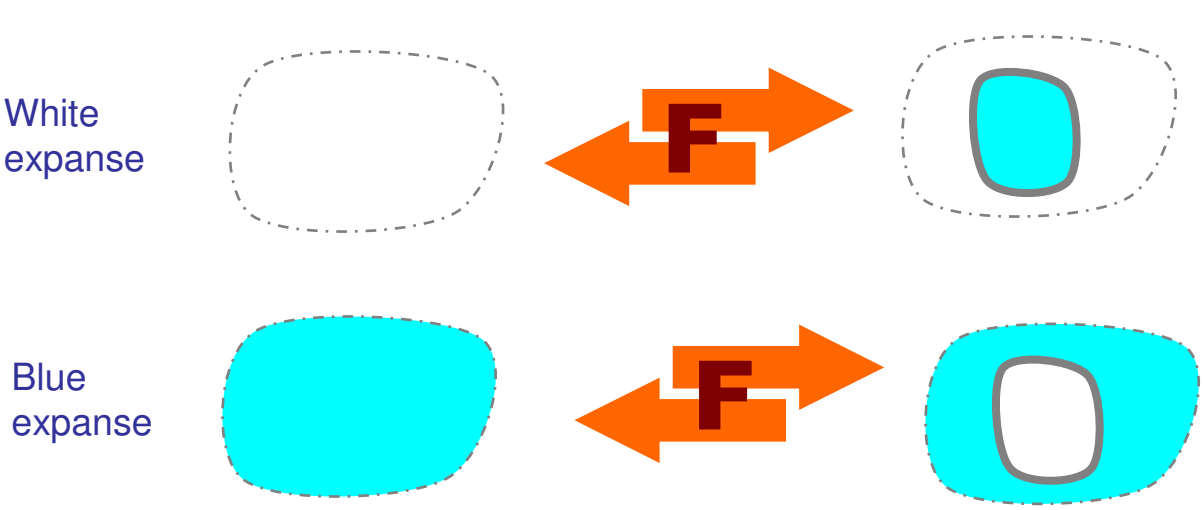


Reactions in Context

by duality

✓ Froth/Fizz Reaction

The spontaneous appearance/disappearance of empty bubbles (of the correct tonality).



Preserves bitonality and is stable.

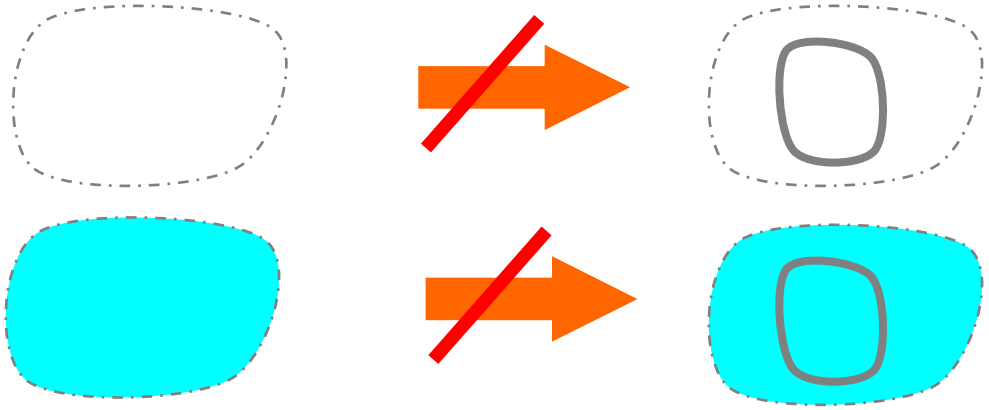
* Phospholipid molecules automatically assemble into closed membranes.

N.B. non-empty membranes should not “spontaneously” be created or deleted: usually only very deliberate processes cause that. However, spontaneous froth/fizz seems be harmless; it means that empty membranes are not observable.

× Bad Bubbles

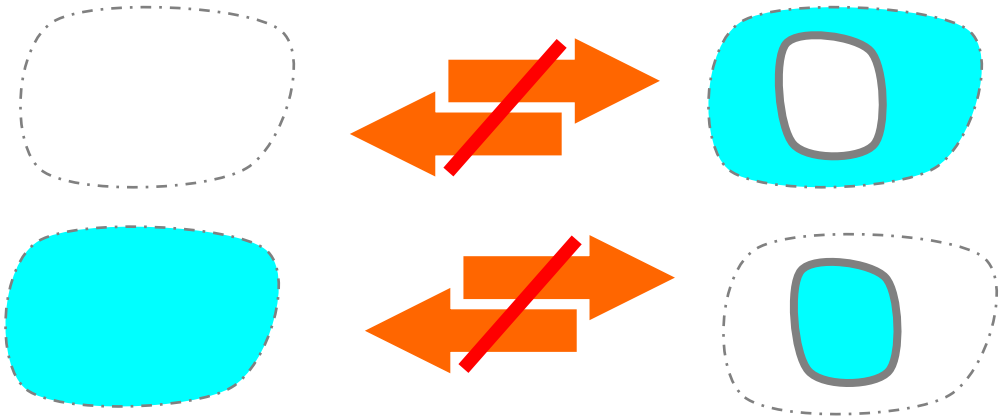
Wrong bubbles:

Violates bitonality.



Bubble catastrophe:

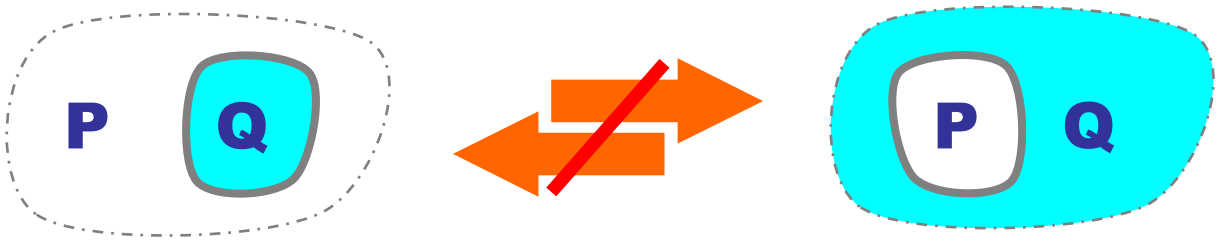
Violates bitonality in context.
Also, ill-toned reaction arrow.



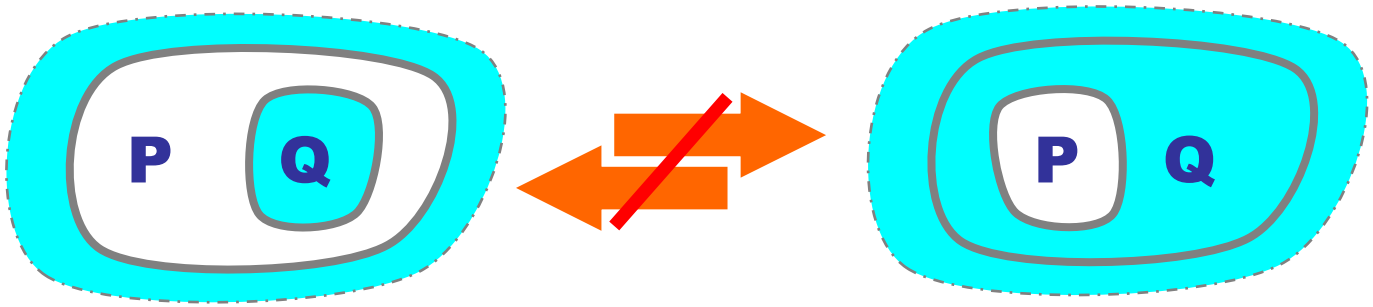
× Flooding

Flooding

Violates bitonality in context.
Also, ill-toned reaction arrow.



Flooding in context violates bitonality:



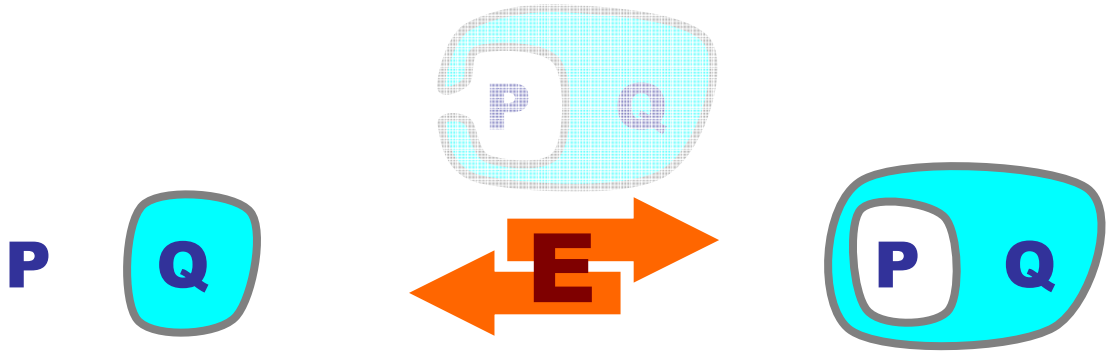
× Ambients

Violate bitonality

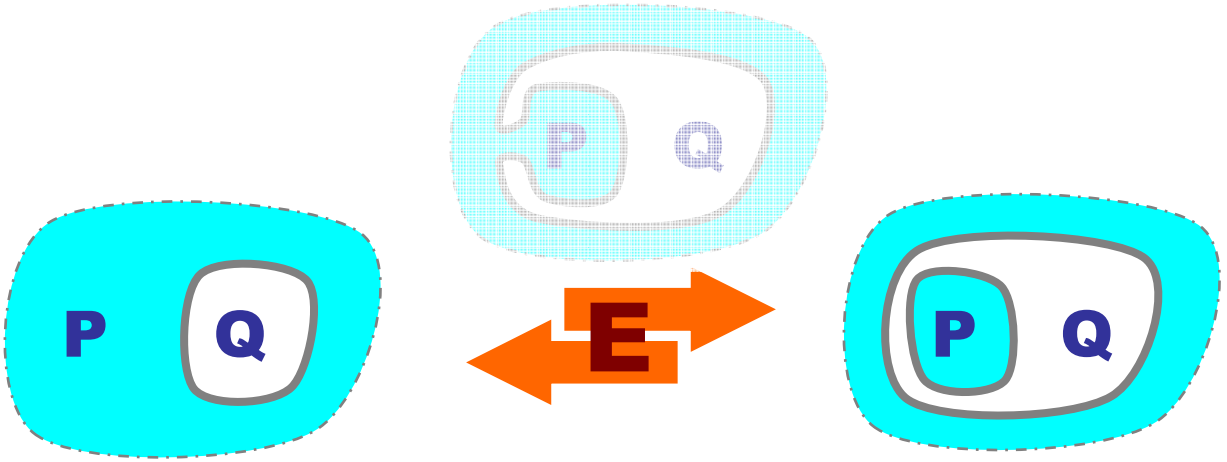
Preserve bitonality, but violate stability for subsystem P.

✓ Endo/Exo Reaction

Preserves bitonality and is stable: the tonality of P and Q does not change.

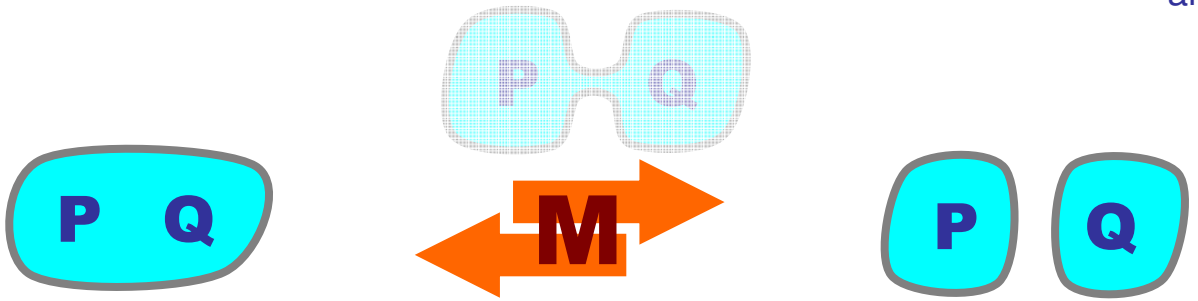


Dual:

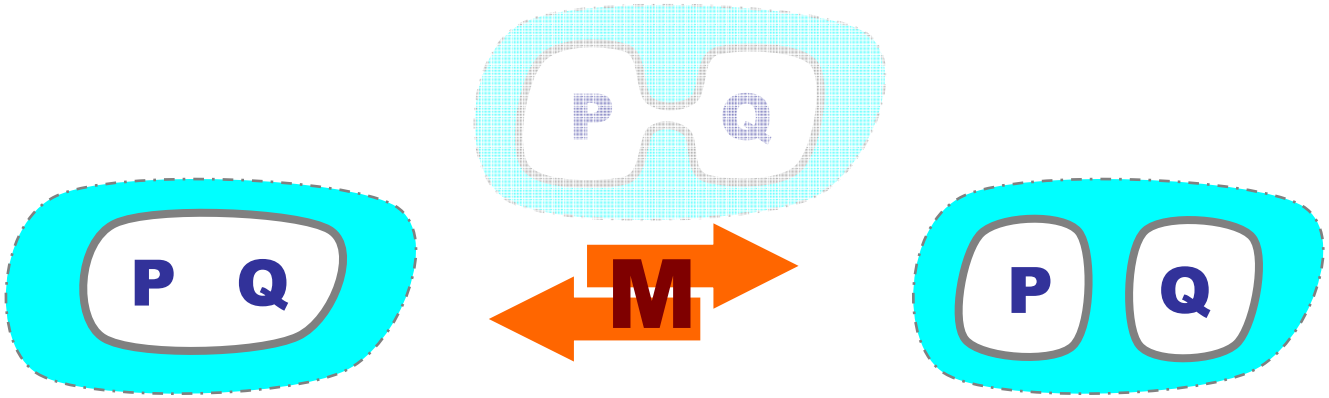


✓ Mito/Mate Reaction

Preserves bitonality and is stable.



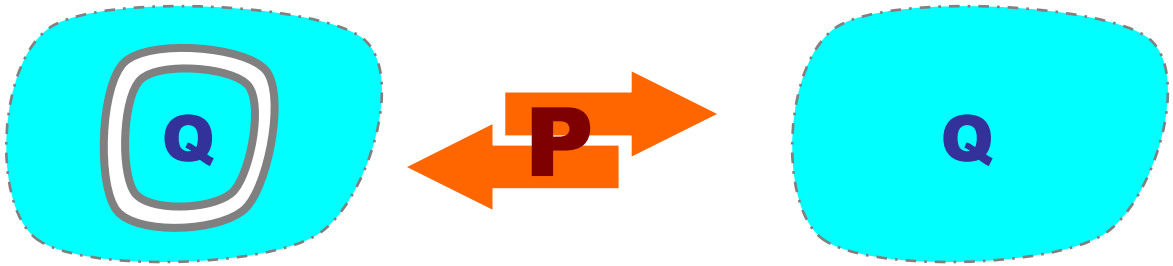
Dual:



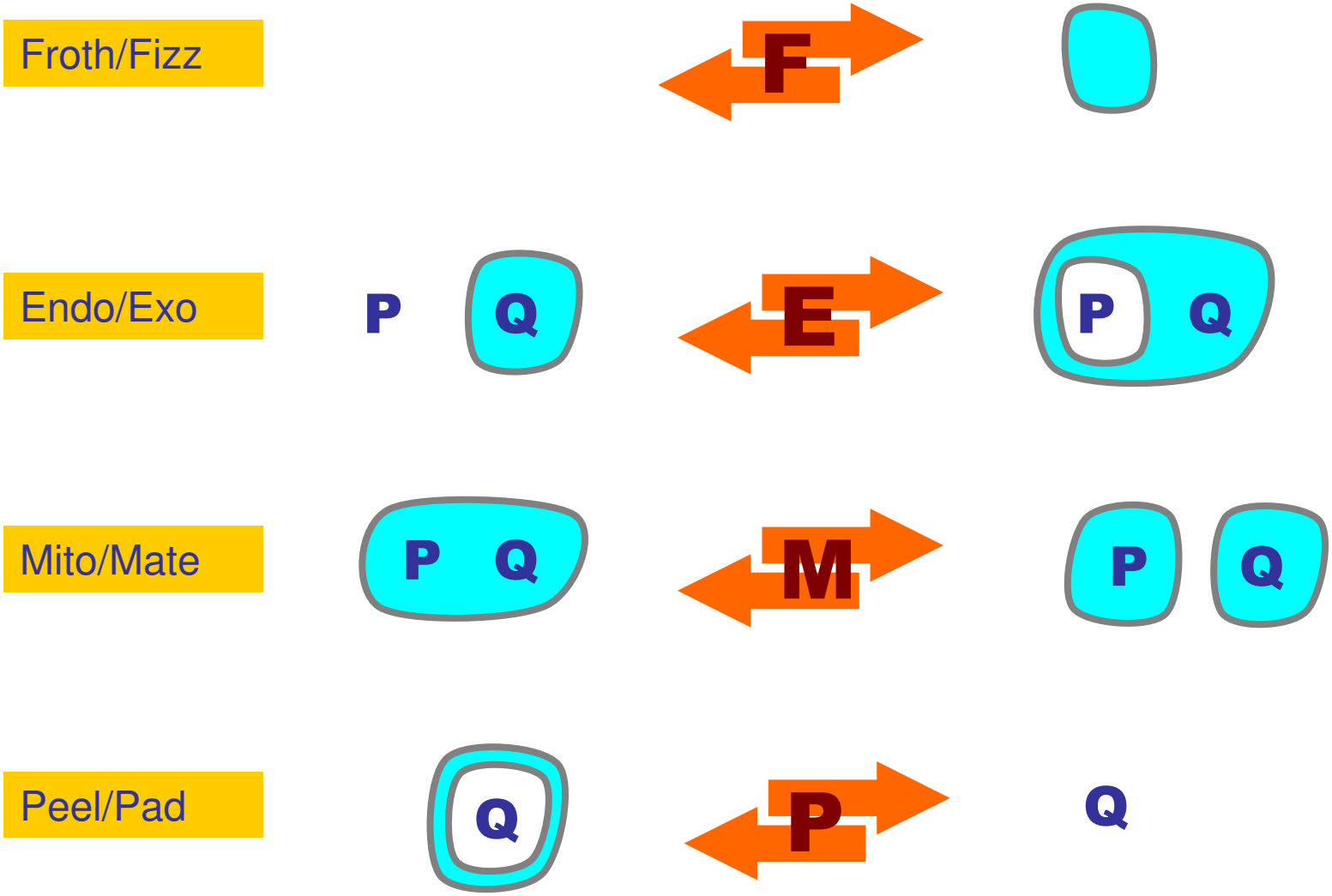
✓ Peel/Pad Reaction

Preserves bitonality
and is stable.

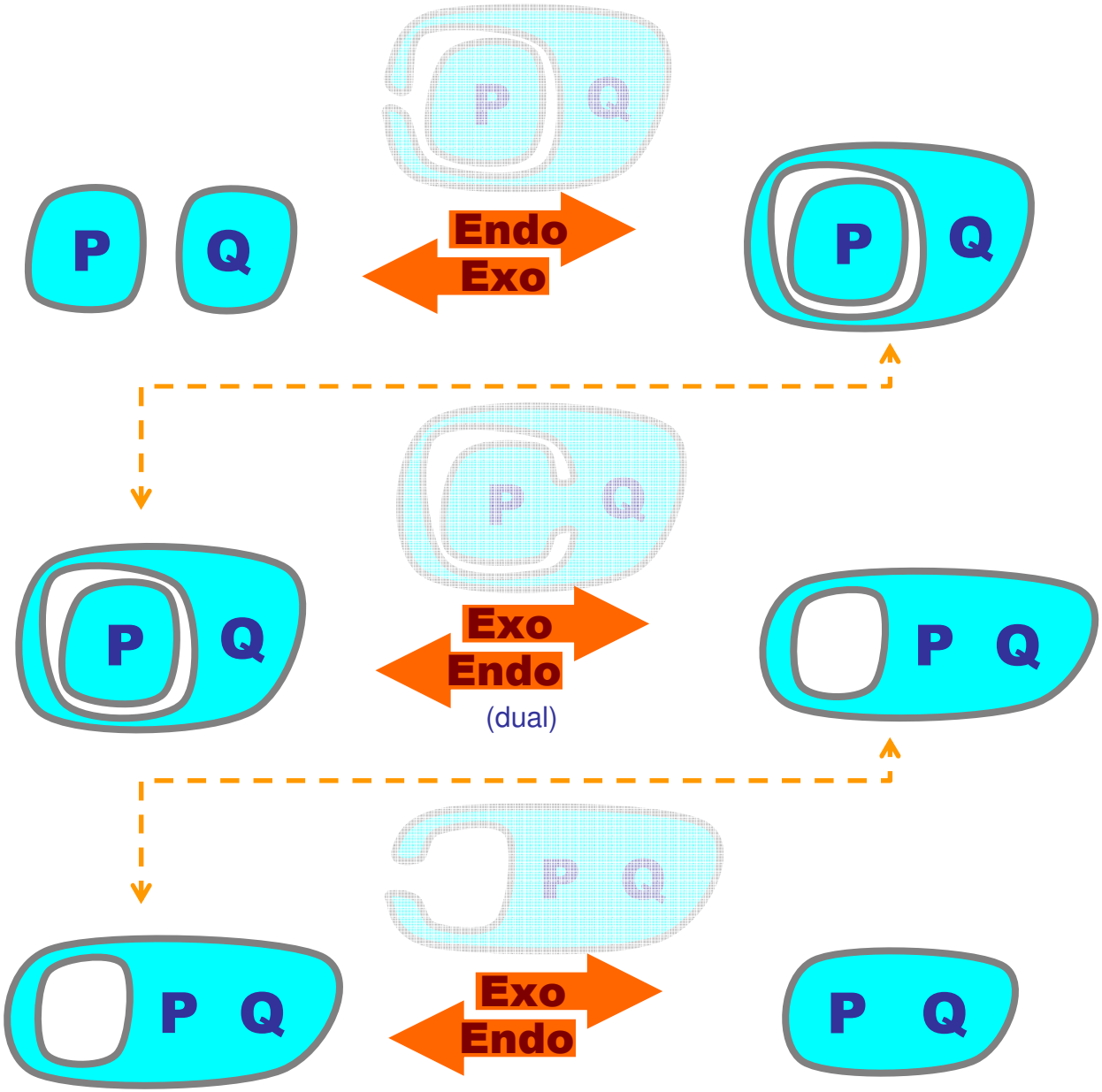
Dual:



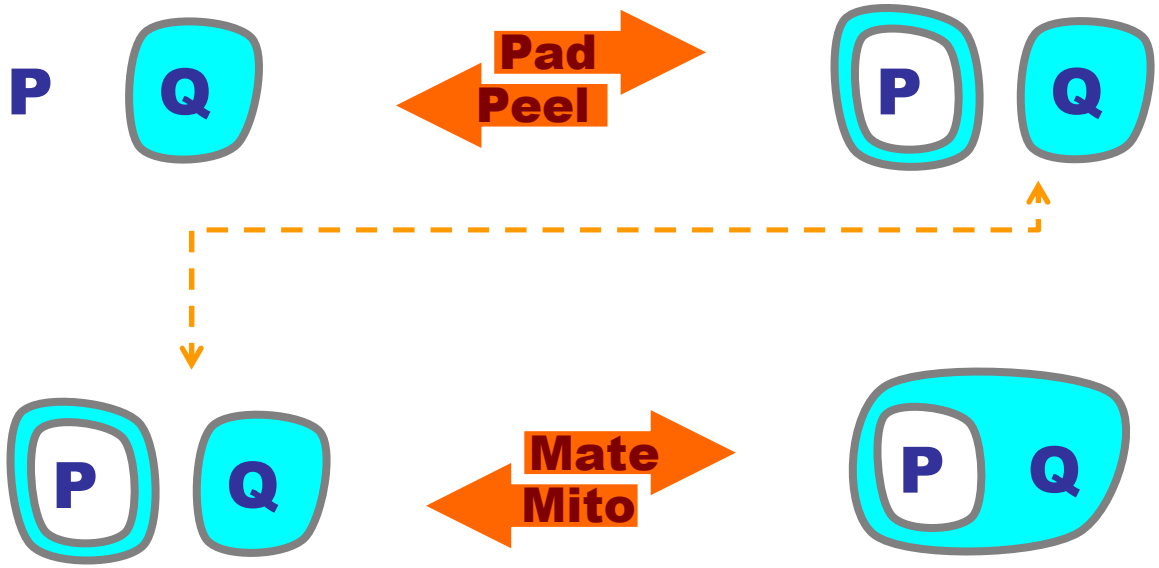
Summary: Four Good Reactions



Mito/Mate by 3 Endo/Exo

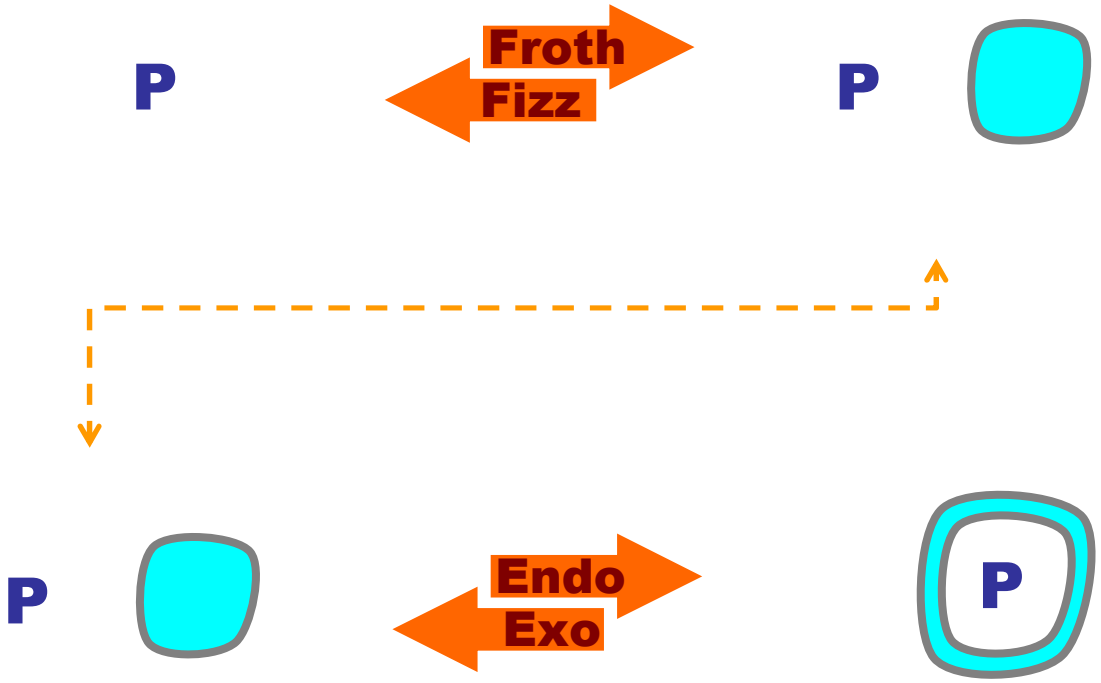


Endo/Exo by Mito/Mate and Peel/Pad



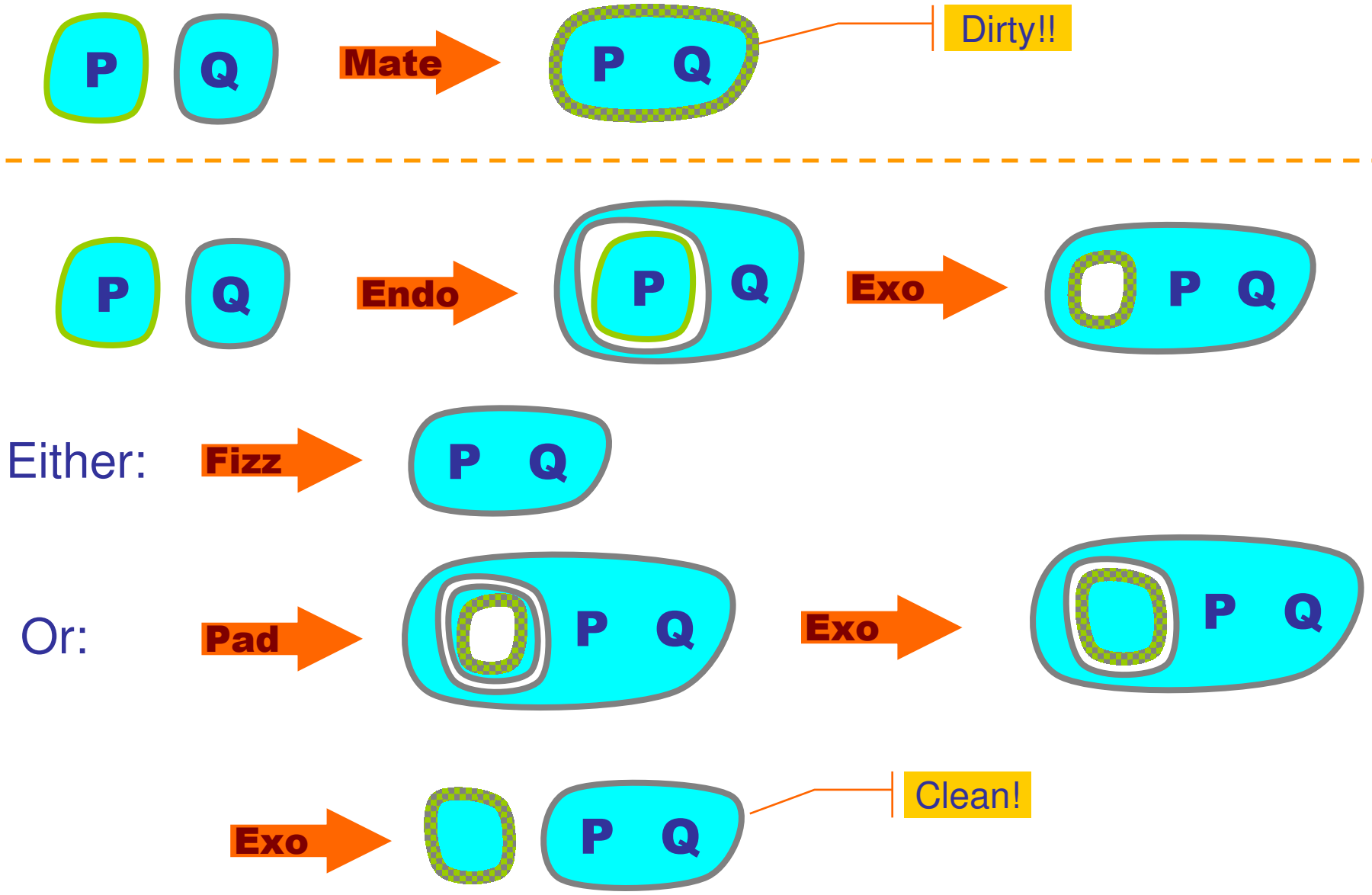
Endo/Exo from Mito/Mate only? No: depth of nesting is constant in Mito/Mate.

Peel/Pad by Froth/Fizz and Endo/Exo



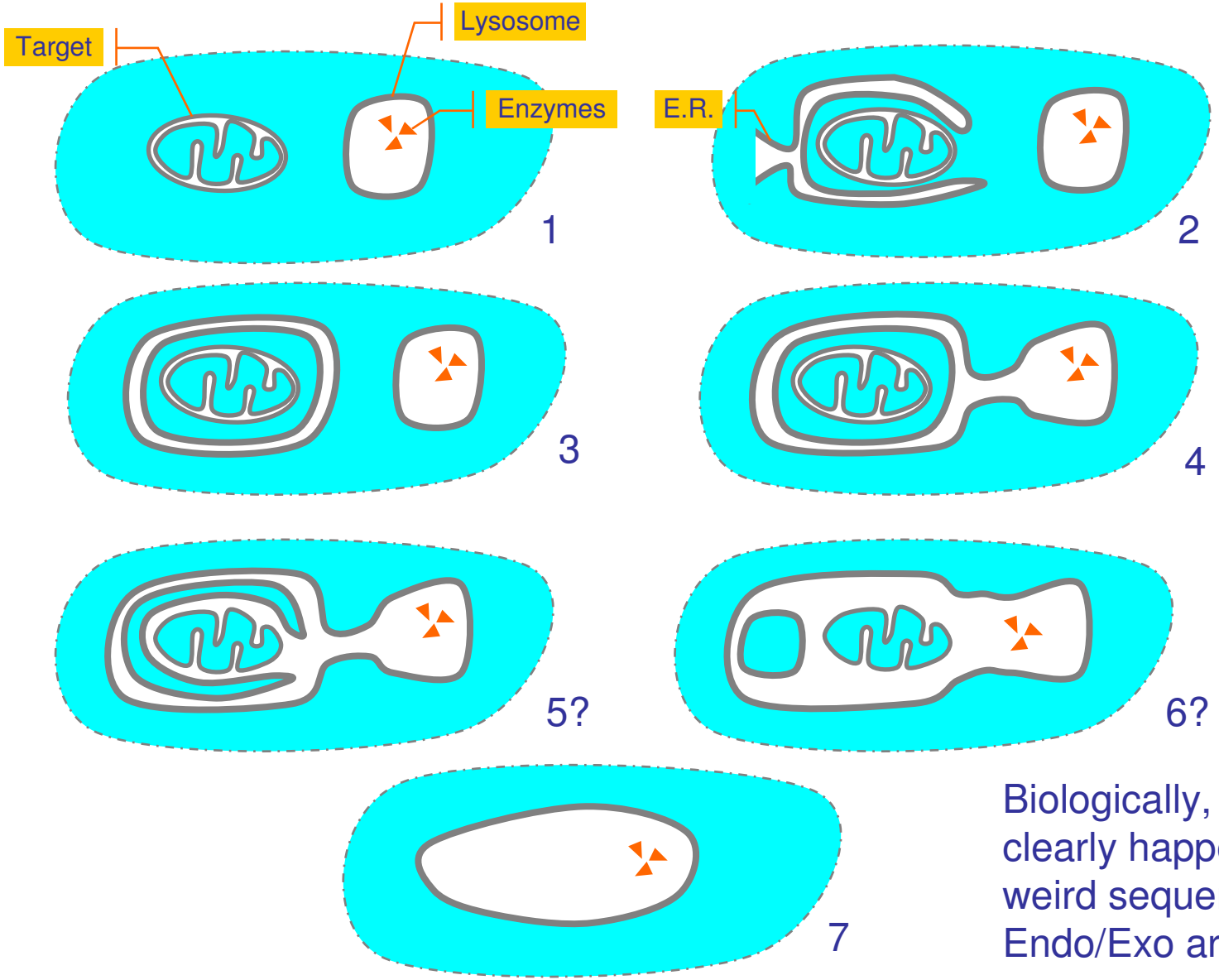
Ex: Clean Eating

(why Endo/Exo is "healthier" than Mito/Mate)



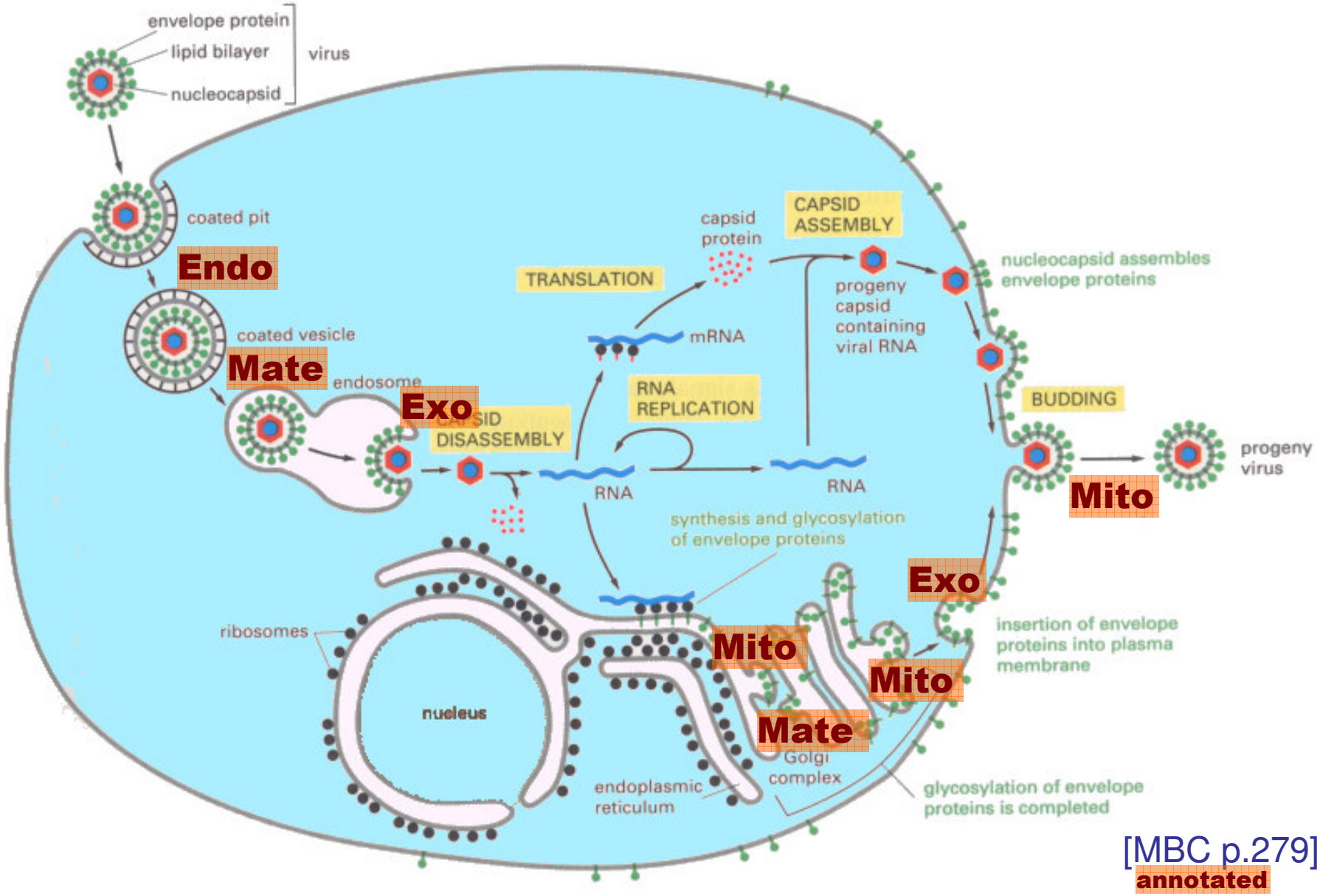
(Real) Ex: Autophagic Process

Lysosome and target don't just merge.



Biologically, Mito/Mate clearly happens. However, weird sequences of Endo/Exo are also common.

(Real) Ex: Viral Reproduction



[MBC p.279]
annotated

Bitonal Algebra

eXosol $X ::= \diamond \mid X \circ X \mid \mathrm{cyto}(Y)$

cYtosol $Y ::= \blacklozenge \mid Y \bullet Y \mid \mathrm{exo}(X)$

We look at this algebra as a preliminary abstraction of process calculi one may devise. Algebraic symmetries will soon be broken, but are still inspiring.

Axioms

$\diamond$ $\circ$ is a comm. monoid

$\blacklozenge$ $\bullet$ is a comm. monoid

F/F: $\diamond = \mathrm{cyto}(\blacklozenge)$ $\quad \blacklozenge = \mathrm{exo}(\diamond)$

E/E: $X \circ \mathrm{cyto}(Y) = \mathrm{cyto}(\mathrm{exo}(X) \bullet Y)$ $\quad \mathrm{exo}(X) \bullet Y = \mathrm{exo}(X \circ \mathrm{cyto}(Y))$

Facts

(without using commutativity)

M/M:

$$\mathrm{cyto}(Y)\ \mathrm{cyto}(Y') = \mathrm{cyto}(\mathrm{exo}(\mathrm{cyto}(Y))\ Y') = \mathrm{cyto}(\mathrm{exo}(\diamond\ \mathrm{cyto}(Y))\ Y')$$
$$= \mathrm{cyto}(\mathrm{exo}(\diamond)\ Y\ Y') = \diamond\ \mathrm{cyto}(Y\ Y') = \mathrm{cyto}(Y\ Y')$$

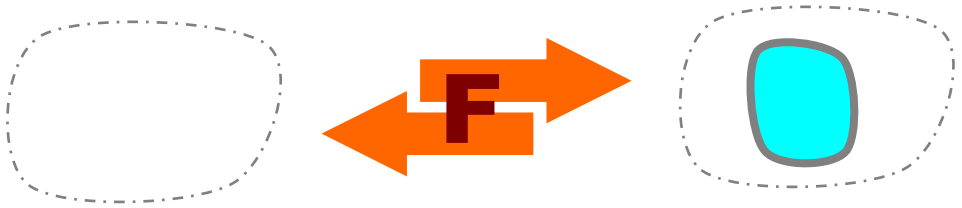
$\mathrm{exo}(X)\ \mathrm{exo}(X') = \mathrm{exo}(X\ X')$ symmetrically

P/P:

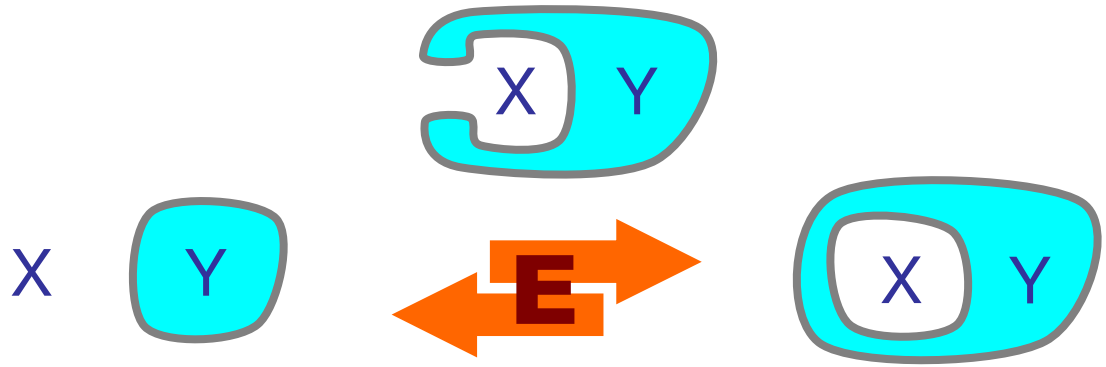
$$X = X \diamond = X\ \mathrm{cyto}(\blacklozenge)$$
$$= \mathrm{cyto}(\mathrm{exo}(X)\ \blacklozenge) = \mathrm{cyto}(\mathrm{exo}(X))$$

$Y = \mathrm{exo}(\mathrm{cyto}(Y))$ symmetrically

Axioms Illustrated



$$\diamond = (\blacklozenge)$$



$$X \circ (Y) = ((X) \bullet Y)$$

Bitonal Algebra v2

eXosol $X ::= \diamond \mid X \circ X \mid \mathrm{cyto}(Y)$

cYtosol $Y ::= \blacklozenge \mid Y \bullet Y \mid \mathrm{exo}(X)$

cyto brackets

exo brackets

Axioms

$\diamond$ $\circ$ is a comm. monoid

$\blacklozenge$ $\bullet$ is a comm. monoid

M/M: $\mathrm{cyto}(Y \bullet Y') = \mathrm{cyto}(Y') \circ \mathrm{cyto}(Y)$ $\quad \mathrm{exo}(X \circ X') = \mathrm{exo}(X) \bullet \mathrm{exo}(X')$

P/P: $\mathrm{cyto}(\mathrm{exo}(X)) = X$ $\quad \mathrm{exo}(\mathrm{cyto}(Y)) = Y$

Facts

E/E:

$X \circ \mathrm{cyto}(Y) = \mathrm{cyto}(\mathrm{exo}(X)) \circ \mathrm{cyto}(Y) = \mathrm{cyto}(\mathrm{exo}(X) \bullet Y)$

$\mathrm{exo}(X) \bullet Y = \mathrm{exo}(X \circ \mathrm{cyto}(Y))$ symmetrically

F/F:

$\mathrm{cyto}(\blacklozenge) = \diamond \circ \mathrm{cyto}(\blacklozenge) = \mathrm{cyto}(\mathrm{exo}(\diamond) \bullet \blacklozenge) = \mathrm{cyto}(\mathrm{exo}(\diamond)) = \diamond$

$\mathrm{exo}(\diamond) = \blacklozenge$ symmetrically

Ex: Viral Infection

⦇capsid⦈∘⦇⦇endosome⦈•cytosol⦈ $=_{Endo}$

⦇⦇⦇capsid⦈⦈•⦇endosome⦈•cytosol⦈ $=_{Mate}$

⦇⦇⦇capsid⦈∘endosome⦈•cytosol⦈ $=_{Exo}$

⦇⦇endosome⦈•capsid•cytosol⦈

Equivalent to a single Mate step, but that's not what "really happens". To explain what "really happens" a bit better, we need to move to a lower level of abstraction.

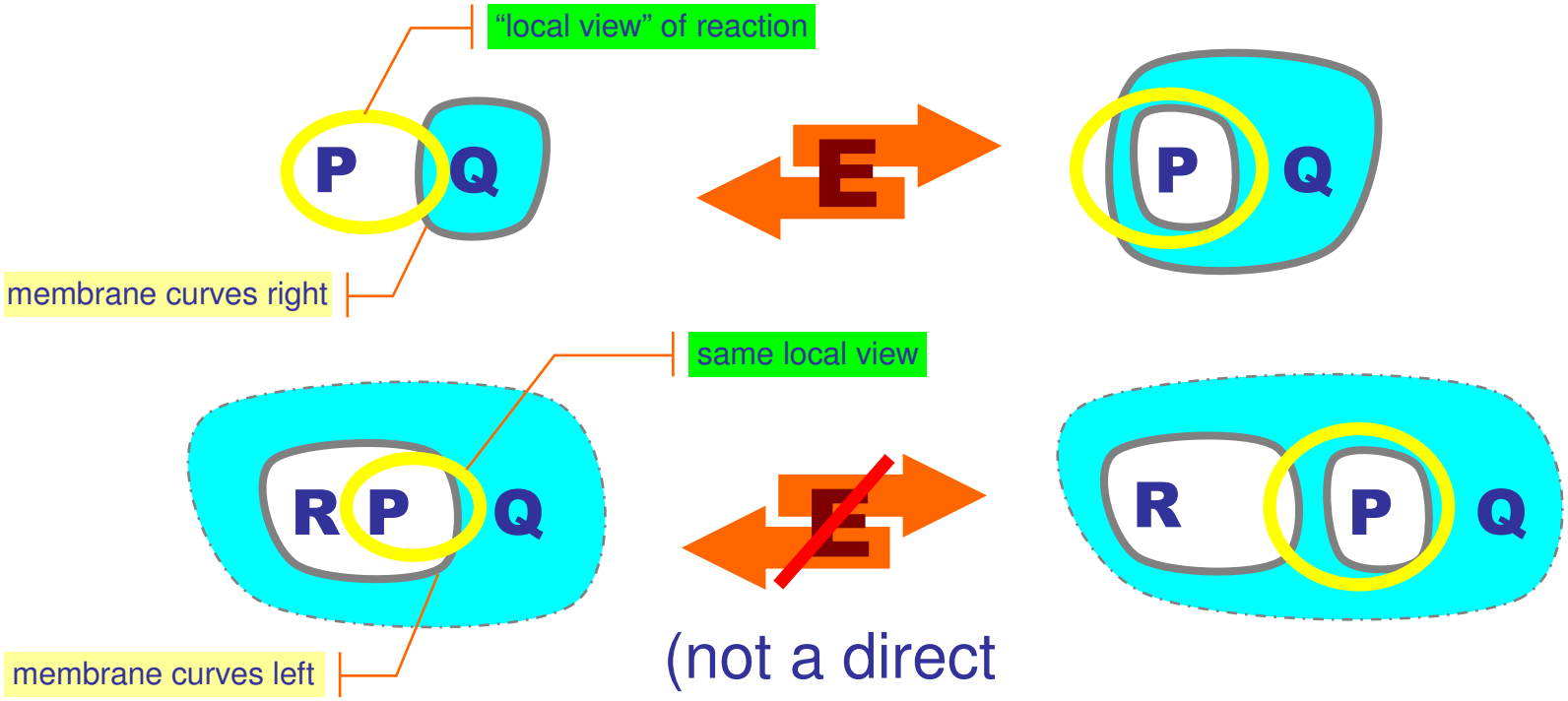
Back to Postulates

Locality Postulate

Interactions should be local to small membrane patches.

E.g., independent of global membrane properties such as overall curvature.

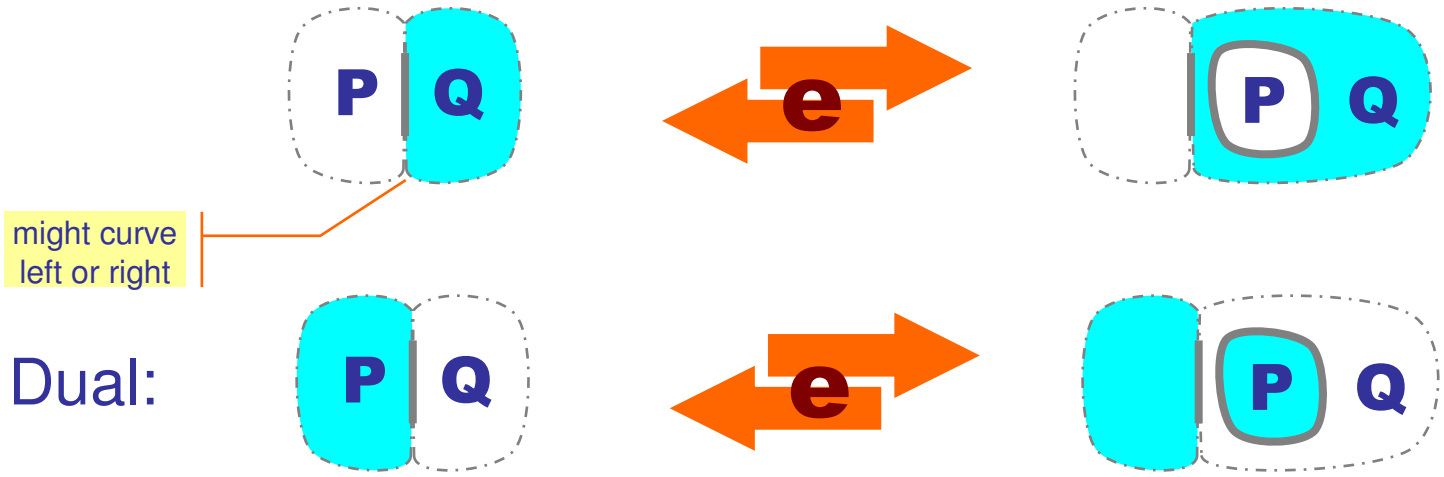
Endo/Exo Violates Locality



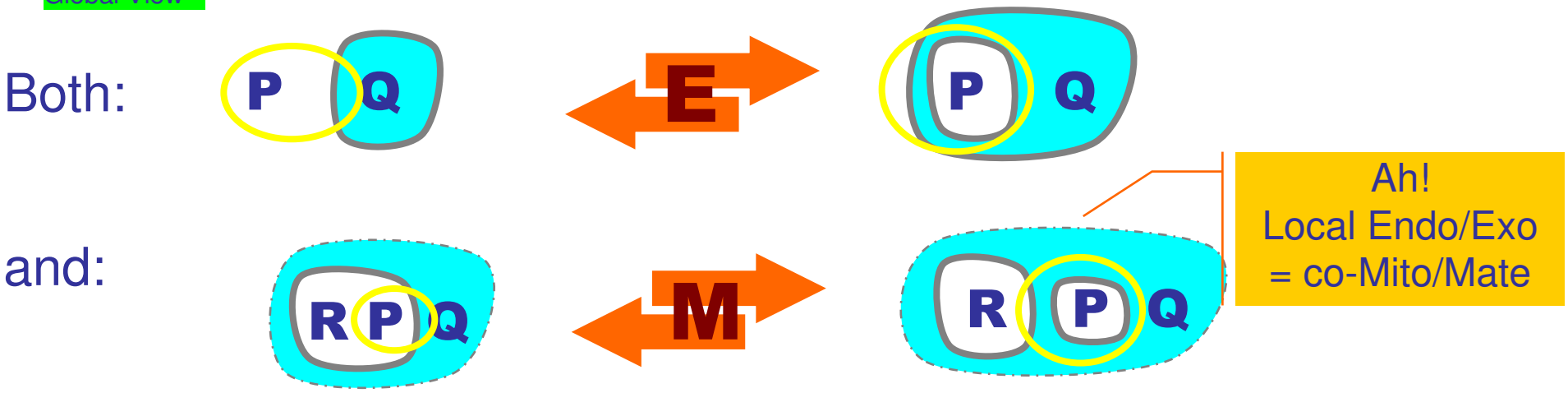
(not a direct instance of endo/exo)

Oops…

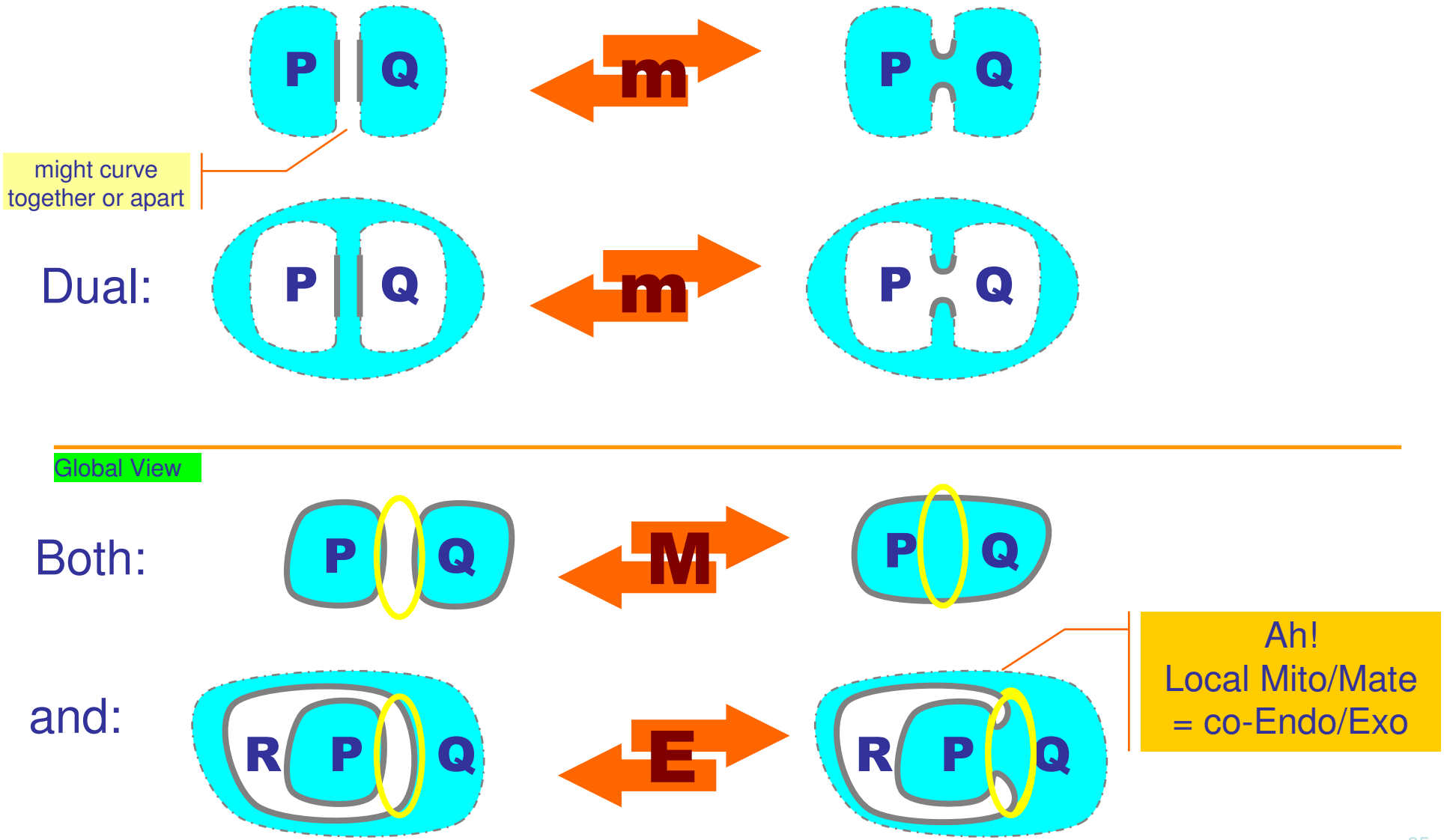
✓ Local-view Endo/Exo Reaction



Global View



✓ Local-view Mito/Mate Reaction



Locality is Not Violated

- Hence, even though Endo/Exo and Mito/Mate strictly violate locality, locality is indirectly preserved in a bigger system that includes them both and their duals.
- Problem: how to formally represent the local-view reactions?

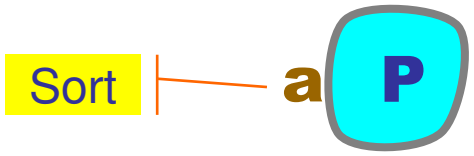
Assessment

- High-level: Algebras
  - Abstraction level still too high; we want to talk about “different sorts” of membranes.
  - We need to be a bit more deterministic.
- Mid-level: Graph Rewriting
  - Abstractly talk about the “sort” of a membrane, and how it changes into other abstract sorts.
- Lower-Level: Calculi
  - Model individual membrane proteins.

Different Kinds of Membranes

Sorted Membranes

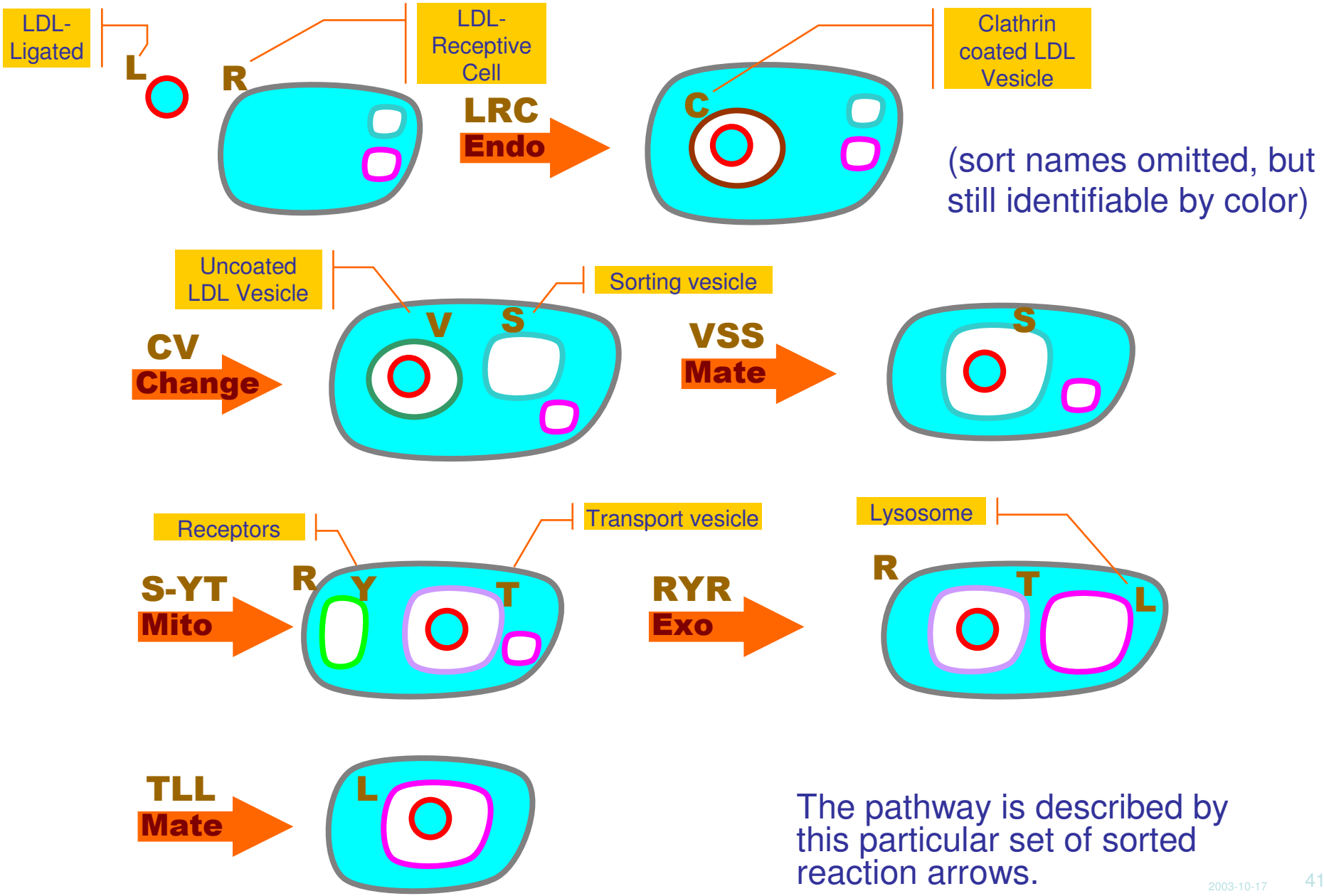
- Different kinds of membranes.
  - Lipid bilayer is universal. All membranes can in principle merge, but the lipid compositions vary.
  - The set of proteins bound to a membrane confer unique characteristics to it and its contents.
- Each membrane is uniform.
  - Membrane proteins diffuse rapidly through the surface of a membrane; they are not localized (unless held together).
- Hence: sorts of membranes.
  - A single name will characterize the collection of features of a membrane; its *sort*.
  - Each sort is meant to be “implemented” by lower level mechanisms.



Sorted Membrane Rewrites

- Rewriting systems
  - We can describe sorted membrane reactions as labeled rewrites (such as labeled versions of endo/exo).
  - E.g. as a special case of Milner's BiGraphs, where the "sort" is the "control". This is possible because each node in a bigraph has a single control. (However, extensions to multi-patch membranes may not fit easily in the BiGraph framework.)

Receptor-Mediated Degradation Pathway



Part III

Brane Calculi

What makes Endo happen?

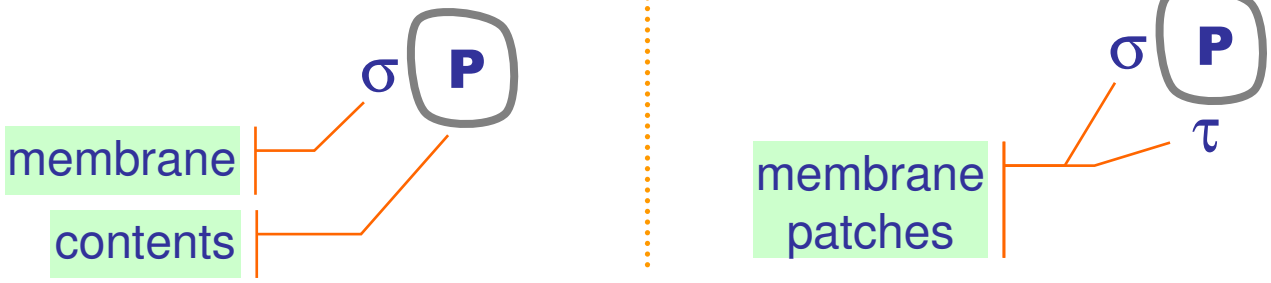
- Moving down a level, to explain “why” certain reactions like endo/exo happen: they do not happen magically.
- Describe membranes as composed of independently active “patches” or membrane proteins (not characterized by a single sort).
- Can be formalized pretty much as action/coaction interactions in process calculi.
- But with actions “on” the membranes, not “inside” them!

Brane Calculi

systems	$P,Q ::= \diamond \mid P \circ Q \mid !P \mid \sigma(\!|P|\!)$	nests of membranes
branes	$\sigma,\tau,\alpha,\beta ::= 0 \mid \sigma\mid\tau \mid !\sigma \mid a.\sigma$	combinations of actions
actions	$a ::= 1 \mid \ldots$	(a great variety of possibilities)

1D fluids (σ) inside a 2D fluid (P)

$\sigma(\!|P|\!)$ $\qquad$ $\sigma\mid\tau(\!|P|\!)$ $\qquad$ $a.\sigma\mid\tau = (a.\sigma)\mid\tau$



N.B. Restriction (νn) could be added to both systems and branes. It usually originate in branes, but may extrude to whole systems.

Structural Congruences

$P \circ Q \equiv Q \circ P$

$P \circ (Q \circ R) \equiv (P \circ Q) \circ R$

$P \circ \diamond \equiv P$

$!P \equiv P \circ !P$ etc.

$\sigma | \tau \equiv \tau | \sigma$

$\sigma | (\tau | \rho) \equiv (\sigma | \tau) | \rho$

$\sigma | 0 \equiv \sigma$

$!\sigma \equiv \sigma | !\sigma$ etc.

$0(\!|\diamond|\!) \equiv \diamond$

$1.\sigma \equiv \sigma$

$P \equiv Q \Rightarrow P \circ R \equiv Q \circ R$

$P \equiv Q \Rightarrow !P \equiv !Q$

$P \equiv Q \wedge \sigma \equiv \tau \Rightarrow \sigma(\!|P|\!) \equiv \tau(\!|Q|\!)$

$\sigma \equiv \tau \Rightarrow \sigma | \rho \equiv \tau | \rho$

$\sigma \equiv \tau \Rightarrow !\sigma \equiv !\tau$

$\sigma \equiv \tau \Rightarrow a.\sigma \equiv a.\tau$

$P \equiv P' \wedge P' \rightarrow Q' \wedge Q' \equiv Q \Rightarrow P \rightarrow Q$

Bitonal Mobility Actions

actions

$a ::= \ldots \mid \circlearrowright_n \mid \circlearrowright^{\perp}_n(\sigma) \mid \circlearrowleft_n \mid \circlearrowleft^{\perp}_n \mid \circledcirc(\sigma)$

phago $\circlearrowright$, exo $\circlearrowleft$, pino $\circledcirc$

,

Old "spontaneous" endo splits into phagocytosis (phago, often still pronounced endo) and pinocytosis (pino).

Bitonal Mobility Actions

actions

$a ::= \ldots \mid \circlearrowleft_n \mid \circlearrowleft^{\perp}_n(\sigma) \mid \circlearrowright_n \mid \circlearrowright^{\perp}_n \mid \circledcirc(\sigma)$

phago $\circlearrowleft$, exo $\circlearrowright$, pino $\circledcirc$

,

Old "spontaneous" endo splits into phagocytosis (phago, often still pronounced endo) and pinocytosis (pino).

Bitonal Mobility Actions

actions

a ::= … | $\circlearrowright_n$ | $\circlearrowright^{\perp}_n(\sigma)$ | $\circlearrowleft_n$ | $\circlearrowleft^{\perp}_n$ | $\circledcirc(\sigma)$

phago $\circlearrowright$, exo $\circlearrowleft$, pino $\circledcirc$

Old “spontaneous” endo splits into phagocytosis (phago, often still pronounced endo) and pinocytosis (pino).

Phago $\nu_n.\alpha|\sigma(\!|P|\!) \circ \nu^{\perp}_n(\rho).\beta|\tau(\!|Q|\!) \rightarrow \beta|\tau(\!|\rho(\!|\alpha|\sigma(\!|P|\!)|\!)\circ Q|\!)$

Exo $\nu^{\perp}_n.\beta|\tau(\!|\nu_n.\alpha|\sigma(\!|P|\!)\circ Q|\!) \rightarrow P \circ \alpha|\sigma|\beta|\tau(\!|Q|\!)$

Pino $\circledcirc(\rho).\alpha|\sigma(\!|P|\!) \rightarrow \alpha|\sigma(\!|\rho(\!|\diamond|\!)\circ P|\!)$

N.B.: the parity of nesting of P and Q is preserved;
this makes the reactions preserve bitonality.

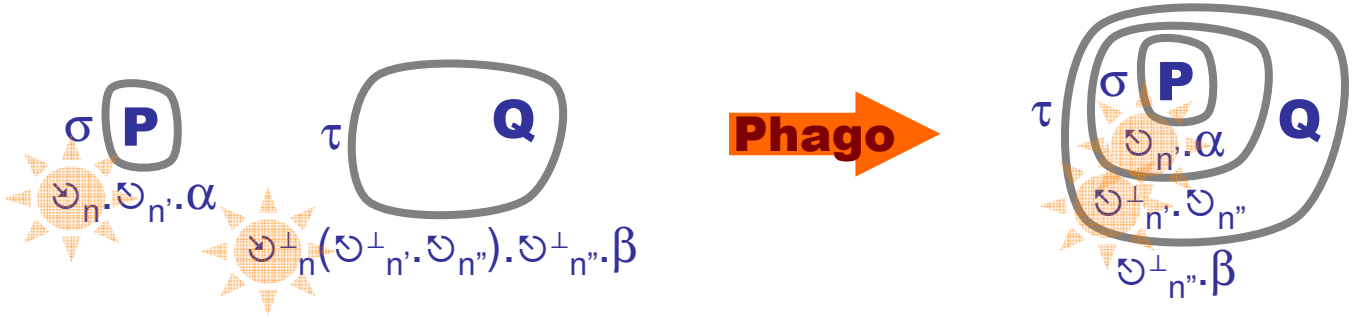
N.B.: in Phago (and Pino), one could perhaps require ρ to be,
conservatively, a piece of τ, by a non-linear rewrite:

CPhago $\nu_n.\alpha|\sigma(\!|P|\!) \circ \nu^{\perp}_n(\rho).\beta|\rho|\tau(\!|Q|\!) \rightarrow \beta|\tau(\!|\rho(\!|\alpha|\sigma(\!|P|\!)|\!)\circ Q|\!)$

Abbreviations: Mate

Mate $\text{mate}_n.\alpha = \overline{n}.\overline{n'}.\alpha$

$\text{mate}^{\perp}_n.\beta = \overline{n}^{\perp}(\overline{n'}^{\perp}.\overline{n''}).\overline{n''}^{\perp}.\beta$

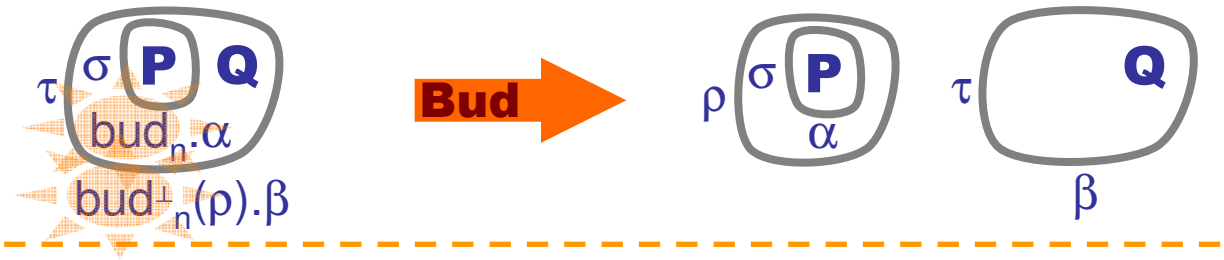


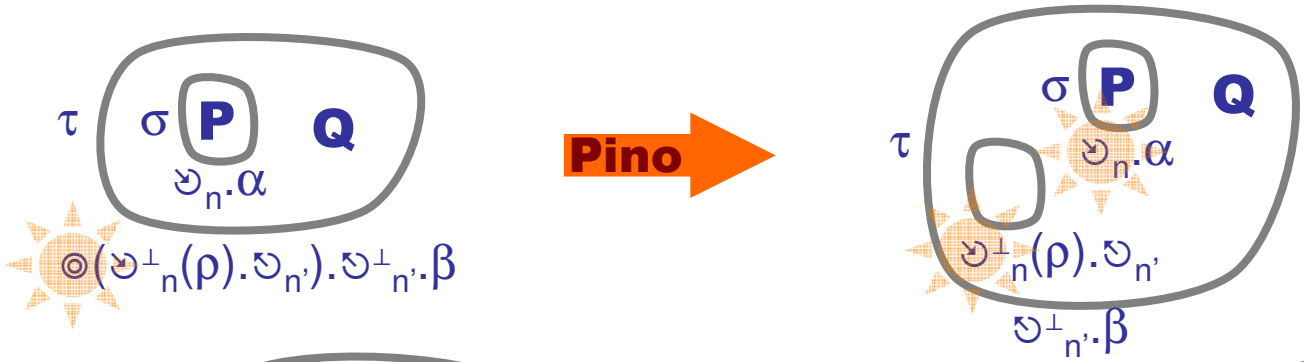
Abbreviations: Bud

Bud $bud_n.\alpha = \circlearrowright_n.\alpha$

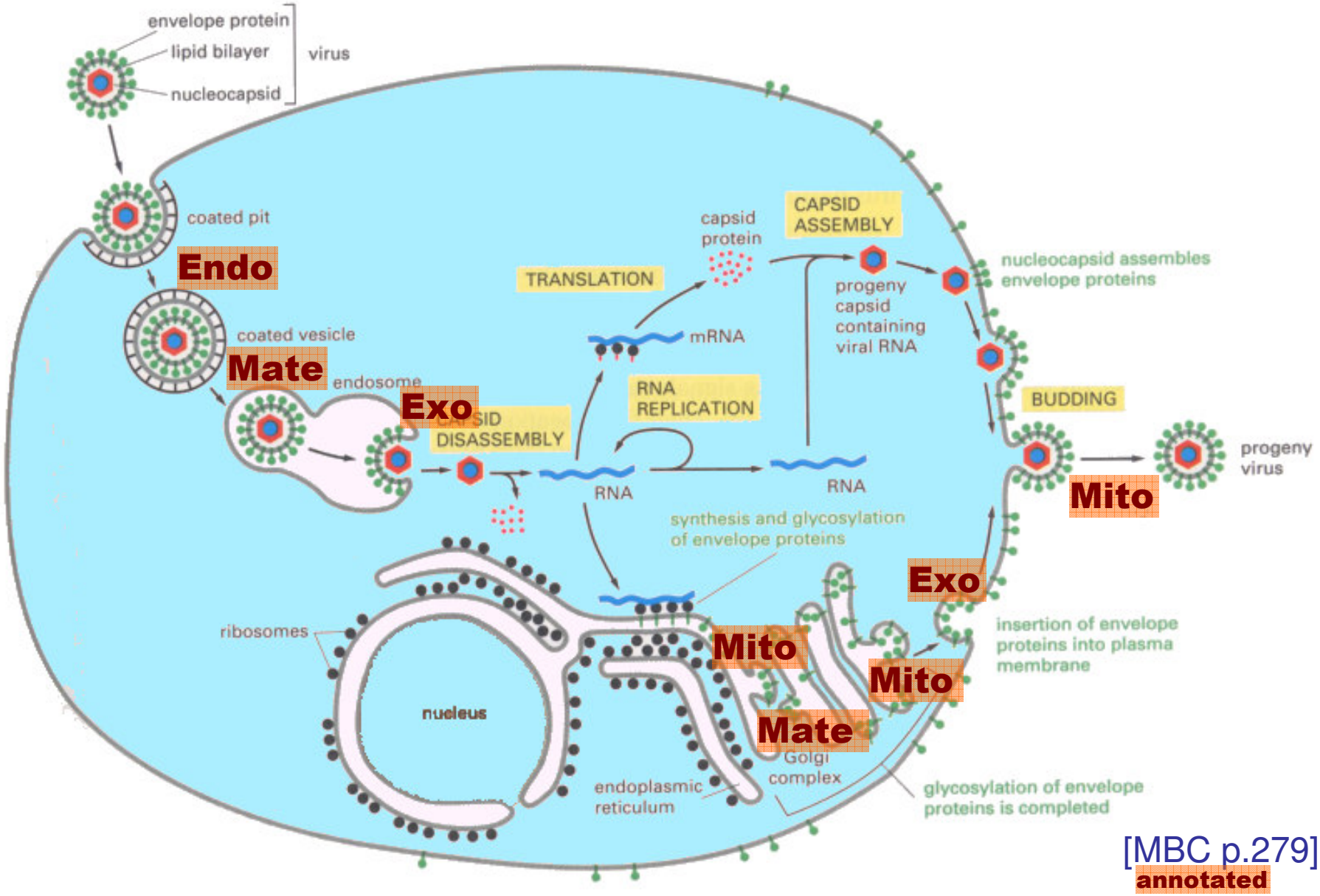
$bud^{\perp}_n(\rho).\beta = \circledcirc(\circlearrowright^{\perp}_n(\rho).\circlearrowright_{n'}).\circlearrowright^{\perp}_{n'}.\beta$

A budding version of old "spontaneous" mito, to avoid arbitrary splits. Follows the pattern of inverse-mate.



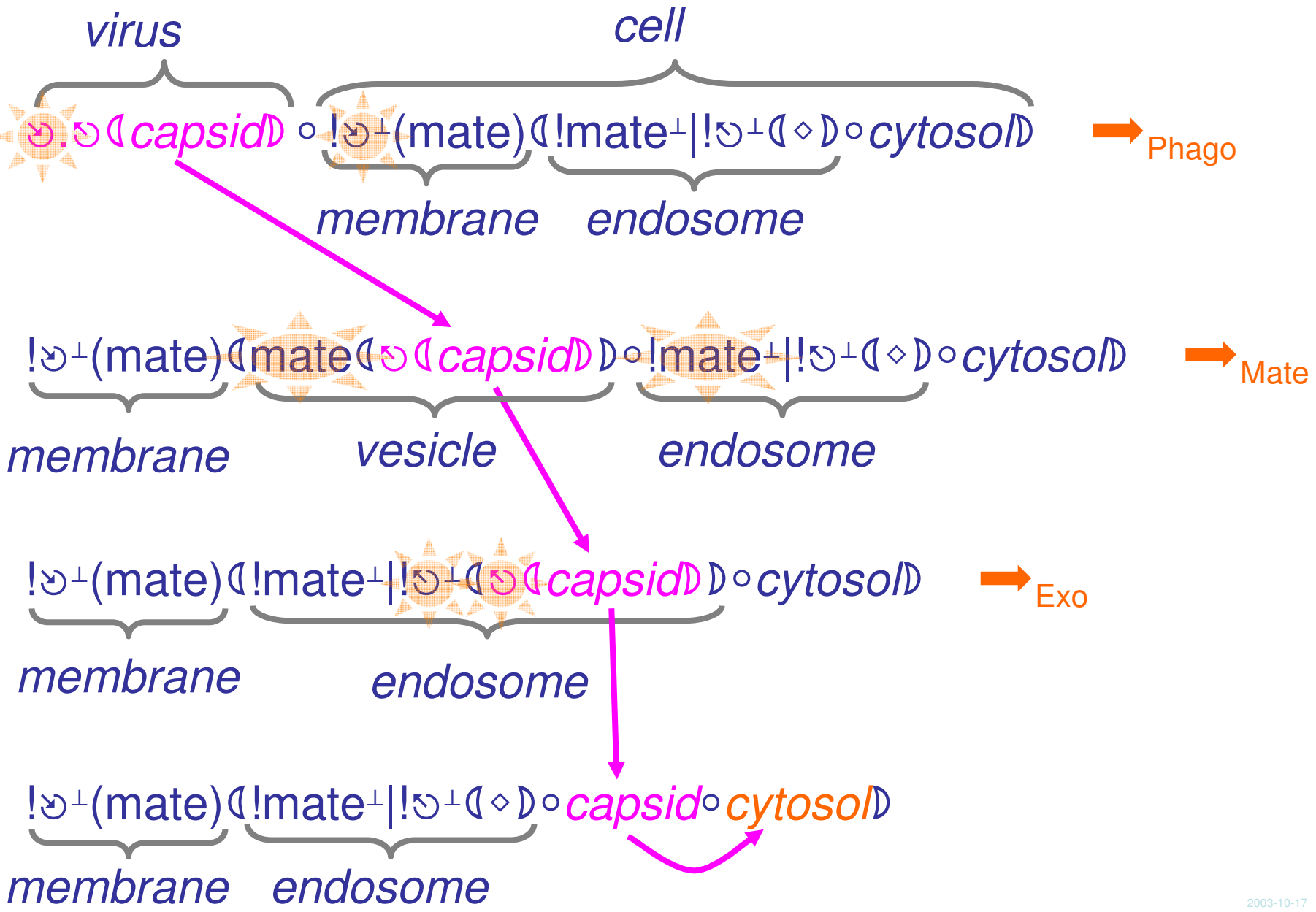


(Real) Ex: Viral Reproduction



[MBC p.279]
annotated

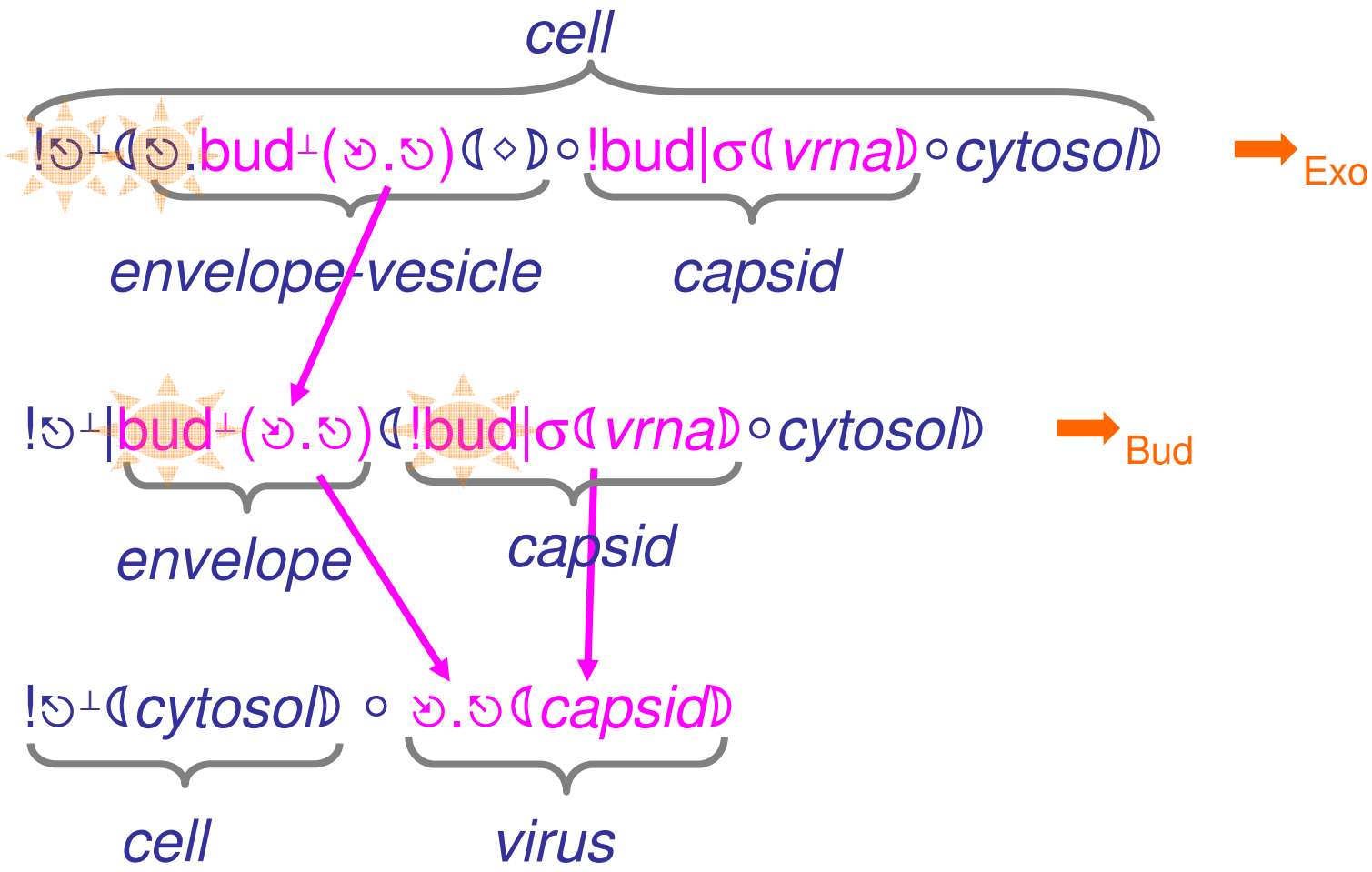
Ex: Viral Infection



Ex: Viral Progeny

capsid ∘ *cytosol* ➡➡ !*envelope-vesicle* ∘ !*capsid* ∘ *cytosol*

by available cellular machinery



Ex: LDL Degradation Pathway

Compartments
Membranes

LigatedLdl = LdlLigand◖LDL◗
Cell = CellBrane◖!Lysosome ∘ !SortingVesicle◗
Lysosome = LysoBrane ◖LysoBody◗
SortingVesicle = SortingBrane◖⋄◗

LdlLigand = $\searrow_{ldlReceptor}$.$bud_{xferVesicle}$

CellBrane = !$\searrow^{\perp}_{ldlReceptor}$(VesicleBrane) | !$\circlearrowleft^{\perp}_{recycle}$

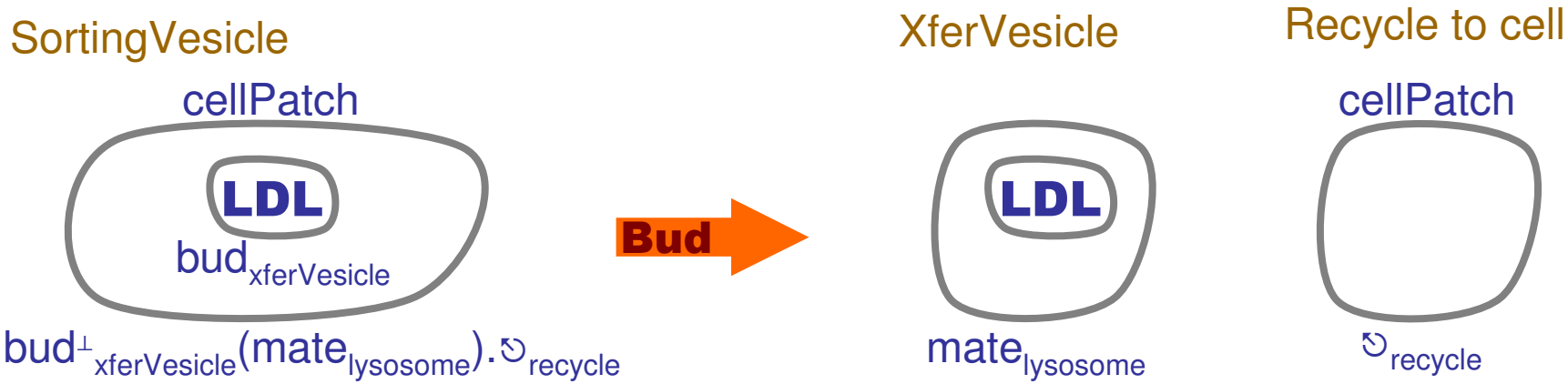
VesicleBrane = $mate_{sortingVesicle}$ | cellPatch(1)

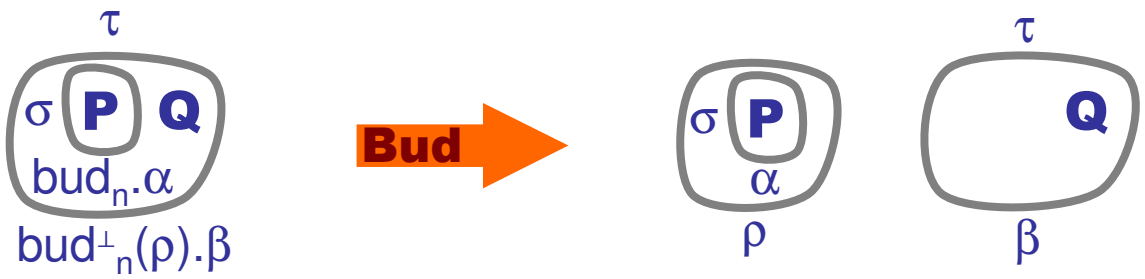
SortingBrane = $mate^{\perp}_{sortingVesicle}$. $bud^{\perp}_{xferVesicle}$(XferBrane). $\circlearrowleft_{recycle}$

XferBrane = $mate_{lysosome}$
LysoBrane = !$mate^{\perp}_{lysosome}$

(1)whatever gets dragged by phago from the cell membrane, e.g. more LDL receptors.

... the critical Bud step





Ex: LDL Degradation Pathway in BioAmbients

Compartments
Processes

LigatedLdl = [LdlLigand | LDL]
Cell = [CellBrane ∘ !Lysosome ∘ !SortingVesicle]
Vesicle(n) = [VesicleBrane(n)]
SortingVesicle = [SortingBrane | XferVesicle]
XferVesicle = [XferBrane]
Lysosome = [LysoBrane | LysoBody]

$LdlLigand = s2s_{ldlBind}^{\perp}(n).in_n.in_n.merge_{xferVesicle}$
$LdlReceptor = (\nu n)\ s2s_{ldlBind}(n).in^{\perp}_n \mid Vesicle(n)$
$CellBrane = !LdlReceptor \mid !pop^{\perp}_{recycle}{}^{(1)}$
$VesicleBrane(n) = in^{\perp}_n.\ merge_{sortingVesicle} \mid cellPatch^{(2)}$
$SortingBrane = merge^{\perp}_{sortingVesicle}.out^{\perp}_{bud}.pop_{recycle}$
$XferBrane = merge^{\perp}_{xferVesicle}.out_{bud}.merge_{lysosome}$
$LysoBrane = !merge^{\perp}_{lysosome}$

(1)pop is out + merge. (2)cellPatch is cell membrane to be recycled

Encoding Brane Calculi?

$$\sigma(\!|P|\!)^\dagger \triangleq s[\sigma^\dagger \mid P^\dagger] \quad ?$$

This encoding confuses membrane with contents, so that the exo encoding is problematic:

Exo $\circlearrowleft^\perp_n.\beta|\tau(\!|\circlearrowleft_n.\alpha|\sigma(\!|P|\!)\circ Q|\!) \rightarrow P \circ \alpha|\sigma|\beta|\tau(\!|Q|\!)$

That is, find $\circlearrowleft^\dagger$ encodings such that:

$s[\circlearrowleft^{\perp\dagger}_n.\beta \mid s[\circlearrowleft^\dagger_n.\alpha \mid \sigma \mid P] \mid \tau \mid Q] \rightarrow P \mid s[\alpha \mid \sigma \mid \beta \mid \tau \mid Q]$

but the split $\sigma \mid P$ is *arbitrary* here: some reactions could not be reflected back to legal brane calculus reactions ($P^\dagger \rightarrow Q \Rightarrow \exists R.\ P \rightarrow R \wedge Q \rightarrow^* R^\dagger$), and it would be in any case difficult to define $\circlearrowleft^\dagger$ so that it splits σ from P.

One cannot easily represent the Exo reaction in (Bio)Ambients, nor can one easily add it as a new primitive!

For exo at least, we need to explicitly identify the membrane.

either $\sigma(\!|P|\!)^\dagger \triangleq s[m[\sigma^\dagger] \mid P^\dagger]$

or $\sigma(\!|P|\!)^\dagger \triangleq s[\sigma^\dagger \mid c[P^\dagger]]$

The second option should be chosen to avoid crossing 4 brackets in s2s reactions, so:

Exo $\circlearrowleft^{\perp}_{n}.\beta|\tau(\!|\circlearrowleft_{n}.\alpha|\sigma(\!|P|\!)\circ Q|\!) \rightarrow P \circ \alpha|\sigma|\beta|\tau(\!|Q|\!)$

$s[\circlearrowleft^{\perp}_{n}.\beta \mid \tau \mid s[\circlearrowleft_{n}.\alpha \mid \sigma \mid c[P]] \mid c[Q]] \rightarrow P \mid s[\alpha \mid \sigma \mid \beta \mid \tau \mid c[Q]]$

But this emulation interferes badly with concurrent Phago's (emulated by at least two "in" steps because of the double bracketing): neither emulations is atomic.

One cannot easily emulate atomic Phago/Exo in (Bio)Ambients.

Conversely, in (Bio)Ambients one can use an action to create a whole new filled-in membrane:

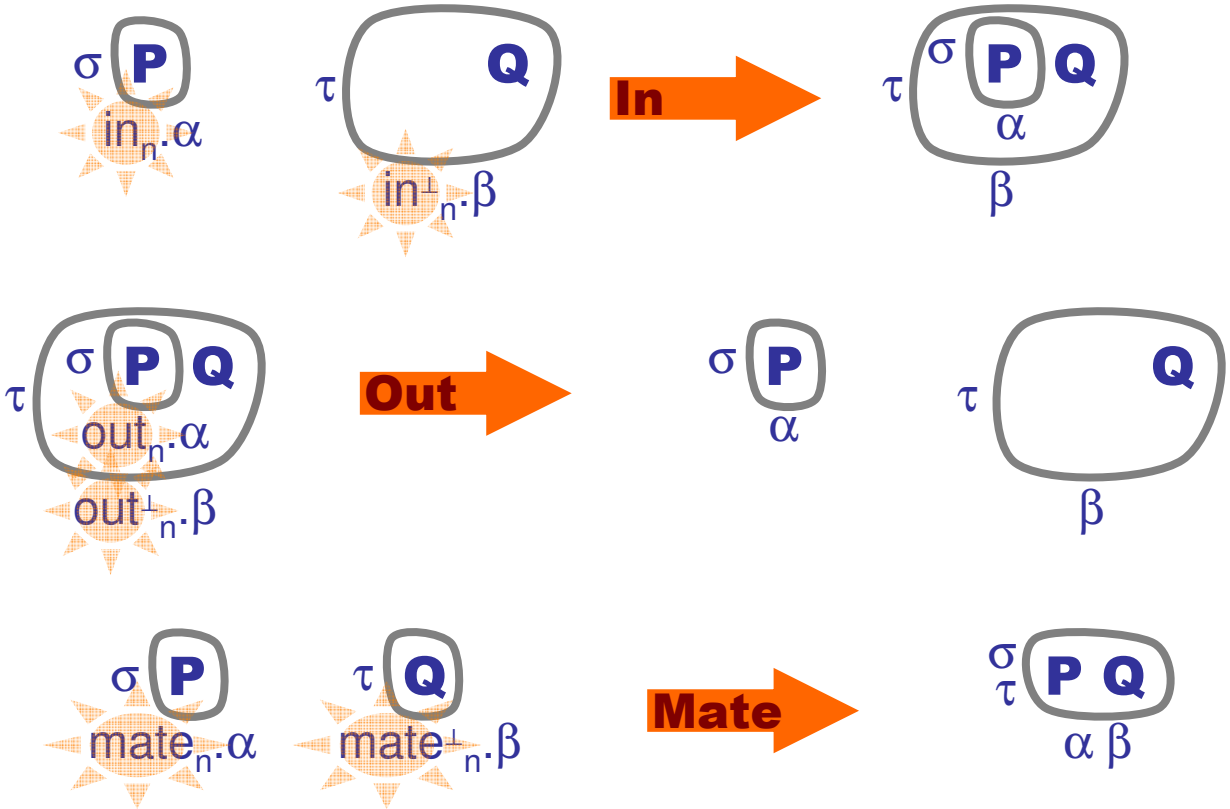
$$a.s[\sigma \mid P] \;=\; a.(\sigma(\!|P|\!))$$

this is not allowed, nor easily representable, in brane calculi.

This is a power that real membranes do not seem to have.

BioAmbients-like Mobility Actions

actions $a ::= \ldots \mid in_n \mid in^{\perp}_n \mid out_n \mid out^{\perp}_n \mid mate_n \mid mate^{\perp}_n$



...

In $in_n.\alpha|\sigma(P) \circ in^{\perp}_n.\beta|\tau(Q) \rightarrow \beta|\tau(\alpha|\sigma(P)\circ Q)$

Out $out^{\perp}_n.\beta|\tau(out_n.\alpha|\sigma(P)\circ Q) \rightarrow \alpha|\sigma(P) \circ \beta|\tau(Q)$

Mate $mate_n.\alpha|\sigma(P) \circ mate^{\perp}_n.\beta|\tau(Q) \rightarrow \alpha|\sigma|\beta|\tau(P\circ Q)$

N.B.: out + mate gives a “melt” primitive that is a good membrane-preserving approximation of “open”:

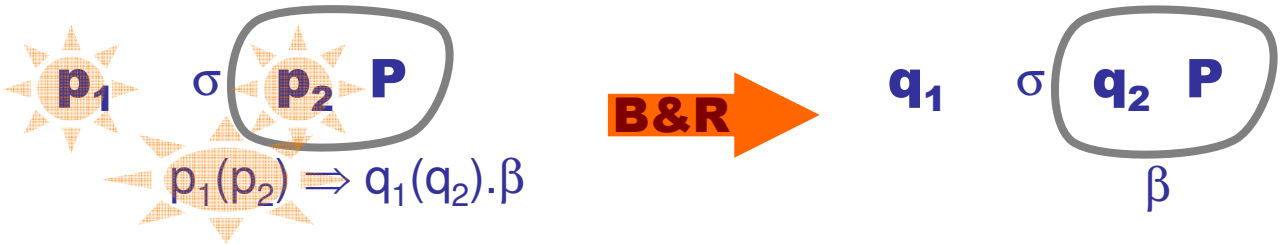
$melt^{\perp}_n.\beta|\tau(melt_n.\alpha|\sigma(P)\circ Q) \rightarrow \alpha|\sigma|\beta|\tau(P \circ Q)$

Molecular Actions

systems $P,Q ::= \ldots \mid m$ $m \in M$ molecules

$p,q ::= m_1 \circ \ldots \circ m_k$ molecule multisets

actions $a ::= \ldots \mid p_1(p_2) \Rightarrow q_1(q_2)$ bind&release



B&R $p_1 \circ p_1(p_2) \Rightarrow q_1(q_2).\alpha|\sigma(p_2 \circ P)$ ➡ $q_1 \circ \alpha|\sigma(q_2 \circ P)$

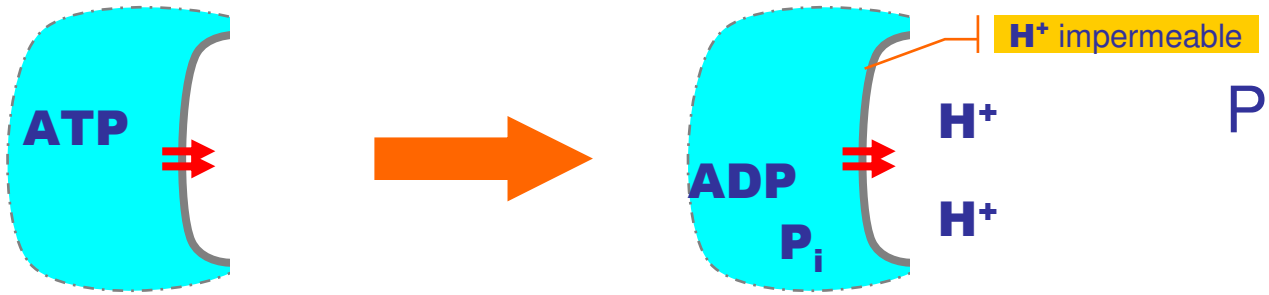
(multiset rewriting, inside and outside membranes)

Special cases: "$\diamond(\diamond)$" is omitted

$m(\diamond)\Rightarrow$	bind out	$\Rightarrow m(\diamond)$	release out
$\diamond(m)\Rightarrow$	bind in	$\Rightarrow \diamond(m)$	release in

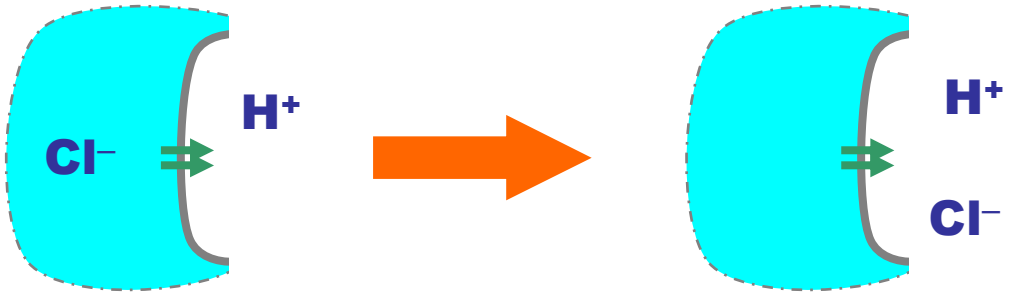
Ex: A Specialized Membrane

E.g. plant vacuole (white).

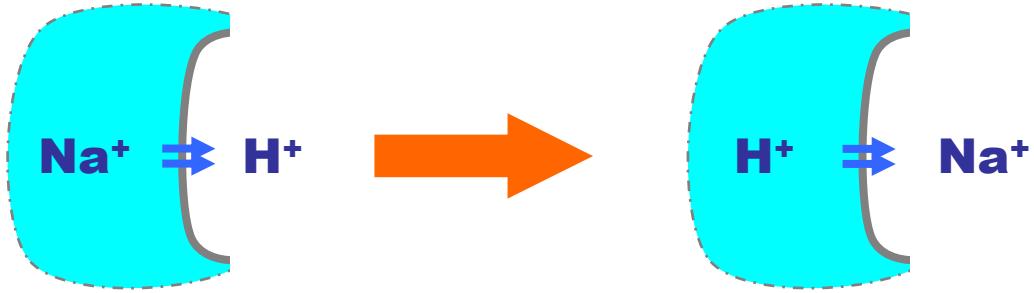


Proton Pump

ATP charges up the vacuole with H^+. Several other pumps work off that charge.



Ion Channel



Proton Antiporter

A membrane of sort “PlantVacuole” has all those things on it.

$$\mathrm{ProtonPump} = !\ \mathrm{ATP}(\diamond) \Rightarrow \mathrm{ADP} \circ \mathrm{P_i}(\mathrm{H^+} \circ \mathrm{H^+})$$

$$\mathrm{IonChannel} = !\ \mathrm{Cl^-}(\mathrm{H^+}) \Rightarrow \diamond(\mathrm{H^+} \circ \mathrm{Cl^-})$$

$$\mathrm{ProtonAntiporter} = !\ \mathrm{Na^+}(\mathrm{H^+}) \Rightarrow \mathrm{H^+}(\mathrm{Na^+})$$

$$\mathrm{PlantVacuole} = \mathrm{ProtonPump} \mid \mathrm{IonChannel} \mid \mathrm{ProtonAntiporter}\ (\diamond)$$

Diffusion (CCS-like channels)

actions $a ::= \ldots \mid df_n(m) \mid df^{\perp}_n(m)$ diffusion (within membrane)

BioAmbients-like Channels

actions $a ::= \ldots \mid s2s_n(m) \mid s2s^{\perp}_n(m)$ sibling to sibling

$\mid p2c_n(m) \mid p2c^{\perp}_n(m)$ parent to child

$\mid c2p_n(m) \mid c2p^{\perp}_n(m)$ child to parent

$s2s_n(m).\alpha|\sigma\langle P\rangle \circ s2s^{\perp}_n(p).\beta|\tau\langle Q\rangle$

$\rightarrow \alpha|\sigma\langle P\rangle \circ \beta\{p \leftarrow m\}|\tau\langle Q\rangle$

$p2c_n(m).\alpha|\sigma\langle p2c^{\perp}_n(p).\beta|\tau\langle Q\rangle \circ P\rangle$

$\rightarrow \alpha|\sigma\langle \beta\{p \leftarrow m\}|\tau\langle Q\rangle \circ P\rangle$

$c2p^{\perp}_n(p).\beta|\tau\langle c2p_n(m).\alpha|\sigma\langle Q\rangle \circ P\rangle$

$\rightarrow \beta\{p \leftarrow m\}|\tau\langle \alpha|\sigma\langle Q\rangle \circ P\rangle$

Implementabilty?

- An implementable “instruction set” could consist of:
  - Bitonal mobility operators, including bud/mate (possibly restricting the ρ in $\circlearrowleft^{\perp}_{n}(\rho)$ and ⊚(ρ)).
  - Selected bind&release pumps.
  - Selected s2s/p2c/c2p operators.
- N.B. BioAmbients in/out do not seem as likely to be directly implementable.

Bitonal Calculi?

- Oriented actions:

white-pointing receptor on oriented membrane may interact with P on white but not with Q on blue

That is, $\sigma(\!|P|\!)$ should have different reactions than $\sigma(\!|P|\!)$.

Bitonal calculi TBD.

Conclusions

- What's different about "bio"-calculi?
  - Orientability and bitonality invariants inspire new, and possibly more bio-realistic, operators.
  - Low-dimensional fluids inside high-dimensional fluids: two commutative monoids.
  - Computing *on* the membrane, not *inside* of it.

References

[MCB] Molecular Cell Biology, Freeman.

[MBC] Molecular Biology of the Cell, Garland.